Module 11

Quantifying Late Detection in MC&A for Bulk Processes

Learning Objective

- Familiarization with the probability with which the Safeguards systems can detect a diversion of material due to insider activity
- Become familiar with the concept of Statistical Process Control

Material Balance Evaluation from the Insider Perspective

- **Goal is to determine the probability with which the Safeguards systems can detect a diversion of material due to insider activity.**
- **Generally relates to bulk processing.**

  "Bulk Material – Material in any physical form that is not identifiable as a discrete item and therefore must be accounted for by weight, volume, sampling, chemical analysis, or non-destructive analysis"
- **Statistical Process Control is the key underlying concept or discipline**

Overview

- Inventory differences (IDs)
- Process Control – statistical methods and control charts
- Discuss with respect to the hypothetical process
- Discuss the calculation of Limits-of-Error for inventory differences (LEIDs)

Inventory Difference Calculation

The ID is calculated as follows:

ID = BI + TI – TO – EI

Where,

ID = Inventory Difference

BI = Beginning Inventory (prior period physical inventory value)

TI = Transfers In (additions) during the current inventory period

TO = Transfers Out (removals) during the current inventory period

EI = Ending Inventory (current period physical inventory value)

Inventory Differences

- For an <u>item</u> facility, the ID should always be zero unless an item is missing.
- For a <u>bulk</u> facility,
  - The ID is typically different from zero due to measurement uncertainties, unmeasured holdup, and losses.
  - The holdup can be treated by estimating the amount in various pieces of equipment, perhaps based on experiments.
  - Under ideal conditions, the bulk facility IDs should vary about zero. (note: systematic differences may cause this not to be the case).

Simplified ID Example for an Item Facility

Reactor Core:

BI = 60 core fuel assemblies
TI = 20 fresh fuel assemblies
TO = 20 spent fuel assemblies
EI = 60 core fuel assemblies

ID = 60 + 20 - 20 - 60 = 0

Simplified ID example for a bulk facility

Fabrication Plant Process Area:

BI = 203.5 kg U in beginning process inventory
TI = 600.1 kg U in oxide powder feed
TO = 604.8 kg U in pellet product
EI = 195.6 kg U in ending process inventory

ID = 203.5 + 600.1 - 604.8 - 195.6 = 3.2 kg U

Making sense of the ID with respect to the Insider

- What techniques can be used to monitor it?
- How does it relate to the insider analysis in the vulnerability assessment?
- When is it or how much of an ID is significant?

MOST IMPORTANTLY….

- What are we trying to detect?

What is significant? Was 3.2 kg from the example in a previous slide significant?

Process A – Not significant		Process B - Significant	
Inventory	Period ID	Inventory	Period ID
1	-1.5	1	-.15
2	2.3	2	.23
3	-3.1	3	-.31
4	0.7	4	0.07
5	3.2	5	3.2
6	-0.8	6	-.08
7	-1.6	7	-.16
8	-0.6	8	-.06
9	1.1	9	.11
10	2.1	10	.21
11	-3.2	11	-.32
12	1.5	12	.15

What about process C? Significant or not significant?

Process C – Not significant

Inventory	Period ID
1	-1.0
2	2.0
3	-1.5
4	0.7
5	3.2
6	-0.8
7	-1.3
8	-0.6
9	0.7
10	1.5
11	-2.1
12	1.2

Key concepts

- What are we trying to detect?
  - Single abnormally large or statistically significant inventory difference
  - Repetitive losses over time that could indicate insider activity
- How will they manifest themselves?
  - Large loss similar to Process B in the previous slide for the single occurrence.
  - Repetitive losses over time or in laymen’s terms a loss trend or in statistical terms a shift in the mean of the inventory difference over time

Process or Product Monitoring and Control

- Shewart Control Charts
- Cumulative Sum (CUSUM) Control Charts
- Exponentially Weighted Moving Average (EWMA) Control Charts

