Scaling Analysis for the Direct Reactor Auxiliary Cooling System for AHTRs

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INTRODUCTION

The Direct Reactor Auxiliary Cooling System (DRACS), shown in Fig. 1 [1], is a passive heat removal system proposed for the Advanced High-Temperature Reactor (AHTR). It features three coupled natural circulation/convection loops completely relying on the buoyancy as the driving force. A prototypic design of the DRACS employed in a 20-MWth AHTR has been discussed in our previous work [2]. The total height of the DRACS is usually more than 10 m, and the required heating power will be large (on the order of 200 kW), both of which make a full-scale experiment not feasible in our laboratory. This therefore motivates us to perform a scaling analysis for the DRACS to obtain a scaled-down model. In this paper, theory and methodology for such a scaling analysis are presented.

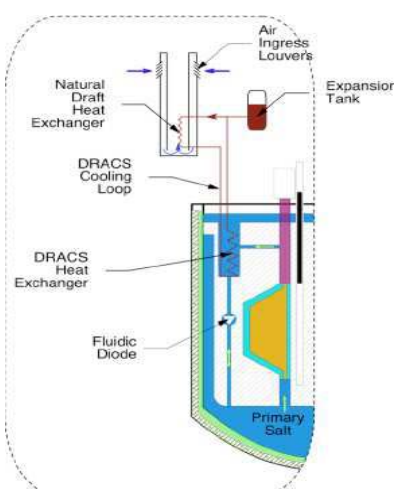

Fig. 1. Schematic of the DRACS [1]

SIMILARITY CRITERIA

In this analysis, the Boussinesq approximation is applied, which states that the fluid is considered incompressible except the buoyancy term in the momentum equation. The one-dimensional governing equations for the DRACS system are given as [3]:

Continuity equation:

$$u_i a_i = u_r a_0 \tag{1}$$

Integral momentum equation:

$$\rho \frac{\mathrm{d}u_r}{\mathrm{d}t} \sum_i \left(\frac{a_0}{a_i} l_i \right) = g\rho \oint \beta (T - T_0) \mathrm{d}z - \frac{\rho u_r^2}{2} \sum_i \left(\frac{fl}{d} + K \right)_i \left(\frac{a_0}{a_i} \right)^2 \tag{2}$$

Fluid energy equation for the ith section:

$$\rho c_p \left\{ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right\} = \frac{4h}{d} (T_s - T_i) \tag{3}$$

Solid energy equation for the ith section:

$$\rho_s c_{ps} \frac{\partial T_s}{\partial t} - k_s \nabla^2 T_s - \dot{q}_s = 0 \tag{4}$$

The boundary condition between the fluid and structure for the ith section is given as:

$$-k_s \frac{\partial T_s}{\partial y} = h (T_s - T_i). \tag{5}$$

Here, subscripts i, r, 0, and s are the ith component, representative variable, reference constant value, and solid structure, respectively. u, a, ρ, t, l, β, T, g, f, K, d, c_p, h, k, and $\dot{q}$ are the velocity, flow area, density, time, axial length, thermal expansion coefficient, temperature, gravitational acceleration, friction factor, form loss factor, hydraulic diameter, specific heat capacity, heat transfer coefficient, thermal conductivity, and volumetric heat generation rate, respectively. The transverse and axial coordinates are denoted by y and z, respectively.

The steady-state fluid velocity and temperature in each loop are chosen to define the dimensionless parameters, as summarized in Table 1. Here, the hydraulic diameter d and conduction depth δ are respectively

defined as $d = 4a/\xi$ and $\delta = a_s/\xi$, where ξ and a_s are the wetted perimeter and cross-sectional area of the structure, respectively.

Table 1 Definitions of dimensionless parameters

	Reference values	Dimensionless parameters
Velocity	u_0 : steady-state salt velocity in the pipe	$U_i = u_i / u_0$ $U_r = u_r / u_0$
Length	l_0 : vertical distance between thermal centers of DHX & core in primary loop or that between thermal centers of NDHX & DHX in secondary loop	$L_i = l_i / l_0$ $Z = z / l_0$
	δ : conduction depth	$\nabla^{*2} = \delta^2 \nabla^2$ $Y = y / \delta$
Area	a_0 : cross-sectional area of the pipe	$A_i = a_i / a_0$
Time	l_0 / u_0	$\tau = t u_0 / l_0$
Temperature	ΔT_0	$\theta = (T - T_0) / \Delta T_0$

The non-dimensional governing equations are obtained by introducing the dimensionless parameters in Table 1 into Eqs. (1)-(5) as:

$$U_i A_i = U_r, \tag{6}$$

$$\frac{\mathrm{d}U_r}{\mathrm{d}\tau}\left(\sum_i \frac{L_i}{A_i}\right) = \mathrm{R}\oint \theta \mathrm{d}Z - \frac{U_r^2}{2}\left(\sum_i \frac{\mathrm{F}_i}{A_i^2}\right), \tag{7}$$

$$\frac{\partial \theta_i}{\partial \tau} + \frac{U_r}{A_i}\frac{\partial \theta_i}{\partial Z} = \mathrm{St}_i(\theta_{si} - \theta_i), \tag{8}$$

$$\frac{\partial \theta_{si}}{\partial \tau} - \mathrm{T}_i^* \nabla_i^{*2} \theta_{si} - \mathrm{Q}_{si} = 0, \tag{9}$$

$$\frac{\partial \theta_{si}}{\partial Y_i} = \mathrm{B}_i\left(\theta_i - \theta_{si}\right). \tag{10}$$

