HIDDEN MARKOV MODELING FOR WEIGH-IN-MOTION ESTIMATION

Robert K. Abercrombie
Erik M. Ferragut
Shane D. Boone[1]
Oak Ridge National Laboratory (ORNL)[2]
Computational Sciences and Engineering Division
Oak Ridge, TN USA

Abstract

This paper describes a hidden Markov model to assist in the weight measurement error that arises from complex vehicle oscillations of a system of discrete masses. Present reduction of oscillations is by a smooth, flat, level approach and constant, slow speed in a straight line. The model uses this inherent variability to assist in determining the true total weight and individual axle weights of a vehicle. The weight distribution dynamics of a generic moving vehicle were simulated. The model estimation converged to within 1% of the true mass for simulated data. The computational demands of this method, while much greater than simple averages, took only seconds to run on a desktop computer.

Keywords: Weigh-in-Motion, WIM, LS-WIM, Markov, Multiple sensors.

---

[1] Current Address for Shane D. Boone: Non-Destructive Evaluation Research Program Manager, Infrastructure Management Team, Federal Highway Administration, Turner-Fairbank Highway Research Center, 6300 Georgetown Pike, McLean, VA 22101.

[2] The submitted manuscript has been authored by a contractor of the U.S. Government under contract DE-AC05-00OR22725. Accordingly, the U.S Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes.

1. Introduction

Previous work has been performed to estimate static axle weights from multiple sensor WIM systems. These works include multiple algorithms varying from Maximum Likelihood (Stergioulas et al. 2000) to Kalman Filtering techniques (Sainte-Marie et al. 1998). More recent developments include a new Statistical Spatial Repeatability (SSR) algorithm that removes bias due to pavement (O'Brien et al. 2010) and impact factors (O'Brien et al. 2008, 2009). To this end, an effort to perform probabilistic modeling on Weigh-in-Motion (WIM) Gen II sensor data was performed in order to reduce error in the static weight measurement due to vehicle oscillation. These errors arise from oscillations as a vehicle traverses the WIM system and are a result of true dynamic forces acting on the sensors. These dynamics occur, because a vehicle is (i) a multi-body system of discrete masses (e.g., body, load, wheels) that are (ii) interconnected by springs (e.g., cab-load coupling, wheel suspensions) and are (iii) excited by various aperiodic external forces (e.g., uneven terrain, steering changes, acceleration, wind variability, load shifts in liquids, engine vibration) with (iv) nonlinear damping by slip-stick friction and shock absorbers. Lower-frequency oscillations (1-5 Hz) arise from vehicle dynamics (e.g., side-to-side rocking, front-to-back rocking, vertical bouncing of the load on the suspension, load-bed flexure, twisting about coupling points, and nonlinear couplings among them). Higher-frequency oscillations (9-14 Hz) depend on vehicle size (e.g., tire rotation). Accurate weights require minimization of these oscillations, which are presently reduced via a combination of: (a) minimal excitations by a smooth, flat, level approach, weighing, and exit; (b) constant, slow speed driving in a straight line; (c) several single-axle weight measurements as the vehicle crosses multiple weigh pads; and (d) continuous motion to foster dynamic friction, which reduces the slip-stick friction (Abercrombie et al., 2008).

The work focused on modeling simulated data of a Ford F-250 driving slowly in a straight line across a portable low speed Gen II measurement system shown in Figure 1 (Abercrombie et. al., 2008). Rather than focusing on reconstructing the original signal, the emphasis was to estimate the hidden parameters of the vehicle dynamics. A simplified model of the moving vehicle was created. The relevant differential equation for the simplified model was:

$$m\ddot{x}_t + \gamma\dot{x}_t + kx_t = Ft \tag{1}$$

where x_t is the distance of the vehicle's center of mass below its equilibrium point at time t, $\dot{x}_t$ and $\ddot{x}_t$ are the first and second time derivatives of x_t, m is the mass of the vehicle, γ is the damping coefficient, k is the spring coefficient, and F_t is the external (broadly defined) force pushing down on the vehicle at time t. The observations are not directly of x_t, but rather the readout of a sensor that has its own dynamics and includes measurement error. We chose a further simplification: to subsume the sensor dynamics into the sensor measurement error.

