Reduce Numerical Diffusion in TRACE Using the High-Resolution Numerical Method ENO

Dean Wang

Oak Ridge National Laboratory
P.O. Box 2008 MS6167, Oak Ridge, TN 37831
wangda@ornl.gov

INTRODUCTION

Nowadays most reactor systems analysis codes such as TRACE and RELAP, etc., solve two-phase flow equations using the first-order upwind differencing numerical scheme for spatial discretization because of its robustness [1]. While very robust, first-order upwinding is only first-order accurate and leads to excessive numerical diffusion. Numerical diffusion becomes a serious issue in BWR stability analysis where non-uniform nodalization for the core fuel channels sometimes has to be used to mitigate numerical damping [2].

High-order numerical methods can effectively reduce numerical diffusions, but often produce spurious oscillations for steep gradients. In recent years, essentially non-oscillatory (ENO) schemes [3] have been successfully used in CFD by a number of researchers. The high-resolution ENO schemes exhibit much more robustness than the centered difference method and more accuracy than the first-order upwind method. In this study, the second-order ENO scheme has been tested within the TRACE code.

IMPLEMENTATION OF ENO IN TRACE

In TRACE, the two-phase mass equations are discretized as

$$\frac{(\alpha\rho\psi)_j^{n+1}-(\alpha\rho\psi)_j^n}{\Delta t}+\frac{\langle(\alpha\rho\psi)^n u^{n+1}\rangle_{j+\frac{1}{2}}-\langle(\alpha\rho\psi)^n u^{n+1}\rangle_{j-\frac{1}{2}}}{\Delta x_j}=\mathrm{R}_j^{n+1} \quad (1)$$

Where α and ρ are the phase volume fraction and density, $\psi=1$ for the mass equation and $\psi=e$ (internal energy) for the energy equation, u the velocity, Δt and Δx the time and space steps, n and j the temporal and spatial indices, and R the mass source term for the mass equation and the heat source term plus the pressure term for the energy equation [1]. The angle brackets denote the mass fluxes calculated according to the upwinding scheme:

$$\langle(\alpha\rho\psi)^n u^{n+1}\rangle_{j+1/2}=u_{j+1/2}^{n+1}\begin{cases}(\alpha\rho\psi)_j^n & if\ u_{j+1/2}^{n+1}\geq 0\\ (\alpha\rho\psi)_{j+1}^n & if\ u_{j+1/2}^{n+1}<0\end{cases} \quad (2)$$

In this study, the second-order ENO scheme has been implemented for the mass and energy flux terms in TRACE Version 5.509. Based on this scheme, the edge value of $(\alpha\rho\psi)$ is extrapolated from the known values at the center of the node by

$$\langle(\alpha\rho\psi)^n u^{n+1}\rangle_{j+1/2}=u_{j+1/2}^{n+1}\begin{cases}(\alpha\rho\psi)_j^n+\frac{\Delta x_j}{2}\cdot Minmod\left\{\frac{(\alpha\rho\psi)_{j+1}^n-(\alpha\rho\psi)_j^n}{(\Delta x_{j+1}+\Delta x_j)/2},\frac{(\alpha\rho\psi)_j^n-(\alpha\rho\psi)_{j-1}^n}{(\Delta x_j+\Delta x_{j-1})/2}\right\} & if\ u_{j+1/2}^{n+1}\geq 0\\ (\alpha\rho\psi)_{j+1}^n-\frac{\Delta x_{j+1}}{2}\cdot Minmod\left\{\frac{(\alpha\rho\psi)_{j+2}^n-(\alpha\rho\psi)_{j+1}^n}{(\Delta x_{j+2}+\Delta x_{j+1})/2},\frac{(\alpha\rho\psi)_{j+1}^n-(\alpha\rho\psi)_j^n}{(\Delta x_{j+1}+\Delta x_j)/2}\right\} & if\ u_{j+1/2}^{n+1}<0\end{cases} \quad (3)$$

Where

$$Minmod\,\{a,b\}=\begin{cases}a\ \ if\ |a|<|b|\ and\ ab>0\\ b\ \ if\ |b|<|a|\ and\ ab>0\\ 0\ \ if\ ab<0\end{cases}$$

The original TRACE V5.509 code is denoted "V5.509 Upwind", and the modified TRACE is denoted "V5.509 ENO". Note that the ENO scheme is only activated for the TRACE CHAN component in this study.

Two TRACE models were employed to test the ENO scheme. The first is the single-CHAN model which consists of a CHAN, an inlet BREAK, and an outlet BREAK. The CHAN component is representative of a typical BWR fuel assembly with 26 uniform axial nodes. It is powered with a uniform axial power shape except for the bottom and top nodes. The flow is determined by the differential pressure between the inlet and outlet BREAKs. The single-CHAN standard model was renodalized with 120 fine nodes for the powered section. It is found that TRACE results of this fine-node model are well converged. Therefore, it is used as the reference case.

The second model is the coupled TRACE/PARCS BWR plant model for the Ringhals reactor system including the RPV, steam line, recirculation loop, etc. The reactor core consists of 648 fuel assemblies, modeled with 325 CHAN components in half-core symmetry. Each CHAN has 25 powered axial nodes with uniform 14.72 cm spacing. The reactor power is 1475.5 MWth and the core flow is 3665 kg/s. The PARCS model has 648 radial

nodes (node/assembly), and 25 axial powered nodes with uniform 14.72 cm spacing. For comparison, a fine-node model was developed with all of the CHAN components renodalized with 100 uniform nodes for the powered section.

