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DETERMINING IMPORTANCE WEIGHTING FUNCTIONS FOR CONTRIBUTON THEORY EIGENVALUE SENSITIVITY COEFFICIENT METHODOLOGIES*

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ABSTRACT

This study introduced two new approaches for calculating the $F^*(r)$ importance weighting function for Contributon and CLUTCH eigenvalue sensitivity coefficient calculations, and compared them in terms of accuracy and applicability. The necessary levels of $F^*(r)$ mesh refinement and mesh convergence for obtaining accurate eigenvalue sensitivity coefficients were determined through two parametric studies, and the results of these studies suggest that a sufficiently accurate $F^*(r)$ mesh for calculating eigenvalue sensitivity coefficients can be obtained for the Contributon and CLUTCH methods with only a small increase in problem runtime.

Key Words: Eigenvalue sensitivity coefficients, TSUNAMI, Monte Carlo, SCALE, Shift, CLUTCH, continuous-energy.

1. INTRODUCTION

Recent work on the Contributon and CLUTCH methods for calculating eigenvalue sensitivity coefficients in the Shift Monte Carlo code has shown promising results [1]. These two Contributon Theory–based sensitivity coefficient methodologies require the use of an importance weighting function, known as $F^*(r)$, when calculating sensitivity coefficients. This study presents several methods for calculating the $F^*(r)$ function during a Monte Carlo simulation, and also quantifies the level of accuracy needed in $F^*(r)$ to obtain accurate eigenvalue sensitivity results.

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The Contributon and CLUTCH (**C**ontributon-**L**inked eigenvalue sensitivity/**U**ncertainty estimation via **T**racklength importance **Ch**aracterization) methods for calculating eigenvalue sensitivity coefficients determine the importance of neutron collisions in Monte Carlo simulations by tallying the number of fission neutrons that are created by the incident neutron after it leaves each collision [1]. However, tallying the future fission neutron production of the incident neutron is not sufficient, and these tallies must be weighted by the $F^*(r)$ function in order to take into account the future multiplication of daughter fission neutrons. Consider a neutron source Q that is equal to the fission source of a system

$$Q = \lambda F \phi, \tag{1}$$

where F is the fission operator. Multiplying this source by the adjoint flux and integrating over phase space gives

$$\langle \phi^* Q \rangle = \lambda \langle \phi^* F \phi \rangle. \tag{2}$$

Consider now a neutron emitted in phase space τ_s, such that $Q = Q_s\, \delta(\tau - \tau_s)$. This source definition reduces Eq. (2) and allows the importance of the neutron in phase space τ_s to be calculated by

$$\phi^*(\tau_s) = \frac{\lambda}{Q_s} \int_V G(\tau_s \to r) F^*(r)\, dr, \tag{3}$$

where the transfer function $G(\tau_s \to r)$ is equal to the expected number of fission neutrons generated in all energies and directions at r due to a source neutron emitted at phase space τ_s and is given by

$$G(\tau_s \to r) = \frac{1}{Q_s(\tau_s)} \int_E \int_\Omega \nu \Sigma_f(r, E) \phi(\, r, E, \Omega \mid Q(\tau_s)\,) d\Omega dE, \tag{4}$$

and $\phi(\, r, E, \Omega \mid Q(\tau_s)\,)$ is the flux created in phase space (r, E, Ω) due to the source $Q(\tau_s)$. The weighting function $F^*(r)$ is defined to equal the expected importance generated by a fission neutron emitted at r and is given by

$$F^*(r) = \int_E \int_\Omega \frac{\chi(r, E)}{4\pi}\, \phi^*(r, E, \Omega) d\Omega dE. \tag{5}$$

Although the Contributon Method determines tallies $G(\tau_s \to r)$ by simulating secondary particles and the CLUTCH Method tallies $G(\tau_s \to r)$ by instead cleverly scrutinizing the track-lengths of a particle, both methods require the $F^*(r)$ weighting function when tallying $G(\tau_s \to$

r) and calculating a neutron's importance. Currently, the TSUNAMI-3D code within the SCALE code package has the capability to calculate eigenvalue sensitivity coefficients using the Contributon Method with the assumption that $F^*(r)$ = 1 everywhere, but it is desirable to implement a more rigorous $F^*(r)$ methodology as the next generation of eigenvalue sensitivity coefficient calculations is developed within SCALE [2]. This study presents three approaches for calculating $F^*(r)$ and examines the accuracy and efficiency of each method; the approaches investigated include assuming that $F^*(r)$ = 1 everywhere, calculating $F^*(r)$ with the Differential Operator Method, and calculating $F^*(r)$ with the Iterated Fission Probability Method. For the Differential Operator and Iterated Fission Probability methods, the $F^*(r)$ will be calculated and stored on a spatial mesh. This study will examine the effect of the $F^*(r)$ mesh resolution and accuracy on the accuracy of sensitivity coefficient calculations.

