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Advanced Small Modular Reactor Economics Model Development

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ACRONYMS

DOE	US Department of Energy
DOE-NE	US Department of Energy Office of Nuclear Energy
EIA	Energy Information Administration
MWeh	megawatt (electrical)
NEI	Nuclear Energy Institute
ORNL	Oak Ridge National Laboratory
SMR	small modular reactor

ABSTRACT

The US Department of Energy Office of Nuclear Energy's Advanced Small Modular Reactor (SMR) research and development activities focus on four key areas:

- Developing assessment methods for evaluating advanced SMR technologies and characteristics;
- Developing and testing of materials, fuels and fabrication techniques;
- Resolving key regulatory issues identified by US Nuclear Regulatory Commission and industry; and
- Developing advanced instrumentation and controls and human-machine interfaces.

This report focuses on development of assessment methods to evaluate advanced SMR technologies and characteristics. Specifically, this report describes the expansion and application of the economic modeling effort at Oak Ridge National Laboratory. Analysis of the current modeling methods shows that one of the primary concerns for the modeling effort is the handling of uncertainty in cost estimates. Monte Carlo–based methods are commonly used to handle uncertainty, especially when implemented by a stand-alone script within a program such as Python or MATLAB. However, a script-based model requires each potential user to have access to a compiler and an executable capable of handling the script.

Making the model accessible to multiple independent analysts is best accomplished by implementing the model in a common computing tool such as Microsoft Excel. Excel is readily available and accessible to most system analysts, but it is not designed for straightforward implementation of a Monte Carlo–based method. Using a Monte Carlo algorithm requires in-spreadsheet scripting and statistical analyses or the use of add-ons such as Crystal Ball.

An alternative method uses propagation of error calculations in the existing Excel-based system to estimate system cost uncertainty. This method has the advantage of using Microsoft Excel as is, but it requires the use of simplifying assumptions. These assumptions do not necessarily bring into question the analytical results. In fact, the analysis shows that the propagation of error method introduces essentially negligible error, especially when compared to the uncertainty associated with some of the estimates themselves.

The results of these uncertainty analyses generally quantify and identify the sources of uncertainty in the overall cost estimation. The obvious generalization—that capital cost uncertainty is the main driver—can be shown to be an accurate generalization for the current state of reactor cost analysis. However, the detailed analysis on a component-by-component basis helps to demonstrate which components would benefit most from research and development to decrease the uncertainty, as well as which components would benefit from research and development to decrease the absolute cost.

1. METHODS FOR COST ESTIMATION

Cost estimates are typically used to compare alternatives or to identify cost drivers within specific options. For example, calculating the levelized cost of electricity provides an estimate of the cost to generate electricity using a notional reactor and fuel cycle arrangement. Depending on the detail in the cost estimate, the front end and back end fuel cycle costs for two different fuel cycle options can be compared and contrasted.

However, comparing and contrasting single values for components can lead to overstated confidence in the analytical results. Reporting a difference in component costs without examining the uncertainty associated with each cost can lead to the belief that one alternative is "better" than the other, even for small differences. This is especially magnified for alternatives with small differentials relative to their values. For example, consider two alternatives at $100/megawatt-electric-hour (MWeh) and $101/MWeh. How confidently can an analyst claim the former option is better than the latter?

1.1 USE OF POINT ESTIMATES

A high-level or first-order approximation of a cost estimate divides the system into components with their own separate costs. A notional reactor and fuel cycle combination can typically be separated into reactor capital costs, fuel costs, operation and maintenance costs, and decommissioning costs. Some of these costs are tied to commodity costs, such as uranium or enrichment services, while others are estimated based on industrial and academic reports, such as capital costs.

The most straightforward cost estimate method for nuclear power uses reported point estimates for aggregated components, such as those reported by the Nuclear Energy Institute (NEI) [Ref. 1]. They report estimates of $16.5/MWeh for operations and maintenance, and $7.5/MWeh for fuel, a combined $24/MWeh.

Another resource for point estimates is the Energy Information Administration (EIA) [Ref. 2]. They report similar combined fuel and operations costs of $23.6/MWeh. Unlike the NEI, the EIA estimates a capital recovery cost of $71.4/MWeh. Assuming these are the primary costs of interest, the total cost of nuclear is then $95.0/MWeh.