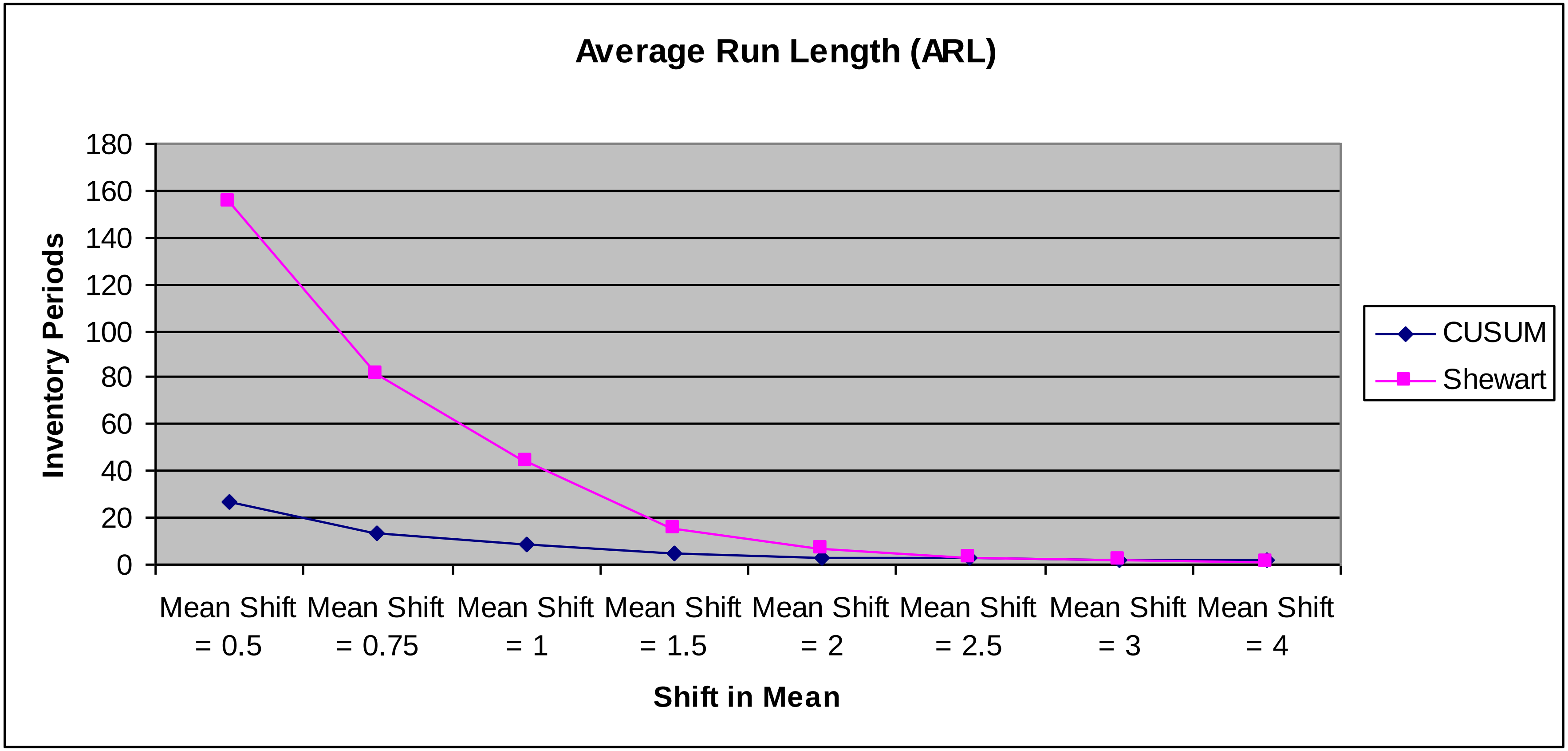
- MOST IMPORTANT PART
  - Average Run Length (ARL)
    - When the process is operating at the target mean and a shift of magnitude δ occurs the number of inventories it takes to detect the shift.
  - Generally CUSUM or EWMA are better at 2 sigma or less while Shewart is better at greater than 2 sigma.

Average Run Length to Detect a 1 Sigma Shift in the Process Average for the Various Techniques

- **Shewart Charts will detect a 1 sigma (standard deviation) shift in the mean in around 42-44 inventories**
- **Both CUSUM and EWMA will detect a 1 sigma shift in about 10 inventories**
- **As the shift in the mean gets smaller all techniques converge on 370 inventories**
- **As the shift in the mean gets larger (e.g., around 2.5 sigma or greater) Shewart is more effective**

Note: Under current DOE regulations an ID > 2 sigma (warning limit) will be investigated regardless.