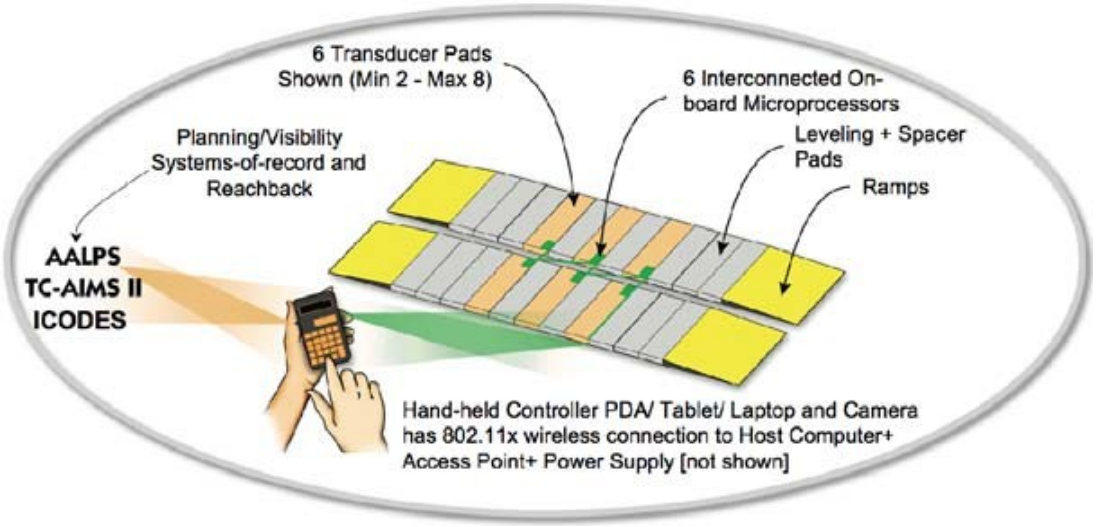

Figure 1. WIM Gen II System diagram showing control/data management pathways

2. Hidden Markov Model

A hidden Markov model is a probabilistic model of observables (sensor readings) given in terms of a hidden state (vehicle position) that is subject to particular transition probabilities. These transition probabilities are determined from the differential equations that define the dynamics of the vehicular system. The observations include error. Our hidden Markov model discretizes both position and time. It differs from a Kalman filter primarily in the method by which the parameters are estimated.

The hidden Markov model is based on a Markov chain of the state $X = \{x_t\}$ which transitions over time according to the differential equation. We allow an arbitrary forcing function to act on the vehicle, but model it as independent identically distributed (i.i.d.) normal random variables with mean zero and known variance. The observations, S, of the hidden Markov model come from the sensor data. Given sensor observations, the approach is as follows: (1) choose an initial value of the parameters P, (m,γ,k), (2) use P and S to find the expected values of the position at each time, (3) use S and X to find P that maximizes the likelihood of the S given X and P (4) go back to step 2 and iterate until P converges. This has been implemented and it is possible to show that better estimates can be derived from poor initial starts.

The position and derivative of the position at time t+1 can be written in terms of the velocity and acceleration at times t and t+1 (as follows from the definition of the time derivative where a small time difference Δ between samples is assumed).

$$x_{t+1} = x_t + \frac{\Delta}{2}\left(\dot{x}_t + \dot{x}_{t+1}\right) \tag{2}$$

$$\dot{x}_{t+1} = \dot{x}_t + \frac{\Delta}{2}\left(\ddot{x}_t + \ddot{x}_{t+1}\right) \tag{3}$$

The second derivative is determined by Newton's second law of motion using Equation 1 for the value of the acceleration:

$$\ddot{x}_t = \left(F_t + mg - kx_t - \gamma\dot{x}_t\right)/m \tag{4}$$

These equations can be used to write a relation between the x_t for three consecutive times, t. This allows for the development of the second derivative, which is necessary to include the force function and the parameters.

$$\left(1+\frac{\Delta\gamma}{2m}+\frac{\Delta^2 k}{4m}\right)x_{t+2}+\left(-2+2\frac{\Delta^2 k}{4m}\right)x_{t+1}+\left(1-\frac{\Delta\gamma}{2m}+\frac{\Delta^2 k}{4m}\right)x_t=\frac{\Delta^2}{4m}\overline{F}_{t+1}+\Delta^2 g \tag{5}$$

where

$$\overline{F}_{t+1}=\frac{F_t+2F_{t+1}+F_{t+2}}{4} \tag{6}$$

As a result, expected values for the sequence of positions and parameter estimates can be used to compute the smoothed forcing function $\overline{F}_t$. The log likelihood of that sequence is monotonically equivalent to the mean squared error of the sequence.

