Parametric Decay Instability (PDI) in Tokamaks

Guangye Chen, Frederick Jaeger, Lee Berry,
Jim Myra (Lodestar)
Theodore Biewer, Phillip Ryan, John Wilgen

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What is PDI?

• In a heating experiment, linear waves can be excited by external sources.

• A high amplitude wave (pump wave @ $f_0$) can non-linearly excite other waves (daughter waves @ $f_1, f_2$).
  – Particles with oscillatory velocity $u(f_0)$.
  – Fluctuating density $n(f_2)$.
  – The above two quantities produce a current $nu$, which can drive a plasma wave at $f_1 = f_2 - f_0$.
  – The plasma wave ($f_1$) and the oscillatory velocity at pump frequency ($f_0$) produces a ponderomotive force that drives the low frequency perturbation.
PDI and Edge Ion Heating in IBW and ICRF Experiments

a  Pinsker 1993, DIII-D

- antenna freq.

b  Rost 2002, CMOD

Probe NPA

antenna freq.

0 20 40 60 80 100
FREQUENCY (MHz)

3 1 0.0
neutral flux (10^3cts/(ms-keV))

f_{rf}/f_{CD} at edge

RF turn on

RF turn off

c  Biewer 2005, NSTX

perpendicular

parallel

0 1 2 3 4 5
P_{RF} (MW)

Edge T_i (eV)

500 400 300 200 100

0 1 2 3 4 5

ERD Reflectometer

Wilgen 2006, NSTX

0 1 2 3 4 5 6 7

Frequency (MHz)

Time (ms)

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UT-BATTELLE
Motivation

• PDI has been widely observed in tokamak ICRF and IBW heating experiments (NSTX, Alcator C-MOD, DIII-D, TST-2, HT-7, etc.).

• PDI in the scrape-off-layer (SOL) can lead to significant power loss, degrading the rf heating efficiency in the bulk plasma (eg. direct IBW).

• On NSTX, edge PDI loss accounts for 30% of total rf power (a lower bound estimate).

• We need a more quantitative understanding of PDI.
Outline

• Kinetic theory and model equations
• Numerical methods
• AORSA 1D simulations and PDI dispersion calculations
  – Alcator C-MOD
  – NSTX
• Summary
• Future work
Oscillating Frame, Kinetic Theory

• Maxwellian plasma in a oscillating frame,
  \[
  \frac{\partial \mathbf{u}}{\partial t} = \mathbf{\Omega} \times \mathbf{b} + \frac{Ze}{m} \mathbf{E}(t), \quad \frac{\partial \xi}{\partial t} = \mathbf{u}
  \]

  dipole pump

• Vlasov equation in the oscillating frame:
  \[
  \frac{\partial f_k}{\partial t} + i k \cdot w f_k - \frac{\partial f_k}{\partial \phi} = -\mathbf{a}_k \cdot \nabla \mathbf{w} F e^{ik \cdot \xi - i\delta_k}, \quad e^{ik \cdot \xi} = \sum_n J_n(\mu) e^{-i\omega_0 t - in\beta}
  \]

• Solutions
  \[
  f_k = \sum_p f_k^{(p)} e^{-i(\omega + p\omega_0) t}, \quad a_k = \sum_m a_k^{(m)} e^{-i(\omega + m\omega_0) t}
  \]

• Electrostatic theory: charge density + Poisson’s equation  Nonlinear Coupling
PDI Model Equations

• Pump wave is solved independently, and assumed constant in the SOL

\[- \frac{c^2}{\omega_0^2} (\nabla \times \nabla \times E_0) + \mathbf{K}_0 \cdot \mathbf{E}_0 = - \frac{i}{\varepsilon_0 \omega_0} \mathbf{J}_{\text{antenna}}\]

• Daughter waves are solved by

\[- \frac{c^2}{\omega_1^2} (\nabla \times \nabla \times E_1) + \mathbf{K}_1 \cdot \mathbf{E}_1 = \mathbf{C}_1 (E_0, E_1, E_2) \frac{i}{\varepsilon_0 \omega_1} \mathbf{J}_{\text{noise},1} \]

\[- \frac{c^2}{\omega_2^2} (\nabla \times \nabla \times E_2) + \mathbf{K}_2 \cdot \mathbf{E}_2 = \mathbf{C}_2 (E_0, E_1, E_2) \frac{i}{\varepsilon_0 \omega_2} \mathbf{J}_{\text{noise},2} \]

• Selection rules:

\[\omega_0 = \omega_1 + \omega_2\]

\[\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2\]
Successive Approximations Diverge for Large Coupling

Iterative method agrees with direct method

Iterative method diverges

Fast wave ➔ IBW+Quasi-Mode

Weak coupling

Stronger coupling

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PDI Simulations for Alcator C-MOD