Average Run Length (ARL)
CUSUM versus Shewart based on shift in process mean



Average Run Length (ARL)
CUSUM versus Shewart based on shift in process mean

Shift in Mean (δ) (k=0.5)	CUSUM (h=4)	Shewart
0.5	26.6	155
0.75	13.3	81
1.0	8.38	44
1.5	4.75	14.97
2	3.34	6.3
2.5	2.62	3.24
3	2.19	2
4	1.71	1.19

Shewart Charts

- **First proposed during the 1920's by Dr. Walter A. Shewart**

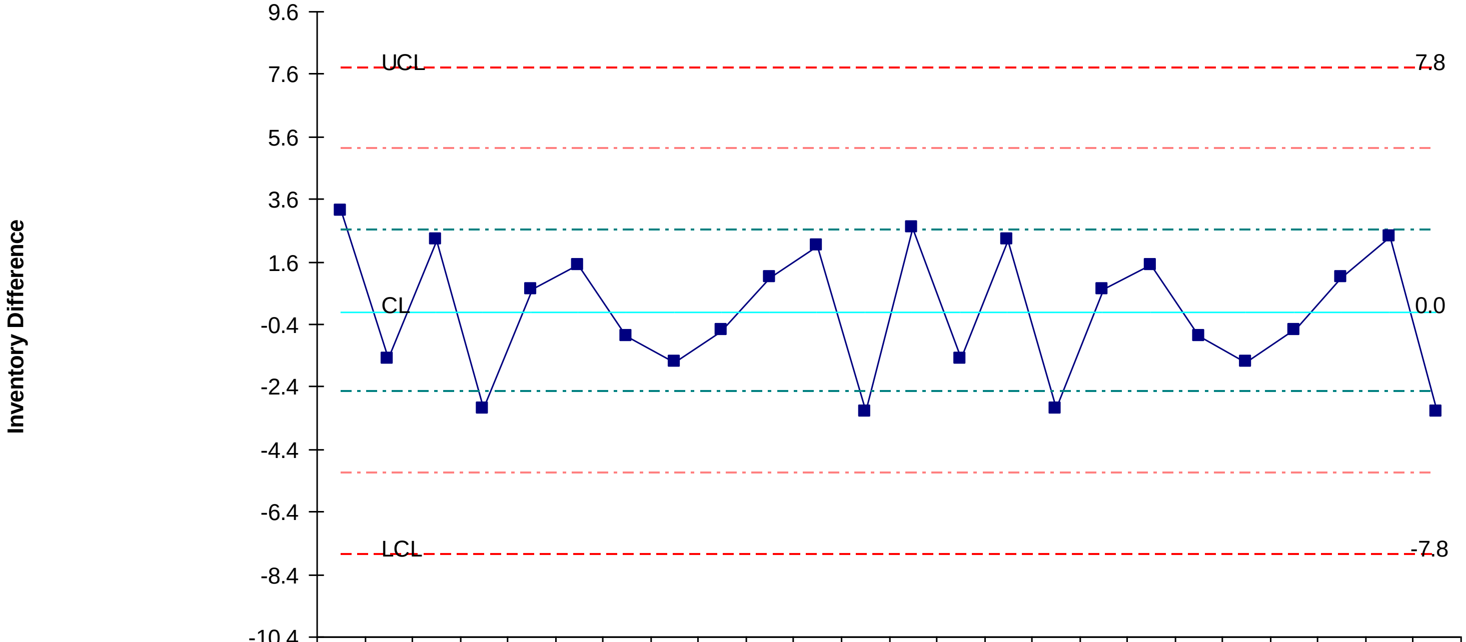
UCL = Process Mean + k (standard deviation)

Center Line = Process Mean or target

LCL = Process Mean – k (standard deviation)