In estimating parameters of a hidden Markov model, it is also necessary to have a probability distribution of the observations given the hidden states. The sensor measurements are modeled as:

$$s_t = k\frac{x_t+x_{t+1}}{2}+\gamma\frac{x_{t+1}-x_t}{\Delta}+\varepsilon_t \tag{7}$$

where the ε_t are the sensor errors, which are modeled as independent and identically distributed (i.i.d.) normal with mean zero and known variance. (This is actually the expected sensor measurement for time t + ½. Using this offset simplifies the analysis.)

(Let $P = (m,\gamma,k)$ be the parameters, $X = \{x_t\}$ be the state, and $S = \{s_t\}$ be the sensor data.)

Now a key observation is that the F_t and ε_t error terms are linear in state, x_t, and also in the parameters m, γ, k after multiplying Equation (5) through by m. Therefore maximizing the likelihood of X given P or of P given X, because of the i.i.d. normality assumptions, is computationally a least squares regression. It is then possible to complete the following computations.

1. Given P and S, find the most likely state sequence X.
2. Given X and S, find the most likely parameters P.

In each case, the likelihood increases. Iterating between the two leads to a locally optimal solution.

3. Simulated Analysis, Experiment, and Results

The analysis used 100 simulated measurements taken over 1 second. The sensor and forcing errors were given standard deviations of 100N, which was used as the known variance in the fitting. The forcing function values were created randomly according to their distribution. These values were then used to update the state x_t according to the update relation, Equation (5). At each point, a sensor observation was simulated using the sensor equation, Equation (7), wherein the error term was randomly generated according to its distribution. The sensor measurements were then input into the hidden Markov model estimation algorithm. The estimation algorithm also requires a preliminary "guess" of the parameters, which were intentionally selected to be very different from the true values.

Figure 2 shows the fit of the estimated forcing function (in red) as compared to the true values (in blue). The forcing function red line essentially overlaps with the horizontal axis in Figure 2. It can be seen that the estimate of the forcing function is generally much smaller than the actual forcing function.

Figure 3 shows the first of the estimated sensor measurements (in red) as compared to the true values (in blue). The initial low estimates of the observations were due to an underestimation of the forcing function. The initial low valley in the forcing function, when underestimated, results in an overestimate of the sensor error, which in turn leads to the low line toward the beginning. Despite the imperfect estimation of the forcing function, the sensor estimation was generally fairly good.

More important than the estimations of position and sensor error are the estimates of the vehicle parameters. The true, initial guess, and final estimate parameters are shown in Table 1. The mass estimate is shown to be within 1% of the true mass. The spring coefficient was not as good, being too high by 33%. The main shortcoming is in the damping coefficient, which was estimated at zero. It seems that to better estimate these values, more advanced methods need to be employed, such as incorporating prior distributions on the parameters.

The model is currently being implemented into a modified Gen II WIM System.

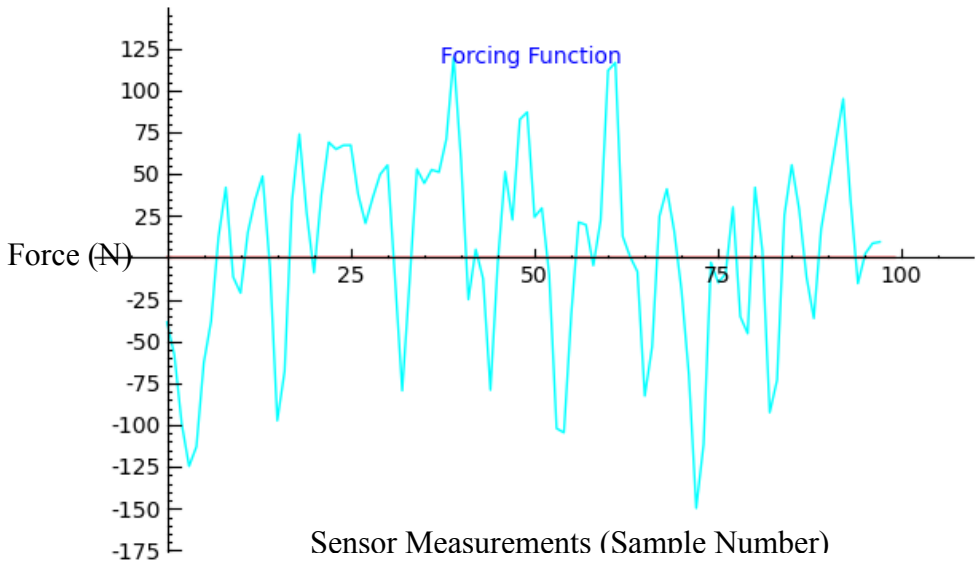

Figure 2. Simulated forcing function data and fitted data (red line overlaying horizontal axis)