2. APPROACHES FOR CALCULATING $F^*(r)$

$F^*(r)$ is defined as the expected importance that is generated by a fission neutron born at location r. As shown in Eq. (5), $F^*(r)$ describes the importance of a neutron only as a function of space and does not require knowledge of any energy or angular dependence. The first approach for calculating $F^*(r)$ simply assumes that $F^*(r)$ is uniformly equal to 1 everywhere; this implies that the neutron importance in Eq. (3) can be determined simply by tallying the number of fission neutrons that are created after an interaction of interest, and that the future multiplication of these fission neutrons does not affect the importance of the original neutron.

The two new methods for calculating $F^*(r)$ both take advantage of the definition of the unconstrained fission neutron emission energy, or χ(E), sensitivity. The unconstrained χ(E) sensitivity is defined, as a function of space, as [3]

$$S_{k,\chi}(r) = \frac{1}{k}\int_0^\infty \int_{4\pi} \bar{\upsilon}\Sigma_f(r,E)\phi(r,E,\Omega)d\Omega dE \int_0^\infty \int_{4\pi} \frac{\chi(r,E')}{4\pi}\ \phi^*(r,E',\Omega')d\Omega' dE'. \tag{6}$$

Because the right-most integral term in the above equation equals $F^*(r)$, Eq. 6 can be rewritten to define $F^*(r)$ as

$$F^*(r) = \frac{S_{k,\chi}(r)}{\int_0^\infty \int_{4\pi} \frac{\bar{\upsilon}\Sigma_f(r,E)\phi(r,E,\Omega)}{k} d\Omega dE}. \tag{7}$$

It should be noted that the term in the denominator for Eq. (7) can be represented by the total weight of fission neutrons that are born at location r. The two new methods for calculating $F^*(r)$ both involve tallying the spatially-dependent fission spectrum sensitivity, as described in Eq. (6), over a spatial mesh and dividing by the total weight of fission neutrons born within that interval. The first of these methods uses the Differential Operator Methodology to calculate chi sensitivities, while the second uses Iterated Fission Probability Methodology [4].

Because $F^*(r)$ is defined as the importance created **per source neutron born at *r***, tallying $F^*(r)$ does not require that the neutron flux is converged to give accurate results, and thus tallies for $F^*(r)$ can begin during inactive neutron generations. It should also be noted that constant multipliers or normalization factors for $F^*(r)$ do not affect the sensitivity coefficients generated by Contributon-based methodologies. These factors ultimately cancel because the $F^*(r)$ function is used to weight terms in both the numerator and denominator of the first-order perturbation equation [3].

3. RESULTS

The Contributon and CLUTCH methods for calculating eigenvalue sensitivity coefficients were implemented into the Shift Monte Carlo code within the Denovo framework of the SCALE code package, and sensitivity coefficients were obtained for two simple test problems using the three approaches for calculating $F^*(r)$. The sensitivity coefficients presented here were calculated using only the CLUTCH method and not the Contributon Method because the two methods are theoretically equivalent and the Contributon Method results in significantly longer runtimes. The test problems examined were a light water reactor (LWR) fuel pin and the Godiva assembly, and both models used reflective boundaries to take advantage of the symmetry present in the systems. Because the Shift continuous-energy physics package is still under development, the sensitivity coefficient Monte Carlo calculations were performed using multigroup neutron energy bins. The multigroup energy structure is given in Tables I and II for the LWR fuel pin and Godiva models, respectively. Neutron cross sections for these energy groups were obtained using 2-D models of the corresponding systems in the TRITON code [2]. Reference sensitivity coefficients were obtained for the two systems through direct perturbations of neutron cross sections.