Shewhart Chart for Process A

Inventory Difference X



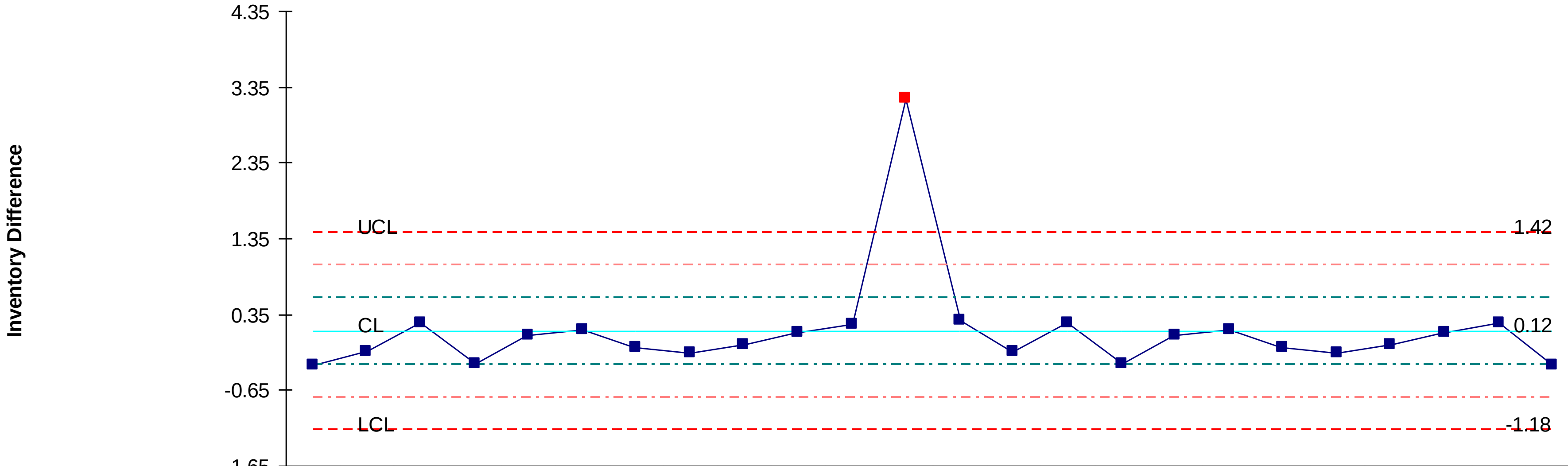
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Inventory Difference	3.2	-1.5	2.3	-3.1	0.7	1.5	-0.8	-1.6	-0.6	1.1	2.1	-3.2	2.7	-1.5	2.3	-3.1	0.7	1.5	-0.8	-1.6	-0.6	1.1	2.4	-3.2
UCL	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8	7.8
+2 Sigma	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2
+1 Sigma	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
Average	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-1 Sigma	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6
-2 Sigma	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2	-5.2
LCL	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8	-7.8

Date/Time/Period

Shewhart Chart – Process B

National Nuclear Security Administration

Inventory Difference X



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Inventory Difference	-0.32	-0.15	0.23	-0.31	0.07	0.15	-0.08	-0.16	-0.06	0.11	0.21	3.20	0.27	-0.15	0.23	-0.31	0.07	0.15	-0.08	-0.16	-0.06	0.11	0.24	-0.32
UCL	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42	1.42
+2 Sigma	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
+1 Sigma	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55
Average	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
-1 Sigma	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31
-2 Sigma	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75
LCL	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18	-1.18

Date/Time/Period

Shewart Charts Advantages/Disadvantages

Advantages

- Very effective at detecting large process shifts

Disadvantages

- Not so good for trends because it only uses information contained in the last data point

Can address some of the disadvantages with respect to trends by applying the Western Electric Company Rules (WECO) for signaling out of control situations.

WECO Rules for evaluating Shewart Charts

- Any point above +3 sigma
- 2 out of the last 3 points above +2 sigma
- 4 out of the last 5 points above +1 sigma
- 8 consecutive points on the positive side of the center line
- 8 consecutive points on the negative side of the center line
- 4 out of the last 5 points below -1 sigma
- 2 out of the last 3 points below -2 sigma
- Any point below -3 sigma

Note: Under current DOE regulations an ID > 2 sigma (warning limit) will always be investigated

