Sister Rod Destructive Examinations (FY23)

Appendix H: Instron Uncertainty Calculations

Spent Fuel and Waste Disposition

Prepared for
US Department of Energy

Spent Fuel and Waste Science and Technology

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SUMMARY

This report documents work performed under the Spent Fuel and Waste Disposition's Spent Fuel and Waste Science and Technology program for the US Department of Energy (DOE) Office of Nuclear Energy (NE). This work was performed to fulfill Level 2 Milestone M2SF-24OR010201024, "FY23 ORNL Testing on Sibling Pins," within work package SF-24OR01020102 and is an update to the work reported in M2SF-23OR010201024, M2SF-22OR010201047, M2SF-21OR010201032, M2SF-19ORO010201026, and M2SF-19OR010201028.

This document discusses the uncertainty of measurements associated with the Instron system and the resulting uncertainty in the acquired measurements and calculated information. The primary parameters measured during the Instron test include the applied load and specimen deflection. The associated primary calculated parameters are the maximum beam deflection at the rod's axial center and modulus of elasticity. Secondary calculated parameters include specimen stress and strain amplitudes, as well as flexural rigidity. The uncertainty associated with the primary and secondary calculated parameters was investigated based on the known uncertainty in the measured parameters.

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REVISION HISTORY

Date	Changes
10/29/2021	Initial document release.
3/31/2022	Updated to include uncertainty related to the use of elastic beam theory in the plastic region. The summary was modified to include all uncertainties in terms of the variable of interest.
1/31/2024	The document ID number and dates were changed for inclusion in the FY23 status report.

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ACRONYMS

ASME	American Society of Mechanical Engineers
DOE	US Department of Energy
ID	inner diameter
NDE	nondestructive examination
NE	Office of Nuclear Energy
NRC	US Nuclear Regulatory Commission
OD	outer diameter
ORNL	Oak Ridge National Laboratory
PWR	pressurized water reactor
SNF	spent nuclear fuel

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H-1. Introduction

Appendix E describes the Instron equipment, the measurements taken, and the basic data processing and extended information derived from those measurements.

This document discusses the uncertainty of measurements associated with the Instron 5967 system and the resulting uncertainty in the acquired measurements and calculated information. The primary parameters measured during the Instron test included the applied load and specimen deflection.

At present time the calculations only address four-point bend (4PB) tests. When axial tension tests are completed, the uncertainty related to those tests will also be calculated.

For 4PB, the associated primary calculated parameters were the bending moment, the maximum beam deflection at the rod's axial center, and the modulus of elasticity. Secondary calculated parameters included specimen stress and strain amplitudes, as well as flexural rigidity. The uncertainty associated with the primary and secondary calculated parameters was investigated based on the known uncertainty in the measured parameters.

H-2. Instrumentation and Equipment That Influence Uncertainty

There are essentially five measurements that are used in the evaluation of 4PB data: bottom fixture support length (L), top fixture loading arm (a), displacement (Δf), load (P), and the rod outer diameter (D). The fixture lengths are measured once and are not changed over the test series. The displacement and load are outputs from the Instron load frame during the test as a function of time.

Information on the Instron equipment and instrumentation is from *Appendix E: Mechanical Testing*. Error in the primary parameters measured was provided by certificates of calibration for displacement and force issued by Instron Calibration Laboratory (see Figures H-1 and H-2). Calibrations were tested in accordance with ASTM E2309/E2309M-2016 for displacement and ASTM E4-16 for force. Table H-1 provides the valid ranges and errors that resulted from calibration testing.

Table H-1. Instron calibration results

Parameter	Measurement range	Maximum Error
Displacement (ascending)	25.0008 to 250.0044 mm	± 0.009%
Displacement (descending)	-24.9996 to -250.0026 mm	± 0.002%
Load (tension)	0.299269 to 29.98002 kN	± 0.37%
Load (compression)	-0.300266 to -29.99315 kN	± 0.19%
Temperature	67.1 to 68.1°F	N/A

The first moment of inertia, I, was calculated based on the fuel specimen's outer diameter, D. Each specimen's diameter was measured and reported in *Sister Rod Nondestructive Examination Final Report* [H-1], and the uncertainty associated with each measurement was ± 0.02 mm. Uncertainty in diameter influences the calculated elastic modulus, stress, strain, and flexural rigidity.