For this study, three cases have been run for each model using the semi-implicit method. Case 1: the fine-node model, run with "V5.509 Upwind"; Case 2: the coarse-node model, run with "V5.509 Upwind"; and Case 3: the coarse-node model, run with "V5.509 ENO".

RESULTS

In the single-CHAN model the transient was initiated by a pressure perturbation at the outlet, resulting in channel flow oscillations as shown in Fig. 1. All the three cases were run at the time-step size of 0.001s, which is well below the Courant limit (CFL) of ~0.018s for the coarse-node model, and 0.0038s for the fine-node model. It is shown that there is considerable numerical diffusion (damping) in the upwind method with the coarse-node model (Case 2). Case 3 has very good agreement with the reference case, which indicates that the ENO method has effectively reduced numerical diffusion. In addition, the steady-state flow is slightly overpredicted in the coarse-mesh model using the upwind method (Case 2) as compared to Cases 1. However, it is well predicted with the ENO method.

For the Ringhals plant model the transient was initiated by a pressure perturbation at the turbine inlet, resulting in core power oscillations (as well as core flow) as shown in Fig. 2. Cases 2 and 3 (the coarse-node model) was run at the time-step size of 0.01s which is below the CFL limit of ~0.025s, and Case 1 (the fine-node model) was run at the CFL limit of ~0.006s. It is shown again that the upwind scheme has significant numerical diffusion. The calculated decay ratio is 0.53 for Cases 1 and 3, and 0.44 for Case 2. Thus the upwind scheme has underpredicted the decay ratio by 17% as compared to the ENO scheme for the same coarse-node model.

CONCLUSIONS

This study has demonstrated that the ENO scheme has a promising potential in reactor thermal-hydraulic analysis where numerical diffusion needs to be kept to a minimum, e.g., BWR core instability analysis.

In addition, it is worth noting that TRACE time-step sizes are restricted by the CFL limit ($\Delta t < \Delta x/v$) for the semi-implicit numerical method, i.e. the minimum ratio of the node size to the edge velocity for all of the nodes in the model. The ENO scheme may need a more restrictive condition on the time-step size than the default CFL limit. It is found with the single-CHAN model that numerical instability occurs for the ENO scheme when the default CFL-based time-step size is used. However, this may not be an issue with large plant models. For example, the transient results of the Ringhals plant model can be well predicted when run with the CFL-based time-step size. It is because the CFL limit occurs in the hottest channel where the vapor velocity at the top is the largest in the core (~6m/s) and the node size is relatively small (~0.15m). So the code will run at the time-step size of ~0.025s. Although numerical instability may occur in this hottest fuel channel with the ENO scheme, it won't take place in most of the core fuel channels since the time-step sizes required by the numerical stability condition for those channels are well below the CFL limit.

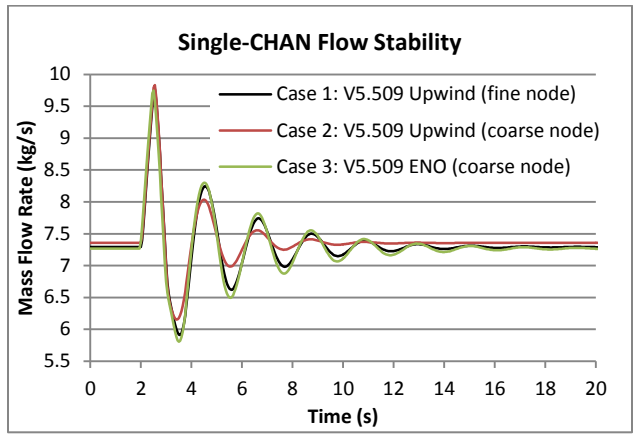

Fig. 1. CHAN Inlet Flow Rate

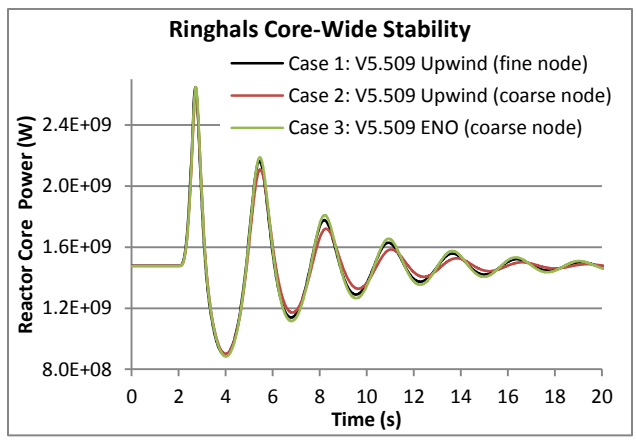

Fig. 2. Ringhals Core Power

REFERENCES

1. TRACE V5.0 Theory Manual, U. S. Nuclear Regulatory Commission.
2. Y. XU, et al, "Application of TRACE/PARCS to BWR Stability Analysis," *Annals of Nuclear Energy* **36** (2009) 317-323.
3. C.W. SHU and S. OSHER, "Efficient Implementation of Essentially Non-oscillatory Shock Capturing Scheme, II," *J. Comput. Phys*., **83**: 32-78, 1989.