Table I. LWR fuel pin model neutron energy group structure

Group Number	Energy Range
1	>1.0 MeV
2	500 keV–1.0 MeV
3	3.0 eV–500 keV
4	0.625 eV–3.0 eV
5	0.1 eV–0.625 eV
6	0.02 eV–0.1 eV
7	< 0.02 eV

Table II. Godiva model neutron energy group structure

Group Number	Energy Range
1	>3.0 MeV
2	1.5 MeV–3.0 MeV
3	1.0 MeV–1.5 MeV
4	500 keV– 1.0 MeV
5	100 keV–500 keV
6	< 100 keV

3.1. Necessity of the $F^*(r)$ Mesh

Sensitivity coefficients were first obtained for the two systems using the assumption that $F^*(r)$ = 1 everywhere in order to quantify the level of accuracy that can be obtained for sensitivity coefficient calculations with no $F^*(r)$ mesh. Because calculating the $F^*(r)$ mesh can consume a significant amount of runtime, it would be advantageous for the methods, most especially the CLUTCH method, to produce accurate sensitivity coefficients without an $F^*(r)$ mesh. The difference between the calculated and reference sensitivity coefficients is given in Tables III and IV for the fuel and moderator regions of the LWR fuel pin, respectively, and in Table V for the Godiva model. The differences given in the Tables are in terms of standard deviations obtained by dividing the difference between the sensitivity coefficients by the effective standard deviation for each coefficient. χ(E) sensitivity coefficients are given for only the first three energy groups of the LWR fuel pin model because χ(E) equals zero for groups four through seven.

Table III. Difference between the calculated and reference fuel region sensitivity coefficients for the LWR fuel pin model for the case when $F^*(r) = 1$

Group Number	Σ_{cap}	Σ_s	Σ_f	$\bar{\nu}$	$\chi(E)$
1	-1.498	1.461	-0.200	1.468	1.156
2	-0.081	-1.029	-0.333	0.105	-0.134
3	0.047	0.864	-1.436	1.998	-0.313
4	0.795	-0.875	1.391	-0.680	---
5	0.803	-0.520	-0.200	0.426	---
6	0.123	-1.105	-1.191	-1.280	---
7	0.945	0.692	0.054	-1.387	---

Table IV. Difference between the calculated and reference moderator region sensitivity coefficients for the LWR fuel pin model for the case when $F^*(r) = 1$

Group Number	Σ_{cap}	Σ_s
1	-1.498	-1.427
2	-0.081	0.655
3	0.047	-0.316
4	0.795	-0.481
5	0.803	-0.746
6	0.123	0.229
7	0.945	-0.140

Table V. Difference between the calculated and reference sensitivity coefficients for the Godiva model for the case when $F^*(r) = 1$

Group Number	Σ_{cap}	Σ_s	Σ_f	$\bar{\nu}$	$\chi(E)$
1	0.984	6.718	1.624	-0.083	-1.317
2	-1.223	5.889	-0.143	-0.295	0.473
3	-0.828	2.428	0.008	-0.825	-0.748
4	1.261	4.765	1.539	2.040	-0.549
5	0.310	7.369	-0.032	1.451	0.310
6	2.210	0.193	-0.836	0.812	0.905

As shown in Tables III and IV, the CLUTCH LWR fuel pin sensitivity coefficients showed very good agreement with the reference sensitivity coefficients. It is expected that about 68% of the sensitivity coefficients will fall within one standard deviation of the reference sensitivities, and 69% of the CLUTCH sensitivity coefficients were within one standard deviation of the reference values. None of the CLUTCH sensitivity coefficients are more than two standard deviations from the reference sensitivity coefficients. This excellent agreement suggests that accurate sensitivity coefficients can be calculated using the assumption that $F^*(r) = 1$ for the LWR fuel pin model. This conclusion is intuitively reasonable because the forward and adjoint spatial flux profiles are very flat across the small, infinitely-reflected fuel pin model.

Although many of the calculated Godiva model sensitivity coefficients agreed well with the reference values, overall agreement was not as good as for the LWR fuel pin. Only 57% of the

calculated Godiva sensitivity coefficients were within one standard deviation of the reference values, and several of the sensitivity coefficients deviated from the reference values by more than six standard deviations. The high neutron leakage and large flux gradient in the Godiva model result in a forward flux, adjoint flux, and $F^*(r)$ function that vary greatly as a function of space. Thus the assumption that $F^*(r)$ = 1 is not valid for sensitivity coefficient calculations for the Godiva model.