Upper fixture length, *a* (1.667 in.), and lower fixture length, *l* (5.000 in.), were measured to the nearest $\frac{1}{16}$ in. Therefore, it was assumed that the nominal uncertainty with each of these measurements was ± $\frac{1}{32}$ in. Fixture length uncertainties have an effect on all calculations discussed.

For the 200°C 4PB tests, the thermal expansion of the fixture lengths and rod diameter was considered. After examining the thermal expansion coefficients of the materials and temperature changes endured, thermal expansion of both materials was determined to be less than 0.5%. Subsequently, the thermal expansion's effect on the bending moment, elastic modulus, stress, strain, and flexural rigidity was also determined to be less than 0.5% and was assumed to have a negligible impact on results.

The tests conducted at ambient hot cell temperature ranged from 70 to 80°F. While these temperatures are slightly higher than the temperature during calibration testing, the Instron 5967 is equipped with sensors inside the load cell to compensate for changes in temperature and can adjust measurement values accordingly.

CERTIFICATE OF CALIBRATION

ISSUED BY: INSTRON CALIBRATION LABORATORY

DATE OF ISSUE: 27-Aug-2019

CERTIFICATE NUMBER: **479082719131225**

NVLAP CALIBRATION
NVLAP Lab Code 200301-0

Instron
825 University Avenue
Norwood, MA 02062-2643
Telephone: (800) 473-7838
Fax: (781) 575-5750
Email: service_requests@instron.com

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APPROVED SIGNATORY

Dan Falkenstein

Digitally signed by Dan Falkenstein
Reason: I attest to the accuracy and integrity of this document
Date: 2019.08.27 19:50:59 -05'00'

Type of Calibration: **Displacement**
Relevant Standard: **ASTM E2309/E2309M-2016**
Date of Calibration: **27-Aug-2019**
Customer Requested Due Date: **27-Aug-2020**

*** * * CALIBRATION RESULTS * * ***

System ID: 5967B13316

Indicator 1. - Service Port (mm)		**System Class: PASSED Class A**
Range: 25.0008 mm to 250.0044 mm - Asc	Starting Position: 800.00 mm	Maximum Error: -0.009%
Range: -24.9996 mm to -250.0026 mm - Desc	Starting Position: 1,000.00 mm	Maximum Error: -0.002%

The starting position is measured from the base beam to the bottom of the crosshead.

Customer

Name:	Oak Ridge National Laboratory
Location:	1 Bethel Valley Road, Building 3525 Oak Ridge, TN 37831
Country:	USA
P.O./Contract No.:	CONT3679
Contact:	Yong Yan
Email:	yany@ornl.gov

Machine/System

Manufacturer:	INSTRON
Condition:	Good

Temperature

Starting Temperature:	68.1 °F
Final Temperature:	67.2 °F

Methodology

The assessment of the testing machine was conducted on site at the above customer location in accordance with ASTM E2309/E2309M-2016 'Standard Practice for Verification of Displacement Measuring Systems and Devices Used in Material Testing Machines' (Follow-the-Displacement Method) using Instron procedure ICA-8-07.

System Classification

The testing machine was verified in the 'As Found' condition with no adjustments or repairs carried out. This is also the 'As Left' condition. The verified range of displacement includes only those displacements which are greater than or equal to the ASTM Lower Limit. Prior to verification, a pre-calibration inspection was conducted and the system was found to be in Good condition. The Simple Acceptance decision rule has been agreed to and employed in the determination of conformance to the identified metrological specification.

The calibration and equipment used conform to a controlled Quality Assurance program which meets the specifications outlined in ANSI/NCSL Z540.1-1994, ISO 10012:2003, ISO 9001:2015 and ISO/IEC 17025:2017.

Figure H-1. Instron certificate of calibration for displacement.