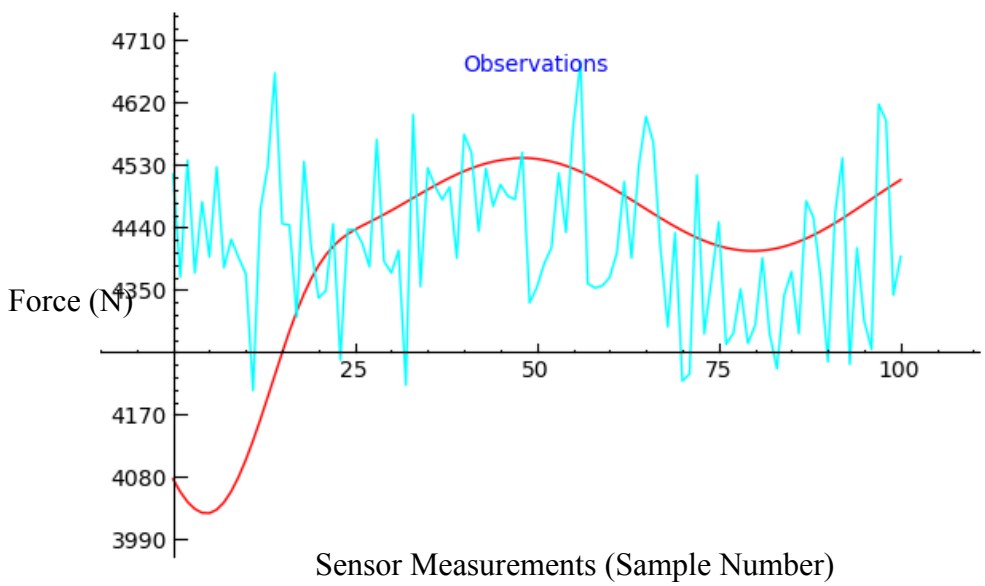

Figure 3. Estimated sensor data (red) superimposed on the simulated sensor data

Table 1. True, initial guess, and final estimate parameters (The initial guess was an arbitrary choice simply selected to be different from the true parameters)

	True	Initial	Final
Mass	450 kg	700 kg	454 kg
Damping Coefficient	7300 N/(m/s)	3000 N/(m/s)	0 N/(m/s)
Spring Coefficient	35000 N/m	70000 N/m	46563 N/m

4. Conclusion

A hidden Markov model was used on simulated data of a Ford F-250 driving slowly in a straight line across a portable low speed Gen II measurement system to assist in the weight measurement error that arises from complex vehicle oscillations of a system of discrete masses. The model estimation converged to within 1% of the true mass for simulated data. The computational demands of this method, while much greater than simple averages, took only seconds to run on a desktop computer. While our method appears to accurately estimate the mass coefficient, the effect of the spring and damping coefficient may be too subtle to estimate without longer runs of measurements. Recent research on sensor corrugated plates (Bin and Xinguo, 2010) may lend itself to the manufacture of larger sensors, thus allowing longer runs of measurements. Further research will investigate data requirements for these parameters and for parameters within more complex models of vehicle dynamics. Ongoing work is investigating extracting better estimates of uncertainty, and exploiting the learned spring coefficients associated with tires, as they may be useful for detecting under and over inflated tires. Additionally, ongoing work will apply this model to data taken from sensor measurements made in the field. Future goals include the implementation of this model into a next generation Weigh-In-Motion system with newly architected load cells. This work is ongoing and being funded under the US Army Small Business Innovation Research (SBIR) program. In cooperation with International Electronic Machine Corporation, the development of a Slow-speed Weigh-in-Motion Error Reduction System (SWIMERS) is being developed that will refine and implement the model discussed in this paper as well as algorithms and refinements used in the modified Gen II WIM System (Abercrombie et al. 2008).

5. Acknowledgements

The authors gratefully acknowledge funding from the U.S. Transportation Command, Department of the Army, and the ORNL Office of Technology Transfer. The authors would like to thank the reviewers for the excellent suggestions during the preparation of this paper.