3.2. Determining Sufficient $F^*(r)$ Mesh Resolution

The necessary spatial refinement for the $F^*(r)$ mesh was next determined by varying the number of mesh intervals and evaluating the effect on the Godiva model sensitivity coefficients. This study was restricted to the Godiva model because the assumption that $F^*(r)$ = 1 was found to produce accurate sensitivity coefficients for the LWR fuel pin model. Table VI gives the difference in number of standard deviations between the calculated and reference sensitivity coefficients as a function of mesh refinement and $F^*(r)$ calculation method. In the table, "DO" represents Differential Operator and "IFP" represents Iterated Fission Probability. Note that "3×3×3" represents a mesh that has three mesh intervals in the x-, y-, and z-dimensions. The model of the Godiva system used for these calculations was a 1/8th model due to x-, y-, and z-symmetry. Table VI also gives the average root mean square (RMS) difference between the calculated and reference sensitivity coefficients in terms of the number of standard deviations. The average RMS difference was used to evaluate the accuracy of a mesh refinement in a single metric, and by definition of the standard deviation, the average RMS difference of the standard deviations should equal 1 if the reference and calculated sensitivities are converging to the same values.

The number of inactive neutron generations was adjusted for each $F^*(r)$ mesh refinement case such that the average relative uncertainty for the $F^*(r)$ mesh intervals was uniform across each mesh refinement case. Although it will not be presented here due to space limitations, the median relative uncertainty for the $F^*(r)$ mesh intervals was slightly over 2%. Based on this $F^*(r)$ mesh value uncertainty and the behavior of the 50×50×50 mesh results, it can be inferred that the uncertainty for the average RMS difference of the results in Table VI is also around 2%.

Table VI. Difference between the calculated and reference sensitivity coefficients for the Godiva model as a function of $F^*(r)$ method and mesh refinement

Sensitivity Coefficient	DO 3×3×3	IFP 3×3×3	DO 10×10×10	IFP 10×10×10	DO 25×25×25	IFP 25×25×25	DO 50×50×50	IFP 50×50×50
Σ_{cap_1}	0.970	0.970	0.967	0.967	0.968	0.967	0.967	0.967
Σ_{cap_2}	-1.272	-1.271	-1.281	-1.282	-1.286	-1.288	-1.281	-1.283
Σ_{cap_3}	-0.893	-0.889	-0.897	-0.899	-0.906	-0.908	-0.903	-0.906
Σ_{cap_4}	1.164	1.176	1.154	1.151	1.159	1.155	1.153	1.149
Σ_{cap_5}	0.291	0.300	0.296	0.296	0.300	0.300	0.296	0.296
Σ_{cap_6}	2.275	2.265	2.284	2.285	2.290	2.293	2.288	2.291