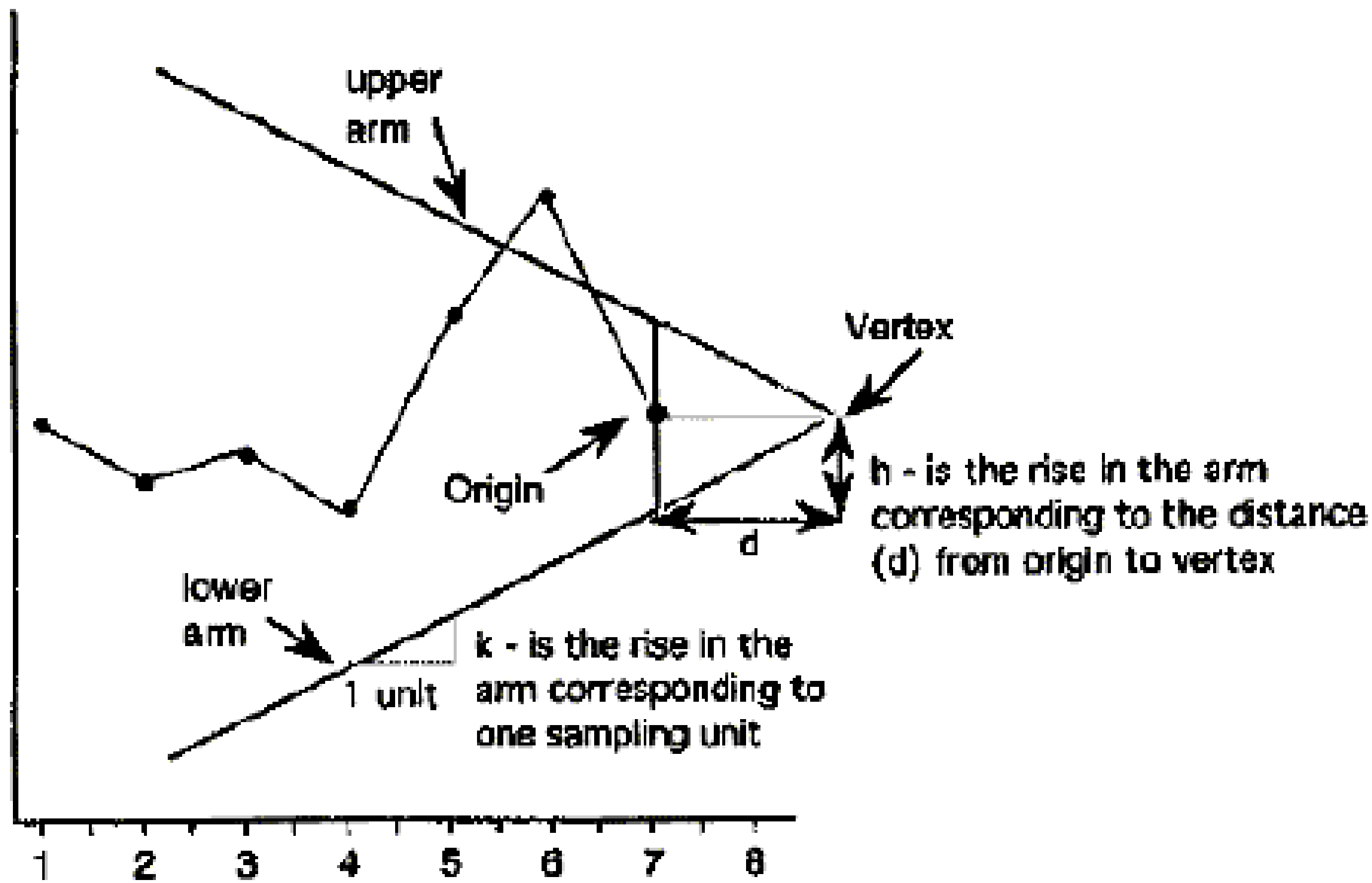
Cumulative Sum (CUSUM) Charts

- **The control chart is formed by plotting the cumulative sums of the difference between the current data and the target value of the "in-control" process.**
- **The cumulative sum response (S_t) for a control chart is calculated using the equation at the right where $\overline{X}_i$ is the subgroup mean and μ_0 is the target value and sigma $_x$ is the standard deviation.**

$$S_t = \sum_{i=1}^{t} \left(\frac{\overline{x}_i - \mu_0}{\sigma_{\overline{x}_i}} \right)$$