CERTIFICATE OF CALIBRATION

ISSUED BY: INSTRON CALIBRATION LABORATORY

DATE OF ISSUE: 27-Aug-2019

CERTIFICATE NUMBER: **479082719143334**

NVLAP CALIBRATION
NVLAP Lab Code 200301-0
ilac-MRA

INSTRON

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Norwood, MA 02062-2643
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APPROVED SIGNATORY

Dan Falkenstein
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Reason: I attest to the accuracy and integrity of this document
Date: 2019.08.27 19:47:37 -05'00'

Type of Calibration: Force
Relevant Standard: ASTM E4-16
Date of Calibration: 27-Aug-2019
Customer Requested Due Date: 27-Aug-2020

Customer

Name:	Oak Ridge National Laboratory
Address:	1 Bethel Valley Road, Building 3525 Oak Ridge, TN 37831 yany@ornl.gov
P.O./Contract No.:	CONT3679
Contact:	Yong Yan

Machine

Manufacturer:	INSTRON
Serial Number:	5967B13316
System ID:	5967B13316
Range Type:	Single

Transducer

Manufacturer:	INSTRON
Transducer ID:	2580-30KN / 133482
Capacity:	30 kN
Type:	Tension/Compression

Classification

Indicator 1. - Service Port - PASSED**

Certification Statement

This certifies that the forces verified with machine indicator(s) (listed above) that are identified as "PASSED" are WITHIN ±1% accuracy, 1% repeatability, and zero return tolerance. All machine indicators were verified on-site at customer location by Instron in accordance with ASTM E4-16. The Simple Acceptance decision rule has been agreed to and employed in the determination of conformance to the identified metrological specification. The certification is based on runs 1 and 2 only. A third run is taken to satisfy uncertainty requirements according to ISO 17025 specifications.

The calibration and equipment used conform to a controlled Quality Assurance program which meets the specifications outlined in ANSI/NCSL Z540.1-1994, ISO 10012:2003, ISO 9001:2015 and ISO/IEC 17025:2017.

**** within ±0.5% accuracy and 0.5% repeatability.**

Method

The testing machine was verified in the 'As Found' condition with no adjustments or repairs carried out. This is also the 'As Left' condition.

Figure H-2. Instron certificate of calibration for force.

H-3. Uncertainty Calculations

The 2σ uncertainties associated with the measurements are estimated by using a sum of squares of the individual variations assuming independence of the variables (no cross correlations):

$$U_f = \left[\left(\frac{\partial f}{\partial x}\right)^2 U_x{}^2 + \left(\frac{\partial f}{\partial y}\right)^2 U_y{}^2 + \left(\frac{\partial f}{\partial z}\right)^2 U_z{}^2 + \cdots\right]^{1/2} \tag{H-1}$$

where:

- U_f is the 2σ uncertainty of the function $f(x,y,z,\ldots)$ and
- $U_x, U_y,$ *and* U_z are the 2σ uncertainties of the measured values $x,y,z,$ respectively.

The derivatives are evaluated at the nominal measured values (effectivity a single-term Taylor series expansion assuming small variations). The notation and variables used in this appendix are the same as those used in the main report and the reader is referred to the report for details.

Throughout these calculations, unless otherwise noted, the units of loads are in N and lengths are in m.

H-3.1 Maximum Bending Moment Uncertainty

The maximum bending moment applied at the rod center is

$$M_{max} = Pa \tag{H-2}$$

where P is the load applied, and a is the distance between the upper loading points. Arm-length uncertainty (U_a) is nominally given as ± 0.03125 in.(0.00079375 m) for a = 1.667 in. (0.0296418 m), and load uncertainty (U_p) is taken as the maximum error measured during Instron calibration compression testing (± 0.19%). Using the approach described in Eq. (H-1) and the bending moment equation in Eq. (H-2), the maximum bending moment uncertainty is

$$U_{M_{max}} = \sqrt{P^2 U_a^2 + a^2 U_p^2} \tag{H-3}$$

Simplifying, the following is obtained:

$$U_{M_{max}} = 8.30 * 10^{-4} * P \;\; (N - m) \tag{H-4}$$

Maximum bending moments calculated ranged from 0 to ~65 N-m. Since load uncertainty varied linearly with load and arm-length uncertainty was constant, the bending moment uncertainty was calculated to be 0.083% of the applied load and 1.96% of M_{max}.

H-3.2 Maximum Beam Deflection Uncertainty

Using classical beam theory, Δ_{max}, the maximum beam deflection at the rod's axial center in the elastic region can be calculated as:

$$\Delta_{max} = \frac{\Delta_f (3l^2 - 4a^2)}{4(3la - 4a^2)} \tag{H-5}$$

As discussed in Appendix E, Section E-2.1.4, elastic beam theory isn't strictly applicable in the plastic region, but the strain at fracture is relatively small and deviation from elastic theory is therefore expected to be small. The potential magnitude of deviation was evaluated by measuring the center beam deflection and the crosshead deflections in test images, and the deviation from elastic beam theory was within 1.9%. For ease of evaluating the uncertainty, the potential deviation from theory was applied over the entire range of deflections (elastic and plastic), although it is truly only applicable in the plastic range.