Sensitivity Coefficient	DO 3×3×3	IFP 3×3×3	DO 10×10×10	IFP 10×10×10	DO 25×25×25	IFP 25×25×25	DO 50×50×50	IFP 50×50×50
$\boldsymbol{\Sigma_{s_1}}$	2.002	2.370	1.342	1.215	1.381	1.212	1.535	1.347
$\boldsymbol{\Sigma_{s_2}}$	1.862	2.173	1.342	1.234	1.327	1.185	1.402	1.231
$\boldsymbol{\Sigma_{s_3}}$	-0.538	-0.344	-0.892	-0.976	-0.905	-1.004	-1.042	-1.166
$\boldsymbol{\Sigma_{s_4}}$	0.375	0.600	-0.310	-0.434	-0.457	-0.625	-0.381	-0.564
$\boldsymbol{\Sigma_{s_5}}$	1.721	1.962	0.810	0.656	0.773	0.579	0.871	0.641
$\boldsymbol{\Sigma_{s_6}}$	-0.507	-0.540	-0.608	-0.631	-0.688	-0.719	-0.645	-0.674
$\boldsymbol{\Sigma_{f_1}}$	0.849	0.975	0.984	0.963	0.572	0.541	0.943	0.912
$\boldsymbol{\Sigma_{f_2}}$	-0.989	-0.795	-1.043	-1.064	-1.095	-1.124	-1.108	-1.143
$\boldsymbol{\Sigma_{f_3}}$	-0.640	-0.582	-0.812	-0.824	-0.567	-0.583	-0.699	-0.721
$\boldsymbol{\Sigma_{f_4}}$	1.103	1.023	0.933	0.918	0.800	0.780	0.881	0.858
$\boldsymbol{\Sigma_{f_5}}$	-0.462	-0.564	-0.633	-0.649	-0.469	-0.483	-0.654	-0.676
$\boldsymbol{\Sigma_{f_6}}$	-0.864	-0.877	-0.856	-0.856	-0.741	-0.740	-0.835	-0.834
$\boldsymbol{\bar{\nu}_1}$	0.054	0.159	0.352	0.355	-0.113	-0.113	0.309	0.313
$\boldsymbol{\bar{\nu}_2}$	-0.240	-0.064	-0.122	-0.115	-0.075	-0.068	-0.191	-0.183
$\boldsymbol{\bar{\nu}_3}$	-0.727	-0.709	-0.882	-0.871	-0.440	-0.427	-0.652	-0.639
$\boldsymbol{\bar{\nu}_4}$	2.360	2.187	2.272	2.277	2.099	2.106	2.224	2.232
$\boldsymbol{\bar{\nu}_5}$	1.110	0.966	0.919	0.901	1.062	1.045	0.893	0.872
$\boldsymbol{\bar{\nu}_6}$	0.639	0.654	0.627	0.623	0.700	0.695	0.633	0.627
$\boldsymbol{\chi(E_1)}$	-1.043	-0.930	-0.679	-0.672	-1.028	-1.024	-0.690	-0.683
$\boldsymbol{\chi(E_2)}$	0.621	0.776	0.804	0.818	1.059	1.079	0.973	0.992
$\boldsymbol{\chi(E_3)}$	-0.401	-0.355	-0.446	-0.437	-0.248	-0.234	-0.402	-0.388
$\boldsymbol{\chi(E_4)}$	-0.515	-0.654	-0.643	-0.645	-0.728	-0.730	-0.781	-0.783
$\boldsymbol{\chi(E_5)}$	-0.233	-0.415	-0.473	-0.493	-0.572	-0.598	-0.564	-0.594
$\boldsymbol{\chi(E_6)}$	0.743	0.784	0.704	0.700	0.748	0.742	0.744	0.739
Average RMS Difference	1.091	1.132	1.000	0.991	0.983	0.973	1.017	1.004

An ideally-refined $F^*(r)$ should give sensitivity coefficients that are mostly within one standard deviation of the reference coefficients, should produce very few coefficients that are greater than 2 standard deviations from the reference values, and should create an average RMS difference in the coefficients of 1.0. Based on the results in Table VI, a 10×10×10 mesh was determined to be the minimum mesh refinement necessary to produce accurate sensitivity coefficients. This refinement corresponds to a mesh interval size of 0.8741 cm, or slightly less than 1 cm. Although the 3×3×3 mesh did not produce poor results, the 10×10×10 mesh resulted in a noticeably lower average RMS difference for the sensitivity coefficients and several fewer

sensitivities that were more than two standard deviations away from the reference values. These results were not significantly improved by moving from a 10×10×10 mesh to a more-resolved mesh. The mesh resolution with the lowest number of sensitivity coefficients within one standard deviation of the reference values was, in fact, the IFP 25×25×25 mesh (with 63% of the sensitivities being within one standard deviation of the reference values).

Compared to the case with no $F^*(r)$ mesh, the 3×3×3 mesh increased the average runtime for each active generation by 9.87%, the 10×10×10 mesh by 13.74%, and the 50×50×50 mesh by 33.85%. Although a 10% increase in problem runtime is significant, it is nowhere near the factor of 300 increase in runtime seen by using the Contributon Method, or the more than 100% increase required for the conventional TSUNAMI Method [3]. Furthermore, this 10% runtime increase will be small compared to continuous-energy cross-section lookups once the continuous-energy physics package has been finished in Shift.

Although the Differential Operator method is considered an "approximate" method for calculating eigenvalue sensitivity coefficients, it performed well compared to the "exact" Iterated Fission Probability method. In fact, the Differential Operator's average RMS differences are generally closer to 1 than those from the Iterated Fission Probability method. This "approximate" method's excellent performance may arise because $F^*(r)$ is only calculated as a function of space and any shortcomings of the Differential Operator method for calculating energy- or angular-dependent sensitivity coefficients are nullified.