1.2 THE NEED TO HANDLE ESTIMATE RANGES

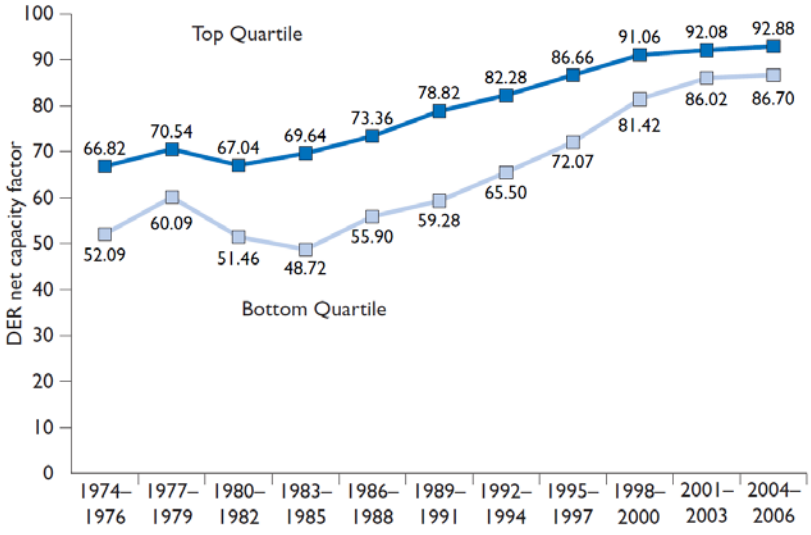
The problem with using point estimates is that they imply a fixed, uniform cost; this is an unreasonable assumption, especially with respect to commodities. Figure 1 shows uranium oxide prices over the last five years [Ref. 3].

The figure shows fluctuation in the price of the material. While the expected or average price of the commodity could be used as a point estimate, a more accurate model allows the price of the commodity to be a random variable.

The recognition that the costs of commodities and services vary with time is almost trivial. However, other components of the total cost also vary. Cost estimation efforts within the US Department of Energy Office of Nuclear Energy (DOE-NE) have attempted to capture these cost ranges for reference in advanced studies, such as in Shropshire, et al [Ref. 4]. Furthermore, other operational parameters can and should be treated as random variables.

For example, Fig. 2 shows the historical distributions of capacity factors in the US nuclear fleet [Ref. 5]. The use of quartiles in Fig. 2 demonstrates that there is a range of potential capacity factors within a single set of years, and the overall capacity factor varies as a function of time.

Fig. 1. Five year uranium oxide prices.



All reactors, top and bottom quartiles. An indication of the improvement of the fleet as a whole is the narrowing gap between the top and bottom quartiles, from more than 20 percentage points to fewer than seven. The chart shows reactors still in service today; if closed reactors were included, the only amounts that would differ by more than two percentage points are the bottom quartiles in 1989–1991 (57.08) and 1995–1997 (68.18), the latter reflecting the reduced output of the last reactors to close.

Fig. 2. US nuclear fleet capacity factor distribution.

2. METHODS FOR UNCERTAINTY ANALYSIS

Having acknowledged the need to handle several parameters as random variables, the next step is to establish the method. Most engineers are familiar with Monte Carlo methods, which are widely used in economic analyses. However, the goal of this modeling effort is to maintain simplicity and relative transparency in the methodology. Therefore, Monte Carlo methods would not be the best choice.

2.1 THE BASIC ASSUMPTION

The primary assumption for the non-Monte Carlo modeling method is that all variables are independently distributed. The primary reason is that the correlation between any two variables is not well understood and would itself introduce more uncertainty into the calculation. This is shown clearly in the next section.

2.2 THE MATHEMATICS OF THE PROPAGATION OF ERROR

Assume a value is the function of *N* variables. That is, $y = f(x_1, x_2, x_3, x_4, \dots, x_N)$. Also assume that each variable x_n has a mean[1] $\bar{x}_n$ and variance ${\sigma^2}_n$. For this analysis, the variance is the measure of uncertainty. Then the variance of y is found by:

$$ {\sigma^2}_y = \sum_{n=1}^{N} \left(\frac{\partial f}{\partial x_n}\right)^2 {\sigma^2}_n + 2 \sum_{n=1}^{N} \sum_{m>n} \frac{\partial f}{\partial x_m} \frac{\partial f}{\partial x_n} \rho_{mn} \sigma_m \sigma_n $$

Using the basic assumption above, the correlation coefficient ρ_{mn} is assumed to be 0 for all $m \neq n$. This simplifies the equation to:

$$ {\sigma^2}_y = \sum_{n=1}^{N} \left(\frac{\partial f}{\partial x_n}\right)^2 {\sigma^2}_n $$

Further, the mean $\bar{y}$ of the function is simply $\bar{y} = f(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4, \dots, \bar{x}_N)$. Therefore, given a multivariable function and knowledge of the basic statistics of its constituent variables, the basic statistics of that function can be calculated. While this does not necessarily give information about the type of distribution (Gaussian, uniform, log-normal, etc.) it does provide more information than a simple point estimate.

