Generalized Modeling of Enrichment Cascades That Include Minor Isotopes

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Ideal Cascade Models Provide Powerful Analysis Capability

- **Good approximation to plant operation**
- **Comparison to facility or environmental samples**
- **Simple calculation—few input parameters**
  - No details of machine performance
  - Not useful for detailed cascade design
- **MSTAR code developed by Ed von Halle**
  - Furnished to IAEA in 1996
  - Matched-abundance-ratio approx. for > 2 isotopes
- **New MSTAR code based on equations of von Halle**

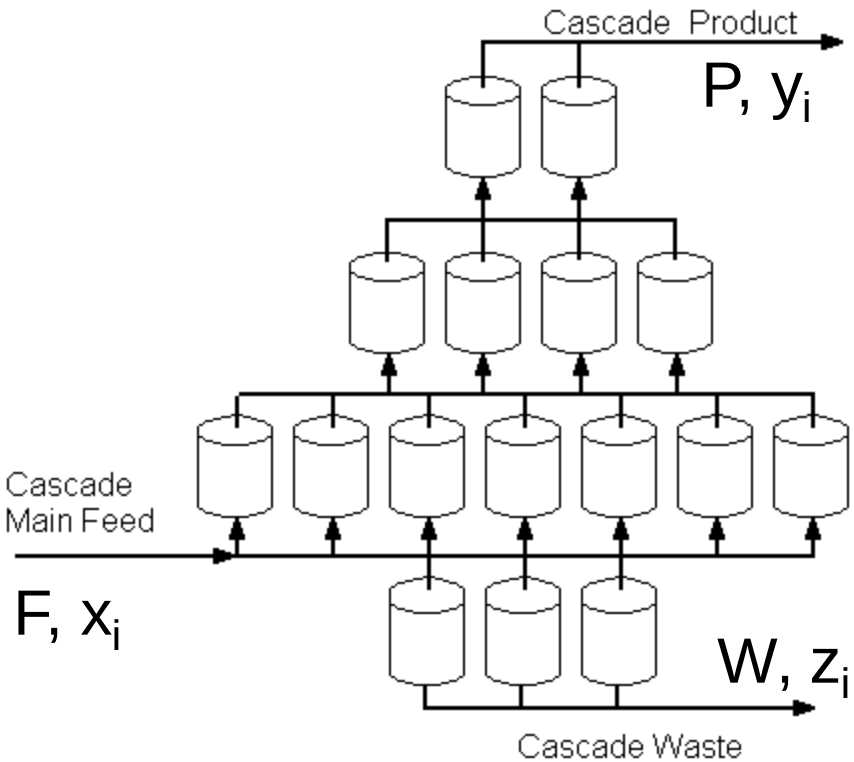
Secondary Information May Be Useful in Facility Monitoring

- **Minor isotopes always present**
  - Fresh fuel involves ^{234}U
  - Reprocessed fuel involves ^{234}U and ^{236}U
  - Other isotopes may be present
- **Minor isotopes undergo enrichment process**
  - Verification of declared enrichment processes
  - Replacement for corrupted or missing data for ^{235}U and ^{238}U
- **Numbers of Stages may be known or observable**
  - Alternatives to enrichment factor (machine performance)

Ideal Enrichment Cascade Uses Few Important Variables

- **Flow rates:**
  - F (feed)
  - P (product)
  - W (waste)
- **Assays (mole fractions) of all isotopes:** x_i, y_i, z_i
- **Number of stages:**
  - N (enriching)
  - M (stripping)
- **Stage separation factor**

$$\alpha_i = \frac{y_i / y_{238}}{z_i / z_{238}}$$



Flexible Input Options Allow More Versatile Applications

Variables	Original MSTAR	New MSTAR
α = sep. factor N, M = No. of stages	α required N, M calculated	α, N, M, or (N+M) Others calculated
x_i, y_i, z_i = assays in feed, product, waste	All feed assays required. y_{235}, z_{235} required	specify > I + 1 indep. assays (I = No. of isotopes)
Over-specified input	Not allowed	Calculates least-squares solution

Sample Problem 1: Enrichment of Natural Uranium to 20% ^{235}U

- **Generic centrifuge: α = 1.5**
- **Original MSTAR assays:**
  - Natural U Feed: ^{234}U = 0.0055%, ^{235}U = 0.711 %
  - Waste: ^{235}U = 0.3 %, Product: ^{235}U = 20 %
  - All other variables determined using this data
- **New MSTAR cases run using different input**
  - All assay combinations (45) for varied cascade inputs (α, N, M, N+M)
  - Verification against Original MSTAR calculations

Sample Problem 1: Most Results for Flexible Input Were Successful

- **Required: 5 input variables**
- **4 assays—all results excellent**
- **2 or 3 assays (substituting α, N, M or N+M)**
  - Most results excellent
  - Failures
    - All assays for same isotope
    - No variable in some section (Feed, Waste, Product)
    - Few isolated cases—heavy reliance on minor isotope ^{234}U