6. References

- Abercrombie, R.K., Hively, L.M., Scudiere, M.B., Sheldon, F.T., (2008), "Sub-0.1 Percent Error in Portable, Low-Speed Weigh-In-Motion," in *Proceedings of the International Conference on Heavy Vehicles (5th International Conference on Weigh-*

in-Motion (ICWIM5) of Heavy Vehicles and 10th International Conference on Heavy Vehicle Transport Technology (HVTT 10), eds. B. Jacob, E. O'Brien, ISTE, Ltd. and John Wiley and Sons, Inc., Paris, France, pp. 447-458.

- Abercrombie, R.K., Beshears, D.L., Hively, L.M., Scudiere, M.B., Sheldon, F.T., Schmidhammer, J.L., Vanvactor, J. (2007), "Prototype Weigh-In-Motion Performance," Oak Ridge National Laboratory, ORNL/TM 2007/039.
- Abercrombie, R.K., Beshears, D.L., Scudiere, M.B., Coats, J.E., Jr., Sheldon, F.T., Brumbaugh, C.W., Hart, E., Mckay, R.J. (2005), "Weigh-In-Motion Research and Development Activities at ORNL," in *Proceedings of. 4th International Conference on Weigh-in-motion (ICWIM4)*, eds. B. Jacob, E. O'Brien, A. González, and C-p. Chou, Tapei, Taiwan National University Publications, 139-149.
- Bin, Ma, & Xinguo, Zou. (2010), "Study of Vehicle Weight-in-Motion System Based on Fiber-Optic Microbend Sensor," Paper presented at the *2010 International Conference on Intelligent Computation Technology and Automation (ICICTA)*, 458-461.
- Ferragut, E.M., and Abercrombie, R.K., (2010), "Weigh-in-Motion Estimation via Probabilistic Hidden Variable Modeling," *ORNL Invention Disclosure Number: 201002373, DOE S-115,419*.
- O'Brien, E. J., González, A., and McInerney, F., (2008) "A statistical Spatial Repeatability (SSR) Algorithm for Multiple Sensor Weigh-In-Motion," in *Proceedings of the International Conference on Heavy Vehicles (5th International Conference on Weigh-in-Motion (ICWIM5) of Heavy Vehicles and 10th International Conference on Heavy Vehicle Transport Technology (HVTT 10)*, eds. B. Jacob, E. O'Brien, ISTE, Ltd. and John Wiley and Sons, Inc., Paris, France, pp. 171-180.
- O'Brien, E.J., González, A. and McInerney, F. (2009), "A statistical Spatial Repeatability (SSR) Algorithm for Multiple Sensor Weigh-In-Motion (WIM*)," European Roads review (ERR)*, 14:32-38.
- O'Brien, E.J., Taheri, A. (2010), "The Influence of Statistical Spatial Repeatability on Remaining Pavement Life," in *Computational Stochastic Mechanics, Proceedings of the 6th International Conference (CSM-6)*, eds. G. Deodatis and P.D. Spanos, Rhodes, Greece.
- Scheuter, F. (1997), "Sources of Error in Dynamic Weight Measurement" in *Traffic Technology International Magazine*, pp. P1215-(E0-E4), (http://www.haenni-scales.ch/e/produkte/pdf/P1215_E.pdf, Last accessed, 14 October 2011).
- Sainte-Marie, J., Argoul, P., Jacob, B., Dolcemascolo, V. (1998), "Multiple Sensor WIM Using Reconstruction Algorithms of the Dynamic Axle Loads", in *2nd European Conference on Weigh-in-Motion of Road Vehicles*, Eds. E.J. O'Brien and B. Jacob, European Commission, 109-117.
- Stergioulas, L.K. Cebon, D. and Macleod, M.D., (2000) Static Weight Estimation and System Design for Multiple-Sensor Weigh-In-Motion *Proceedings of the Institution of Mechanical Engineers, C: Journal of Mechanical Engineering Science*, 214 (8). pp. 1019-1036.
- Scheuter F. (1998), "Evaluation of Factors Affecting WIM System Accuracy" in COST 323 Weigh in Motion of Road Vehicles, *Proc. 2nd European Conf. on WIM*, Lisbon, Sept. 1998, (http://www.haenni-scales.ch/e/produkte/pdf/P1216_E.pdf, Last accessed 14 October 2011).