3.3. Uncertainty in the $F^*(r)$ Mesh Values

The acceptable amount of uncertainty in the $F^*(r)$ mesh intervals was determined by varying the number of inactive generations over which the mesh was tallied and by examining its effect on the difference between the reference and calculated sensitivity coefficient. $F^*(r)$ mesh values were calculated using 50, 200, 2000, 10,000, and 50,000 inactive generations, each of which simulated 5000 neutron histories. These results are presented in Table VII as a function of $F^*(r)$ calculation methodology and the number of inactive generations. Again, "DO" represents the Differential Operator Method, and "IFP" represents the Iterated Fission Probability Method. A 10×10×10 mesh was used for all of the calculations presented in Table VII.

Table VII. Difference in number of standard deviations between the calculated and reference sensitivity coefficients for the Godiva model as a function of the number of inactive generations used for calculating $F^*(r)$

Sensitivity Coefficient	DO 50	IFP 50	DO 200	IFP 200	DO 2000	IFP 2000	DO 10,000	IFP 10,000	DO 50,000	IFP 50,000
Σ_{cap_1}	0.968	0.968	0.969	0.969	0.968	0.967	0.967	0.968	0.968	0.967
Σ_{cap_2}	-1.280	-1.279	-1.283	-1.283	-1.282	-1.284	-1.283	-1.281	-1.279	-1.281
Σ_{cap_3}	-0.905	-0.904	-0.902	-0.902	-0.902	-0.905	-0.905	-0.902	-0.901	-0.904
Σ_{cap_4}	1.157	1.158	1.156	1.157	1.158	1.154	1.147	1.151	1.156	1.152

Sensitivity Coefficient	DO 50	IFP 50	DO 200	IFP 200	DO 2000	IFP 2000	DO 10,000	IFP 10,000	DO 50,000	IFP 50,000
Σ_{cap_5}	0.286	0.284	0.301	0.299	0.307	0.307	0.304	0.303	0.297	0.297
Σ_{cap_6}	2.292	2.292	2.281	2.281	2.279	2.282	2.286	2.284	2.286	2.289
Σ_{s_1}	1.388	1.443	1.441	1.468	1.517	1.331	1.198	1.377	1.699	1.502
Σ_{s_2}	1.427	1.432	1.411	1.424	1.398	1.237	1.385	1.535	1.534	1.367
Σ_{s_3}	-0.905	-0.874	-0.831	-0.831	-0.990	-1.111	-0.851	-0.740	-0.945	-1.067
Σ_{s_4}	-0.413	-0.392	-0.248	-0.241	-0.286	-0.465	-0.533	-0.363	-0.235	-0.420
Σ_{s_5}	0.916	0.885	1.053	1.052	0.891	0.670	0.841	1.046	1.058	0.826
Σ_{s_6}	-0.782	-0.767	-0.654	-0.661	-0.657	-0.685	-0.693	-0.665	-0.616	-0.646
Σ_{f_1}	0.867	0.872	0.782	0.789	0.800	0.770	0.846	0.876	0.969	0.937
Σ_{f_2}	-0.967	-0.962	-1.063	-1.068	-1.124	-1.155	-1.117	-1.087	-1.079	-1.113
Σ_{f_3}	-0.706	-0.709	-0.597	-0.596	-0.637	-0.659	-0.627	-0.608	-0.681	-0.702
Σ_{f_4}	0.909	0.921	0.873	0.876	0.908	0.885	0.763	0.786	0.902	0.877
Σ_{f_5}	-0.704	-0.711	-0.582	-0.587	-0.463	-0.484	-0.574	-0.556	-0.639	-0.662
Σ_{f_6}	-0.853	-0.848	-0.786	-0.786	-0.864	-0.863	-0.850	-0.851	-0.837	-0.835
$\bar{\nu}_1$	0.212	0.207	0.025	0.028	0.128	0.133	0.210	0.207	0.306	0.310
$\bar{\nu}_2$	-0.070	-0.071	-0.092	-0.100	-0.184	-0.174	-0.149	-0.159	-0.195	-0.186
$\bar{\nu}_3$	-0.633	-0.651	-0.538	-0.535	-0.590	-0.577	-0.531	-0.544	-0.659	-0.646
$\bar{\nu}_4$	2.220	2.226	2.190	2.190	2.211	2.218	2.153	2.148	2.222	2.228
$\bar{\nu}_5$	0.892	0.898	0.945	0.946	1.032	1.010	0.937	0.958	0.904	0.882
$\bar{\nu}_6$	0.609	0.614	0.687	0.688	0.631	0.625	0.625	0.631	0.635	0.629
$\chi(E_1)$	-0.860	-0.849	-1.173	-1.179	-0.921	-0.917	-0.909	-0.913	-0.696	-0.687
$\chi(E_2)$	0.791	0.785	1.061	1.064	0.986	1.008	0.951	0.929	0.955	0.975
$\chi(E_3)$	-0.293	-0.301	-0.351	-0.351	-0.366	-0.351	-0.178	-0.192	-0.413	-0.399
$\chi(E_4)$	-0.599	-0.595	-0.603	-0.607	-0.648	-0.650	-0.695	-0.693	-0.779	-0.782
$\chi(E_5)$	-0.529	-0.526	-0.484	-0.480	-0.543	-0.572	-0.651	-0.624	-0.541	-0.571
$\chi(E_6)$	0.722	0.721	0.756	0.756	0.735	0.730	0.727	0.731	0.749	0.744
Average RMS Difference	0.999	0.999	1.004	1.005	1.005	0.998	0.992	0.999	1.016	1.007