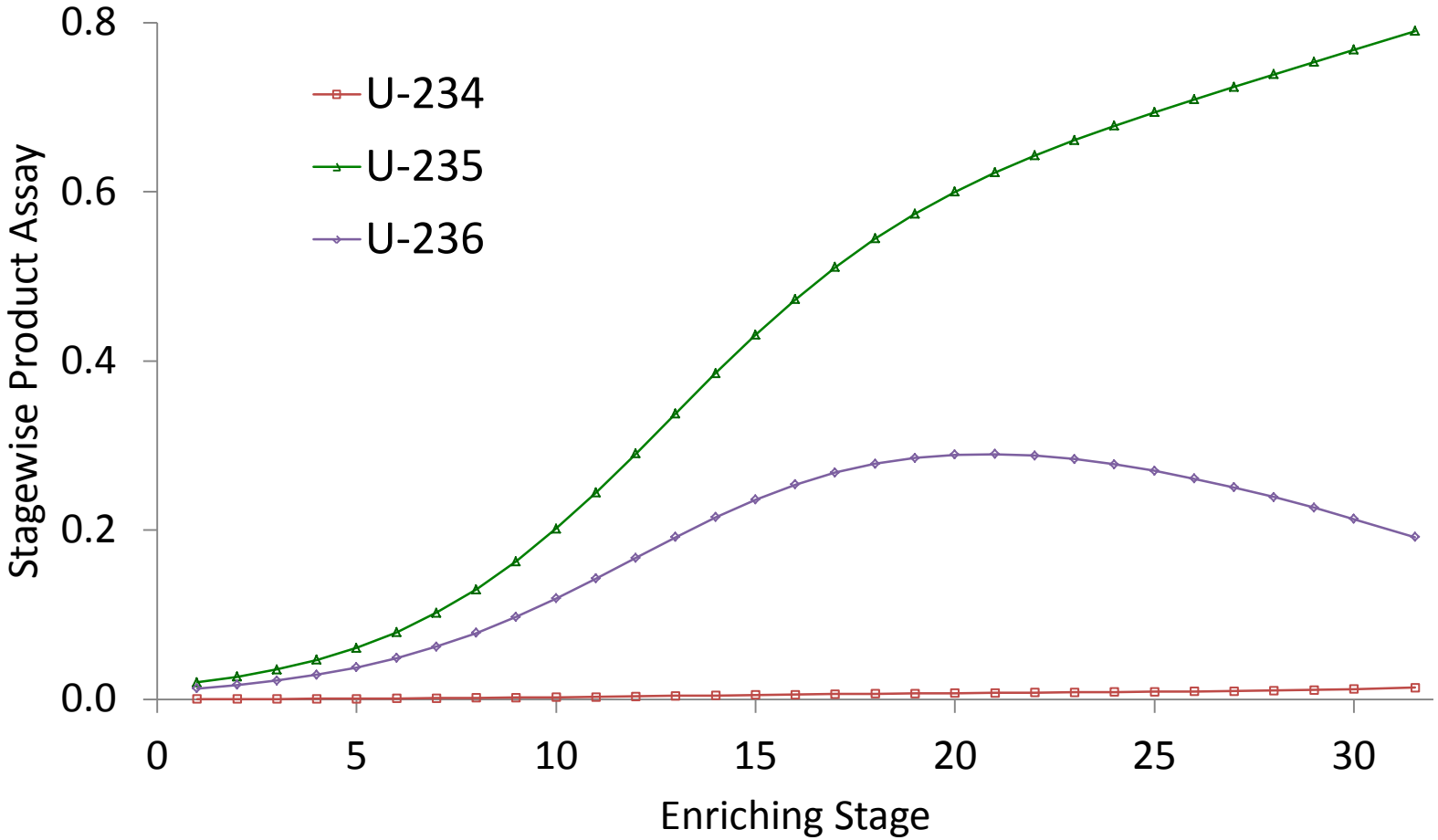
Sample Problem 2: Reprocessed Fuel to Very High Enrichment

- **Generic centrifuge: α = 1.81**
- **Initial MSTAR assays:**
  - Feed: ^{234}U = 0.025 %, ^{235}U = 1.5 %, ^{236}U = 0.55 %
  - Waste: ^{238}U = 99.7 %, Product: ^{235}U = 79 %
  - All other variables determined using this data
- **New MSTAR cases run using different input**
  - Principal assay combinations (180)
    - Required at least 1 assay for each isotope
    - Varied cascade inputs (α, N, M, N+M)
  - Verification against Initial calculations