Exponential growth of the daughter field

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_2$</td>
<td>53.4 MHz</td>
</tr>
<tr>
<td>$f_3$</td>
<td>26.6 MHz</td>
</tr>
<tr>
<td>$T_e$</td>
<td>15 eV</td>
</tr>
<tr>
<td>$T_i$</td>
<td>3 eV</td>
</tr>
<tr>
<td>$n_e$</td>
<td>$1 \times 10^{19}$ m$^{-3}$</td>
</tr>
<tr>
<td>$B_0$</td>
<td>5.3 T</td>
</tr>
<tr>
<td>$r_t$</td>
<td>0.9 m</td>
</tr>
<tr>
<td>$k_y$</td>
<td>2000 m$^{-1}$</td>
</tr>
<tr>
<td>$n_\phi$</td>
<td>40</td>
</tr>
</tbody>
</table>
Which waves to simulate?

- PDI daughter waves satisfy the selection rules.

\[ \omega_0 = \omega_1 + \omega_2 \]
\[ k_0 = k_1 + k_2 \]

- One of the daughter waves is a linear plasma wave.
Two Branches of IBW’s

- $k_\perp (\text{left}) \ll k_\perp (\text{right})$
- Damping(left) << damping(right)
- Slow convergence for large $k_\perp$
IBW Dispersion Relation & Decay Channels

CMOD Minority heating regime

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PDI Dispersion Relation

CMOD Minority heating regime

\[
\varepsilon(\omega_1, k_1)\varepsilon(\omega_2, k_2) = -\frac{\mu_D^2 \mu_D^3}{4}(\chi_e(\omega_1, k_1) - \chi_e(\omega_2, k_2))(\chi_i(\omega_1, k_1) - \chi_i(\omega_2, k_2)),
\]

\[
\mu_D = \mu_e - \mu_i,
\]

\[
\mu = \frac{q}{m} \left[ \left( \frac{E_\perp \cdot k_\perp}{\omega_0^2 - \omega_c^2} + \frac{E_\parallel k_\parallel}{\omega_0^2} \right)^2 + \frac{|E_\perp \times k_\perp|^2 \omega_c^2}{(\omega_0^2 - \omega_c^2)^2 \omega_0^2} \right]^{1/2},
\]

\[
E_\perp = 9\text{ kV/m}
\]

Growth rate

\[
E_\perp = 45\text{ kV/m}
\]

MOST UNSTABLE MODE
Diamond-shape-wave Excitation

\[ g(x) = \frac{a \sin(\text{Re}(k_x)x)}{\sqrt{\text{Re}(k_x)}} e^{\int \text{Im}(k_x)dx} \]
PDI Threshold Consistent with Experiment

CMOD Minority heating regime

- Linear 3D AORSA simulations predict typical fast wave edge amplitude = (5, 12) kV/m @0.5MW rf power.
- Scaling trends for the PDI threshold behavior with density, temperature are consistent with Ref. [Rost 2002].
PDI Simulations for NSTX

- Daughter IBW amplitudes vs. frequency at a fixed point in the edge.
- Both weak and strong PDI are simulated.
- Peaks are separated by the cyclotron frequency at the probe ($\Omega_i$).
- Simulation parameters:
  - $T_e = T_i = 50$ eV,
  - $n_e = 2 \times 10^{18}$ m$^{-3}$,
  - $B_\phi = 0.45$ T
  - $k_y = 7000$ m$^{-1}$, $n_\phi = 15$
PDI Dispersion Relation

Growth rate

\[ \omega_{IBW}/\omega_{CD} \]

MOST UNSTABLE MODE

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Predicted power threshold is consistent with previous reflectometer measurements (100 to 300kW), (Wilgen 2005).
Summary

• Fast wave->IBW+QM is successfully excited using the extended AORSA code.
• The most unstable modes are found to be the waves excited just above the cyclotron harmonic.
• PDI simulations are consistent with the threshold reported in minority heating experiments on Alcator C-MOD.
• PDI simulations show the threshold decreases with $k_\phi$, consistent with the edge ion heating measurements.
• Predicted PDI threshold in NSTX is about 100 KW, consistent with previous reflectometer measurements.
Future Work

Core heating vs. edge ion heating in NSTX

J. Hosea, et al.

Bphi=0.45 T

Bphi=0.55 T

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Future Work

Core heating vs. edge ion heating in NSTX

Bphi=0.45 T

After rf is on

Bphi=0.55 T

Before rf is on

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D and He Plasmas Heated to NSTX Record $T_e(0)$ with ~ 3 MW of $k_\phi = -8$ m$^{-1}$ HHFW

G. Taylor, APS-DPP 2009