Using Eqs. (H-1) and (H-5), uncertainty in maximum beam deflection ($U_{\Delta max}$) is evaluated as

$$U_{\Delta\max} = 1.019\sqrt{\left(\frac{3l^2-4a^2}{4(3la-4a^2)}\right)^2 (U_{\Delta f})^2 + \left(\frac{-3\Delta_f l\left(4a^2-8al+3l^2\right)}{4a^2(4a-3l)^2}\right)^2 (U_a)^2 + \left(\frac{3\Delta_f(3l-2a)(l-2a)}{4a(3l-4a)^2}\right)^2 (U_l)^2} \quad \text{(H-6)}$$

When substituting in known terms, Eq. (H-6) with the included plastic region uncertainty simplifies to

$$U_{\Delta max} = 1.26 * 10^{-2} * \Delta_f \quad (m) \quad \text{(H-7)}$$

For convenience, the uncertainty can be expressed as a percentage of the maximum beam deflection, Δ_{max}

$$U_{\Delta max} = 1.26 * 10^{-2} * \Delta_f / 1.149 = 1.10 * 10^{-2} * \Delta_{max} \quad (m)$$

was ~18 mm, with an uncertainty calculated to be ±0.23 mm.

H-3.3 Modulus of Elasticity Uncertainty

The modulus of elasticity (*E*) can be derived using elastic beam theory, given the approach discussed in Appendix E, using the measured crosshead extension (as corrected for machine compliance), Δ_f ,

$$E = \frac{Pa}{6\Delta_f I}(3la - 4a^2) \quad \text{(H-8)}$$

where *I* is the first moment of inertia about rod diameter *D* ($I = \frac{\pi}{64}D^4$), where *D* is nominally assumed to be 9.5 mm) and the rod OD is that measured using nondestructive methods. Simplifying Eq. (H-8) and following Eq. (H-1) the uncertainty in the modulus of elasticity (U_E) is derived as

$$U_E = \sqrt{\left(\frac{4.169 * 10^8 * (3la^2 - 4a^3)}{\Delta_f}\right)^2 (U_p)^2 + \left(\frac{-4.169 * 10^8 * P(3la^2 - 4a^3)}{\Delta_f^2}\right)^2 (U_{\Delta f})^2 + \left(\frac{2.501 * 10^9 * P(la - 2a^2)}{\Delta_f}\right)^2 (U_a)^2 + \left(\frac{1.251 * 10^9 * Pa^2}{\Delta_f}\right)^2 (U_l)^2} \quad \text{(H-9)}$$

Reducing Eq. (H-9) with known terms, Eq. (H-10) is obtained as shown below:

$$U_E = 3.986 * 10^3 * \frac{P}{\Delta_f}(GPa) \quad \text{(H-10)}$$

The most prominent sources of the elastic modulus uncertainty are the upper fixture loading arm length, *a*, followed by the lower fixture length, *l*, and their associated uncertainties. Figure H-3 depicts elastic modulus uncertainty vs. load for various values of beam deflection, Δ_f, that ranged from 0 to approximately 3 mm (in the elastic regime) during testing. Uncertainties in load and beam deflection measured by the Instron system are statistically insignificant when calculating elastic modulus uncertainty in this test. Uncertainties are plotted up to the maximum elastic modulus calculated (~59 GPa) for each deflection distance. Equation (H-10) and Figure H-3 suggest that more rigid specimens have larger magnitudes of elastic modulus uncertainty. However, the relative uncertainty of the elastic modulus remains constant at 2.5%.