For example, by tracking the different components separately in the analysis, the magnitude of the contribution from each component can be compared to find not only the overall cost driver, but also the overall uncertainty driver. Components with high relative uncertainty may contribute very little to the overall uncertainty; conversely, components with low relative uncertainty may make large contributions to the overall uncertainty. This is a result of the partial derivative for the function for each component of cost.

The calculation of system uncertainty is straightforward. However, the complication in the nuclear fuel cycle calculation lies in the derivative terms. The total levelized cost contains many linear terms, e.g., the capital cost. However, interest rates and uranium enrichment parameters are highly nonlinear. This complicates the calculation of the derivative.

[1]The "bar" convention ($\bar{x}$) is used to denote the mean or expected value, defined by $\int_{-\infty}^{\infty} xp(x)dx$.

2.3 THE MATHEMATICS OF THE DERIVATIVE

Given a function $y = f(x)$, the derivative at a point $x = a$ is the slope of the function at that point. The slope at the point is given by:

$$m = \frac{\Delta f(a)}{\Delta a} = \frac{f(a + \Delta x) - f(a)}{(a + \Delta x) - a} = \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

Alternatively, the slope can also be given by:

$$m = \frac{\Delta f(a)}{\Delta a} = \frac{f(a) - f(a - \Delta x)}{a - (a - \Delta x)} = \frac{f(a) - f(a - \Delta x)}{\Delta x}$$

In either case, the derivative is defined as the slope as $\Delta x \to 0$.

From a numerical perspective, the derivative of the function at $x = a$ can also be found by:

$$m = \frac{\Delta f(a)}{\Delta a} = \frac{f(a + \Delta x) - f(a - \Delta x)}{(a + \Delta x) - (a - \Delta x)} = \frac{f(a + \Delta x) - f(a - \Delta x)}{2\Delta x}$$

Using this combined approach helps to account for the fact that a numerical analysis requires $\Delta x > 0$. An example of this approach is, given $y = kx,^2$ from basic calculus the derivative is $y' = 2kx$. Using the first method gives:

$$m = \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{kx^2 + 2kx\Delta x + k\Delta x^2 - kx^2}{\Delta x} = 2kx + k\Delta x$$

Using the second method gives:

$$m = \frac{f(x) - f(x - \Delta x)}{\Delta x} = \frac{kx^2 - kx^2 + 2kx\Delta x - k\Delta x^2}{\Delta x} = 2kx - k\Delta x$$

Using the third method gives:

$$m = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} = \frac{kx^2 + 2kx\Delta x + k\Delta x^2 - kx^2 + 2kx\Delta x - k\Delta x^2}{2\Delta x} = 2kx$$

The combined method does not have a Δx term in it; this increases numerical stability in the solution, even with a nonlinear function in x. There are still issues with the use of the numerical method with exponential and logarithmic functions. For example, $y = e^x$:

$$m = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} = \frac{e^{(x+\Delta x)} - e^{(x-\Delta x)}}{2\Delta x} = \frac{e^x\left(e^{\Delta x} - e^{-\Delta x}\right)}{2\Delta x}$$

Using $e^{\Delta x} = \cosh[\Delta x] + \sinh[\Delta x]$, $\cosh[-\Delta x] = \cosh[\Delta x]$, and $\sinh[-\Delta x] = -\sinh[\Delta x]$:

$$m = \frac{e^x\left(e^{\Delta x} - e^{-\Delta x}\right)}{2\Delta x} = \frac{e^x\left((\cosh[\Delta x] + \sinh[\Delta x]) - (\cosh[-\Delta x] + \sinh[-\Delta x])\right)}{2\Delta x} = e^x\frac{\sinh[\Delta x]}{\Delta x}$$

.

The $\frac{\sinh[\Delta x]}{\Delta x}$ factor represents the error introduced by using numerical methods; as $\Delta x \to 0$, the slope approaches the correct value of $e.^{x}$ Fig. 3 shows that for small Δx the introduced error is on the order of single-digit percent; this is acceptable considering the relative uncertainty in the estimation of any parameter's statistics is likely in the tens of percent.