Sample Problem 2: Most Alternative Inputs Yield Excellent Results

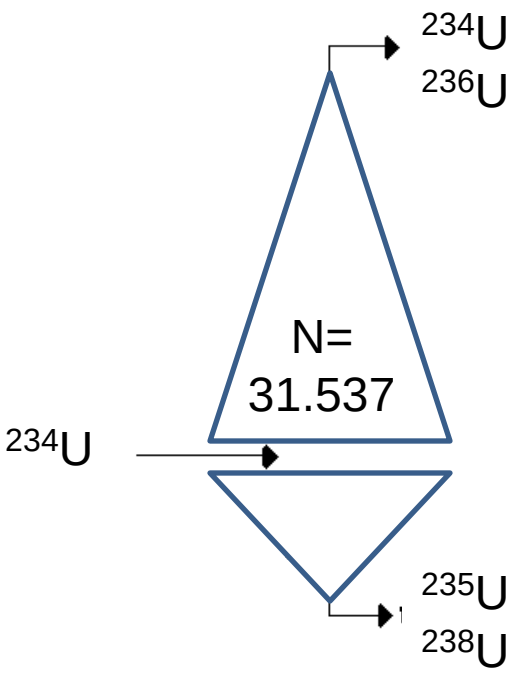
- **Most cases good to 4 or 5 significant figures in all calculated variables**
- **Small number of failures**
  - Converged solution far from exact solution
  - Error residuals very low: Legitimate math. solution
- **Very difficult problem**
  - Product assay of ^{238}U very near zero (0.45%)
  - Non-unique solutions for some input combinations

Enrichment Plot Reveals Peculiar Pattern for Minor Isotope ^{236}U



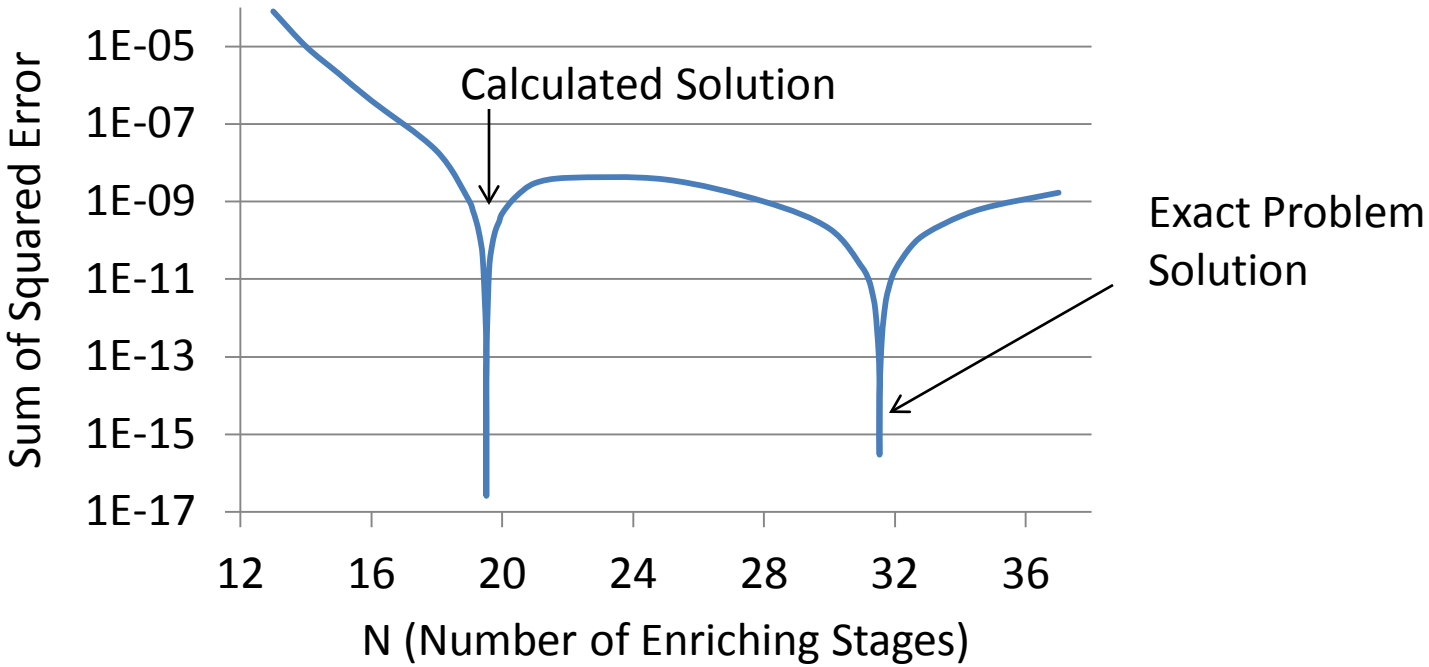
High Reliance on Minor Isotopes May Result in Non-unique Solution

- **Representative calculation:**
  - Minor isotopes in feed and product
- **Enriching stages:**
  - Calculation N = 19.513
  - Exact N = 31.537
- **Calculation has very low residual error**



Residual Error Plot Indicates Multiple Solutions

- **Over-specified system: N added as input variable**
- **Calculation is least-squares solution (nonlinear)**
- **Two sites of very low squared error implies two solutions**



SUMMARY:
Alternative Inputs Provide Flexibility and Additional Verification Capability

- Numbers of stages (N, M, or N+M) can substitute for enrichment factor (α)
- Assays in product or waste streams can replace or verify feed assays
- Minor isotopes can verify enrichment processes
- Difficult cases can arise—must be aware of problem limitations

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