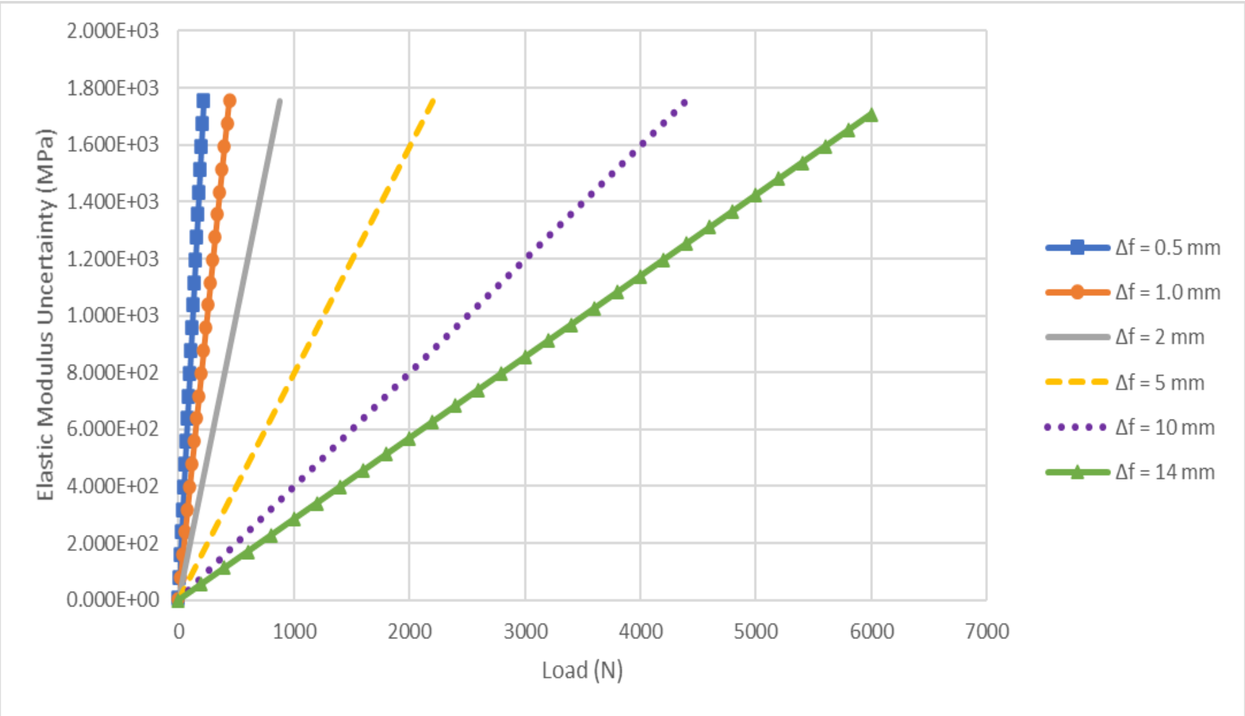

Figure H-3. Elastic modulus uncertainty (U_E) versus load (P) for varying beam deflections.

H-3.4 Flexural Rigidity Uncertainty

Flexural rigidity (EI) is the product of the elastic modulus and the first moment of inertia, and following Eq. (H-1), its uncertainty (U_{EI}) can be expressed as

$$U_{EI} = \sqrt{\left(\frac{3la^2 - 4a^3}{6\Delta_f}\right)^2 \left(U_p\right)^2 + \left(\frac{-P(3la^2 - 4a^3)}{6{\Delta_f}^2}\right)^2 \left(U_{\Delta f}\right)^2 + \left(\frac{P(la - 2a^2)}{\Delta_f}\right)^2 (U_a)^2 + \left(\frac{Pa^2}{6\Delta_f}\right)^2 (U_l)^2} \quad \text{(H-11)}$$

Simplifying and including the 1.9% uncertainty related to the use of elastic beam theory:

$$U_{EI} = 1.019 * 1.44 * 10^{-6} * \frac{P}{\Delta_f} = 1.47 * 10^{-6} * \frac{P}{\Delta_f} \ (N - m^2) \quad \text{(H-12)}$$

The flexural rigidity uncertainty's most dominant sources are the upper and lower fixture lengths. Flexural rigidity uncertainty vs. load for varying beam displacements is plotted in Figure H-4 for $0 < EI < 28$ N-m^2, the range of values calculated during testing. Relative flexural rigidity uncertainty remains constant at 2.3%.