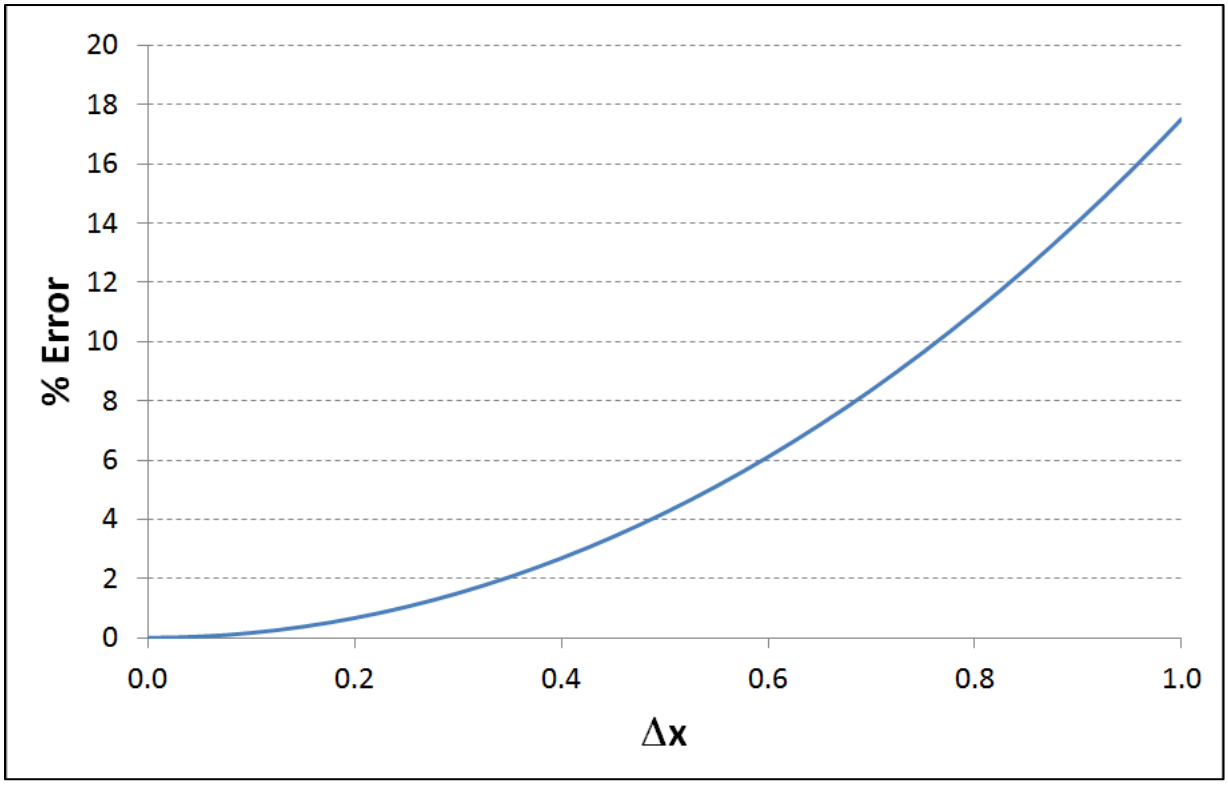


Fig. 3. Error introduced to exponential derivative by linear method.

Values associated with exponential functions in levelized cost calculations are products of interest rates and payback periods. With annual interest rates on the order of a maximum of 15% (0.15) and payback periods on the order of a maximum of 60 years, the product is on the order of 0.15*60=9. Using a 10% change in either interest or payback period would result in a Δx of 0.9. This corresponds to an approximate 14% error in the derivative term. However, most interest rates and payback periods analyzed are around 7% and 40 years, so the Δx term with a 10% variation is closer to 0.28, or less than 2% error in the derivative term.

Now use the function $y = \ln x$. By calculus, the derivative of $y = \ln x$ is $y' = \frac{1}{x}$. Using the method above yields:

$$m = \frac{\ln(x + \Delta x) - \ln(x - \Delta x)}{2\Delta x}$$

The values associated with logarithmic functions in levelized cost calculations are enrichment fraction; these are typically on the order of 0.1 (corresponding to 10% enrichment), but likely much less. Also note that logarithmic functions require $x - \Delta x > 0 \to \Delta x < x$. Since the definition of the derivative is the slope at the point as the Δx term approaches 0, the numerical approximation is nearly identical to the

analytical answer. Table 1 compares the numerical and analytical values (to two decimal places) for selected values of x and Δx.

Table 1. Approximations to the derivative of the natural logarithm

	Exact m	Approximate m			
x		Δx = 1%	Δx = 5%	Δx = 10%	Δx = 20%
10^{-4}	10000	10000.33	10008.35	10008.35	10136.63
10^{-3}	1000	1000.03	1000.83	1003.35	1013.66
10^{-2}	100	100.00	100.08	100.34	101.37
10^{-1}	10	10.00	10.01	10.03	10.14
10^{0}	1	1.00	1.00	1.00	1.01
10^{1}	0	0.10	0.10	0.10	0.10

As the table shows, the error introduced by the numerical approximation for small values of x and Δx is negligible. A 10% Δx for x=10^{-4} introduces a 0.34% error; this is well within the accuracy of the parameter estimations.