CUSUM with a sample V-Mask demonstrating an out of control process

Sample V-Mask demonstrating an out of control process

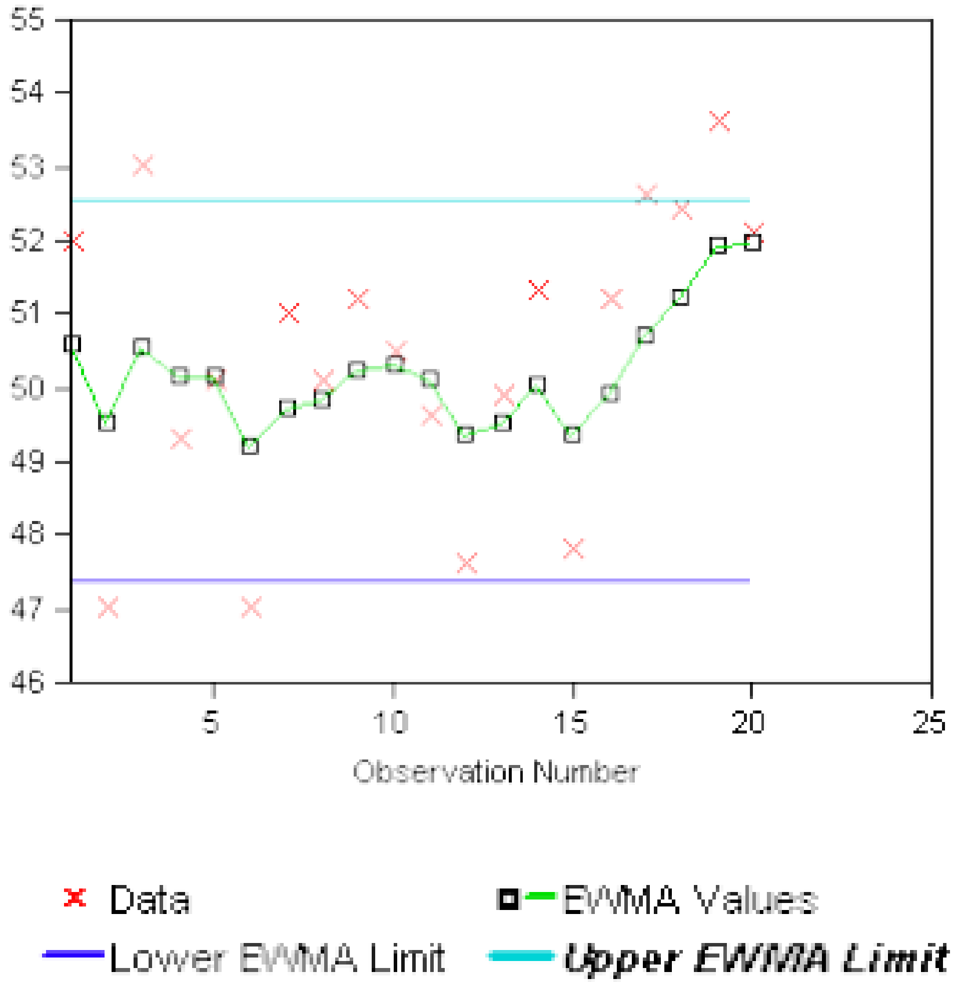


Exponentially Weighted Moving Average (EWMA) Control Charts

- **Uses a weighting factor r ($0 < r \leq 1$) which decreases measurement weighting exponentially going backwards in time.**
- **It is calculated using the equation at the right where $\bar{X}_i$ is the current observation and r is the weight assigned to the most recent measurement.**
- **A small value of r guards against a small shift in the mean.**

$$\text{EWMA}_t = \sum_{i=1}^{t} r(1-r)^{t-1}\bar{x}_i$$

EWMA Chart example showing in-control but a trend in the process



CUSUM and EWMA Charts Advantages/Disadvantages

Advantages

- Very effective at detecting small process shifts

Disadvantages

- Not as effective as Shewart charts (e.g., 2 sigma or greater)
- CUSUM is not as intuitive and generally requires specialized software

EXERCISE

- **Simulate various theft scenarios and how the charting techniques react.**

Summary

- **Process shifts in the inventory of 2 sigma or greater are relatively easy to detect (e.g., significant statistically) within 1 inventory period**
  - **Shewart charting approach most effective**
- **Process shifts in the inventory of 1-2 sigma will be detected within 10 inventory periods**
  - **CUSUM or EWMA tend to be more effective although Shewart with WECO rules applied can be effective**
- **Process shifts in the inventory of 0-1 sigma, while not be statistically significant may be seen using the techniques discussed previously (*note: timeline for Probability of Detection (Pd) variable*)**