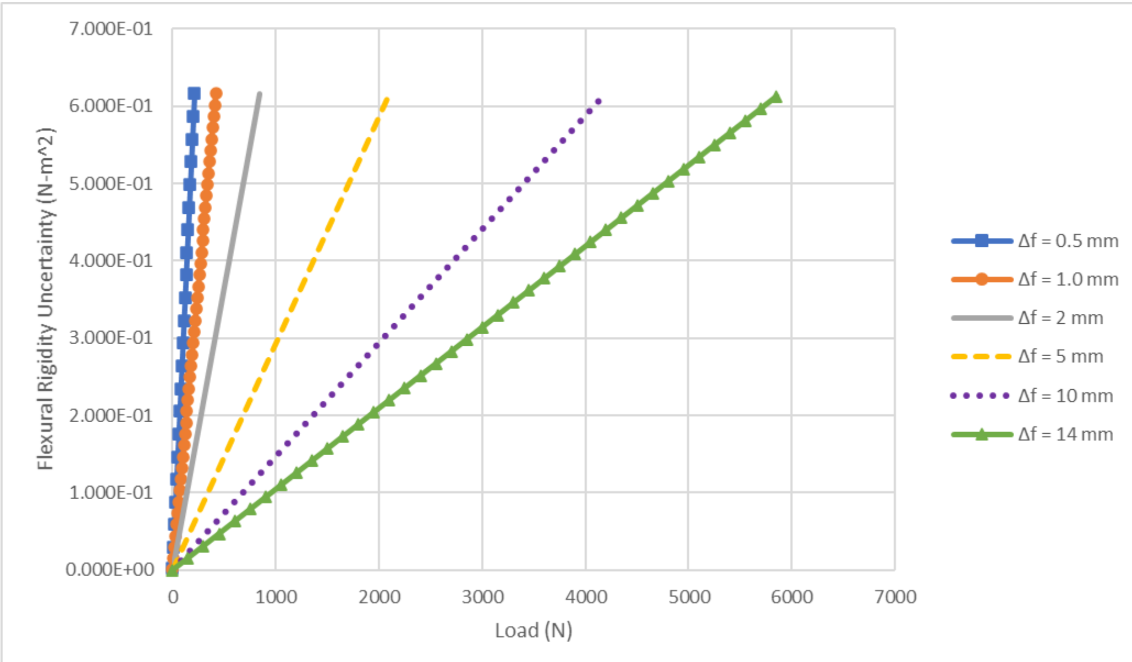

Figure H-4. Flexural rigidity uncertainty (U_{EI}) vs. load (P) for varying beam deflections.

H-3.5 Maximum Stress Uncertainty

Maximum stress (σ_{max}) occurs at the maximum deflection point at the bottom of the specimen and is calculated as

$$\sigma_{\max} = \frac{Pa\,(D/2)}{I} \qquad \text{(H-13)}$$

After substituting $I = \frac{\pi}{64}D^4$ and applying Eq. (H-1), maximum stress uncertainty is given in Eq. (H-14).

$$U_{\sigma max} = 1.188 * 10^7 \sqrt{a^2\left(U_p\right)^2 + P^2(U_a)^2 + \left(\frac{3Pa}{D}\right)^2 (U_D)^2} \qquad \text{(H-14)}$$

which simplifies to

$$U_{\sigma max} = 1.99 * 10^{-2} * \sigma_{max} \ \ (Pa) \qquad \text{(H-15)}$$

Maximum stress values varied from 0 to 768 MPa with a constant relative uncertainty of 1.99%.

H-3.6 Maximum Strain Uncertainty

Maximum strain (ε_{max}) is estimated as

$$\varepsilon_{max} = \frac{D\rho}{2}, \tag{H-16}$$

where ρ is curvature, which is a function of load and displacement. The maximum strain uncertainty ($U_{\varepsilon max}$) is a function of deflection uncertainty and is calculated in the following equation, which includes the 1.9% uncertainty related to use of elastic beam theory in the plastic region:

$$\begin{aligned} U_{\varepsilon max} &= 1.019 * \ 8.795 * 10^{-6} \frac{\Delta_f}{\left(0.0161 \ - \ 5.277{\Delta_f}^2\right)^2} \\ &= 8.962 * 10^{-6} \frac{\Delta_f}{\left(0.0161 \ - \ 5.277{\Delta_f}^2\right)^2} \end{aligned} \tag{H-17}$$

Figure H-5 depicts the relationship between maximum strain uncertainty and displacement, while Figure H-6 plots relative maximum strain uncertainty to maximum strain.