2.4 THE OVERALL METHOD

The overall method then finds the uncertainty in the cost estimate as calculated from the mean value by varying each independent variable around its mean. Combining the numerical derivative method described above with the propagation of error method (with assumed independence of all variables) yields:

$$\sigma^2{}_y = \sum_{n=1}^{N} \left(\frac{\partial f}{\partial x_n}\right)^2 \sigma^2{}_n \approx \sum_{n=1}^{N} \left(\frac{f(\bar{x}_n + \Delta x_n) - f(\bar{x}_n - \Delta x_n)}{2\Delta x_n}\right)^2 \sigma^2{}_n$$

The use of Δx_n denotes the use of variable-specific numerical variations. This is important in minimizing nonlinear error as shown in Fig. 3.

This method can be extended to values other than mean values. For example, a variable with an asymmetric distribution has a median greater or less than the mean; for this report, that value is denoted $\breve{x}_n$. This value may be used as a nominal or anticipated value rather than the mean value. The uncertainty analysis then becomes:

$$\sigma^2{}_y = \sum_{n=1}^{N} \left(\frac{f(\breve{x}_n + \Delta x_n) - f(\breve{x}_n - \Delta x_n)}{2\Delta x_n}\right)^2 \sigma^2{}_n$$

2.5 AN EXAMPLE OF THE METHOD IN USE

This example is illustrative of how the method is applied in a simplified model.

Assume a function $y = x_1 e^{x_2 x_3} + \sin[x_4]\, {x_5}^2 + \ln(x_6)$. The explicit partial derivatives are:

$$\frac{\partial y}{\partial x_1} = e^{x_2 x_3}$$
$$\frac{\partial y}{\partial x_2} = x_1 x_3 e^{x_2 x_3}$$
$$\frac{\partial y}{\partial x_3} = x_1 x_2 e^{x_2 x_3}$$
$$\frac{\partial y}{\partial x_4} = \cos[x_4]\, {x_5}^2$$
$$\frac{\partial y}{\partial x_5} = 2 \sin[x_4]\, x_5$$
$$\frac{\partial y}{\partial x_6} = \frac{1}{x_6}$$

Table 2 lists example values for the mean for each variable.

Table 2. Example values

Variable	Mean
x_1	1000
x_2	0.1
x_3	50
x_4	0.7
x_5	6
x_6	0.02

Table 3 then compares the explicit partial derivatives at the mean with the approximate partial derivatives at the mean and 10% Δx_n. That is, $\Delta x_1 = 100$, *etc*.

Table 3. Partial derivatives at the mean values for 10% variation

Variable	Explicit	Calculate	Percentage of error
x_1	148.413	148.413	0
x_2	7.421E+06	7.734E+06	4.22
x_3	1.484E+04	1.547E+04	4.25
x_4	27.534	27.512	-0.08
x_5	7.731	7.731	0
x_6	50	50.168	0.34

The data in Table 3 show that even at the upper bounds of expected values for exponentials and variations, the maximum error introduced by the numerical method is less than 5%. Further, since the full implementation of the propagation of error formula sums the uncertainties, the overall error is essentially bounded by the largest combined derivative and standard deviation.

Table 4 shows the complete agreement achieved using a 1% Δx_n.

Table 4. Partial derivatives at the mean values for 1% variation

Variable	Explicit	Calculate	Percentage of error
x_1	148.413	148.413	0
x_2	7.421E+06	7.424E+06	0.04
x_3	1.484E+04	1.485E+04	0.07
x_4	27.534	27.534	0
x_5	7.731	7.731	0
x_6	50	50.002	0

3. CONCLUSION AND CURRENT AND FUTURE WORK

The combination of the propagation of error formula with the numerical method for calculating partial derivatives has been shown to be a valid approach for uncertainty analysis in levelized cost analyses. The method has already been implemented in the current test version of the G4ECONS software in use by the Generation IV International Forum.

The main benefit of this method is the ability to implement it in a stand-alone spreadsheet-based analytical tool. Since this does not require a Monte Carlo-based calculation, this method can be used without additional software. This is beneficial from the perspective of validation and verification, as well as transportability. The main drawback is the complexity in calculating the partial derivatives with respect to a large number of variables.

Future work will include pushing the test version into a production-level version. An application of the method is included in a companion letter report [Ref. 6].

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