The average and median uncertainties for the $F^*(r)$ mesh intervals are presented in Table VIII for the Differential Operator–calculated $F^*(r)$ meshes and in Table IX for the Iterated Fission Probability–calculated meshes. Although the Iterated Fission Probability Method is generally considered to be a less-efficient method for calculating sensitivity coefficients, the uncertainties for the method presented in Table IX are very similar in magnitude to the Differential Operator

uncertainties in Table VIII. It is expected that the average and median mesh uncertainties will decrease proportionally in response to the square root of an increase in the number of particles simulated. This behavior is clearly seen for the median mesh uncertainties but not for the average mesh uncertainties, likely because the average mesh uncertainty is biased by meshes that received a very small number of tallies and thus have large uncertainties.

The results presented in Table VII suggest that simulating only 50 inactive generations, or an average of 250 source particles per mesh interval, will produce a sufficiently converged $F^*(r)$ mesh for performing sensitivity coefficient calculations. This is surprising given the greater than 25% uncertainty present in the $F^*(r)$ mesh values after simulating only 50 inactive generations. These results show promise for the CLUTCH and Contributon Methods because 50 generations are lower than the 200 generations that are typically necessary for fission source convergence in Monte Carlo simulations; this suggests that $F^*(r)$ can potentially be calculated without simulating any additional inactive generations.

Table VIII. Differential Operator $F^*(r)$ Mesh relative uncertainty as a function of number of inactive generations

Number of Inactive Generations	50	200	2000	10,000	50,000
Average Mesh Uncertainty	26.70%	16.09%	7.85%	4.77%	2.94%
Median Mesh Uncertainty	19.30%	10.04%	3.24%	1.48%	0.66%

Table IX. Iterated Fission Probability $F^*(r)$ Mesh relative uncertainty as a function of number of inactive generations

Number of Inactive Generations	50	200	2000	10,000	50,000
Average Mesh Uncertainty	28.98%	16.10%	6.23%	3.79%	2.24%
Median Mesh Uncertainty	22.44%	10.37%	3.27%	1.47%	0.67%

4. CONCLUSIONS

This study introduced two new approaches for calculating the $F^*(r)$ function for Contributon and CLUTCH eigenvalue sensitivity coefficient calculations. It was found that although the assumption that $F^*(r) = 1$ is well suited for some applications, the Differential Operator and Iterated Fission Probability methods can both be used to accurately calculate $F^*(r)$ during inactive neutron generations. A mesh refinement of slightly less than 1 cm was found necessary for producing accurate eigenvalue sensitivity coefficients for a 3-D, six-group model of the Godiva system and sufficiently converged $F^*(r)$ values were obtained after only 50 neutron generations. These results suggest that a sufficiently accurate $F^*(r)$ mesh for calculating eigenvalue sensitivity coefficients can be obtained for the Contributon and CLUTCH methods

with only a small increase in problem runtime. In addition to the Differential Operator and Iterated Fission Probability methods, future studies on calculating $F^*(r)$ will likely include an approach where $F^*(r)$ is calculated during a deterministic Denovo calculation before beginning the Monte Carlo simulation, and also a mesh-free approach where $F^*(r)$ is calculated and described using kernel density estimation functions.

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