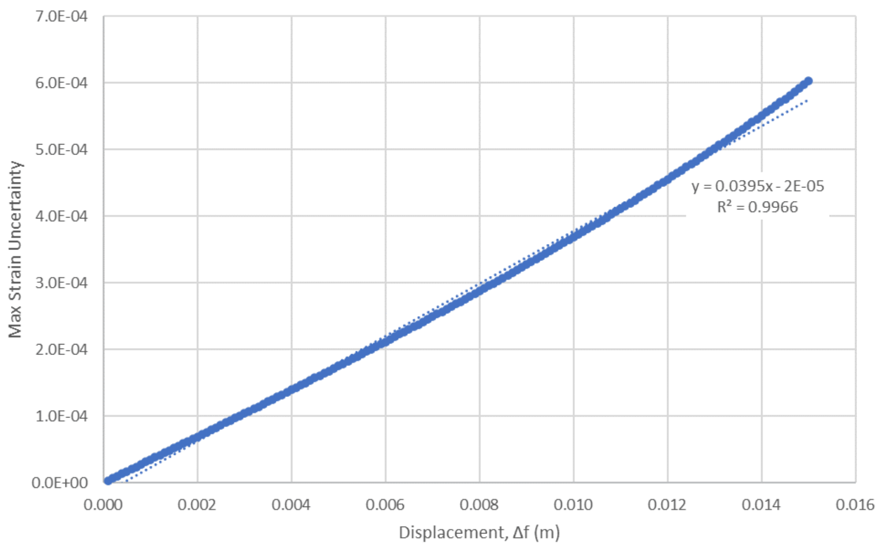

Figure H-5. Maximum strain uncertainty ($U_{\varepsilon max}$) vs displacement (Δ_f).

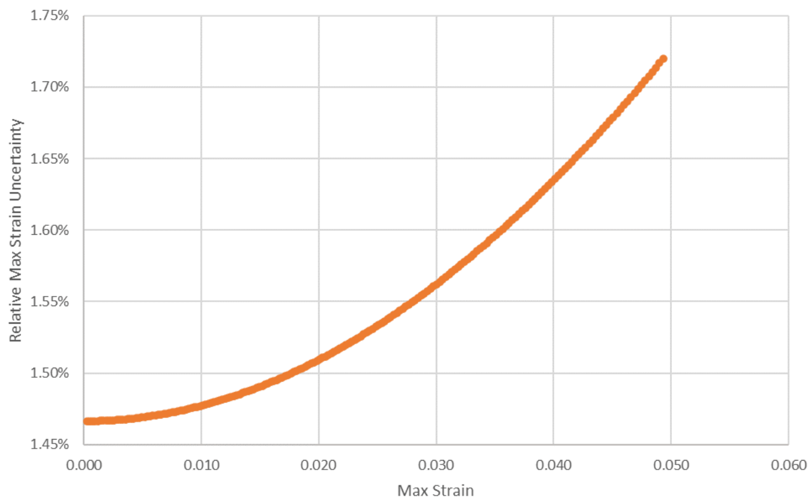

Figure H-6. Relative maximum strain uncertainty vs maximum strain.

Measured strains ranged from 0 to 0.046 with a relative maximum strain uncertainty that ranges from 1.49% to 1.69% for the data.

H-4. Summary of Results

The uncertainty associated with each of the Instron primary and secondary parameters is summarized in Table H-2.

Table H-2. Summary of Instron primary and secondary parameter uncertainties

Parameter	Measurement range	Uncertainty
Bending moment, M_{max} (N-m)	0 to 60.70	1.96% of M_{max}
Maximum beam deflection, Δ_{max} (m)	0 to 0.0141	$1.10\% \; of \; \Delta_{max}$
Modulus of elasticity, E (GPa)	0 to 59.3	$2.5\,\% \; of \; E$
Flexural rigidity, EI (N-m^2)	0 to 23.3	$2.3\% \; of \; EI$
Maximum stress, σ_{max} (MPa)	0 to 768	$1.99\% \; of \; \sigma_{max}$
Maximum strain (m/m)	0 to 0.046	$1.69\% \; of \; \varepsilon_{max}$

REFERENCES

[H-1] R. A. Montgomery et al., *Sister Rod Nondestructive Examination Final Report*, SFWD-SFWST-2017-000003 Rev. 1 (M2SF-17OR010201021) / ORNL/SPR-2017/484 Rev. 1 (ORNL/SPR-2018/801), Oak Ridge National Laboratory, Oak Ridge, TN, 2019.