Here, the dimensionless numbers are defined as:

Richardson number: $\mathrm{R} \equiv \dfrac{\beta g \Delta T_0 l_0}{u_0^2}$,

Friction number: $\mathrm{F}_i \equiv \left(\dfrac{fl}{d} + K\right)_i$,

Stanton number: $\mathrm{St}_i \equiv \left(\dfrac{4hl_0}{\rho c_p u_0 d}\right)_i$,

Time ratio number: $\mathrm{T}_i^* \equiv \left(\dfrac{\alpha_s}{\delta^2}\dfrac{l_0}{u_0}\right)_i$,

Heat source number: $\mathrm{Q}_{si} \equiv \left(\dfrac{\dot{q}_s l_0}{\rho_s c_{ps} u_0 \Delta T_0}\right)_i$,

Biot number: $\mathrm{B}_i \equiv \left(\dfrac{h\delta}{k_s}\right)_i$.

Here, α_s is thermal diffusivity of the solid.

The similarity criteria between the prototype and model are established based on the aforementioned non-dimensional governing equations, i.e., Eqs. (6)-(10). They are summarized as follows:

$$\left(\sum_i L_i / A_i\right)_R = 1, \tag{11}$$

$$\left(\sum_i \mathrm{F}_i / A_i^2\right)_R = \left[\sum_i \left(f_i \frac{l_i}{d_i} + K_i\right) / \left(a_i / a_0\right)^2\right]_R = 1, \tag{12}$$

$$\mathrm{R}_R = \left(\beta \Delta T_0 l_0 / u_0^2\right)_R = 1, \tag{13}$$

$$A_{iR} = \left(a_i / a_0\right)_R = 1, \tag{14}$$

$$\mathrm{St}_{iR} = \left(h l_0 / \rho c_p u_0 d\right)_{iR} = 1, \tag{15}$$

$$\mathrm{B}_{iR} = \left(h\delta / k_s\right)_{iR} = 1, \tag{16}$$

$$\mathrm{T}_{iR}^* = \left(\alpha_s l_0 / \delta^2 u_0\right)_{iR} = 1, \tag{17}$$

$$\mathrm{Q}_{siR} = \left(\dot{q}_s l_0 / \rho_s c_{ps} u_0 \Delta T_0\right)_{iR} = 1, \tag{18}$$

where the subscript R denotes the ratio between the model and the prototype. Equations (11)-(13) are the requirements considering the entire primary/secondary loop of the DRACS, while the rest are applied to each individual component in the loop.

Considering the steady-state momentum and energy conservation of the entire loop, the ratios for the reference velocity and temperature difference are given as:

$$\left(u_0\right)_R = \left(\frac{\beta l_0 \dot{Q}}{\rho c_p a_0}\right)_R^{1/3}, \tag{19}$$

$$\left(\Delta T_0\right)_R = \left(\frac{\dot{Q}}{\rho c_p u_0 a_0}\right)_R. \tag{20}$$

Here, $\dot{Q}$ is the total decay power. The requirement related to the Richardson number is satisfied automatically if Eqs. (19) and (20) are used.

The ratios for the hydraulic diameter, heat transfer coefficient, and conduction depth for each component are provided from Eqs. (15)-(17) as:

$$\left(d_i\right)_R = \left(\frac{\rho_s c_{ps}}{\rho c_p}\right)_{iR} \left(\frac{\alpha_{si} l_0}{u_0}\right)_R^{1/2}, \tag{21}$$

$$\left(h_i\right)_R = \left(k_{si}\right)_R \left(\frac{u_0}{l_0 \alpha_{si}}\right)_R^{1/2}, \tag{22}$$

$$\left(\delta_i\right)_R = \left(\frac{\alpha_{si} l_0}{u_0}\right)_R^{1/2}. \tag{23}$$

SCALING METHODOLOGY

For the primary loop scaling, there are two equations, i.e. Eqs. (19) and (20), but 5 unknowns, $\left(u_0\right)_R$, $\left(l_0\right)_R$, $\left(a_0\right)_R$, $\left(\Delta T_0\right)_R$, and $\left(\dot{Q}\right)_R$. To close the equation system, three assumptions need to be made. Two degrees of freedom are removed with assumptions on the power and loop height ratios, recognizing the constraints of the electric heater power and space available in our experiment. The third assumption is made through the analysis of the core, since the only component that has a detailed design available in the prototypic design is the pebble bed reactor core (shown in Fig. 1).

According to Eq. (22), the convection time ratio is formulated as:

$$\left(\frac{l_0}{u_0}\right)_R = \left(\frac{k_s \rho_s c_{ps}}{h^2}\right)_R. \tag{24}$$

The time ratio in the primary loop is therefore determined based on the information of the heat transfer coefficient and structure material of the core in both the prototype and experiment. This enables us to complete the core and primary loop scaling, as illustrated in Fig. 2. The product of the loop height ratio and pipe area ratio can be calculated either by using Eq. (18) with specified core geometry or from the loop scaling analysis. The heat source number matches if the evaluated values of this product from the two approaches are comparable.

For the secondary loop scaling, the same set of equations are applied, but the number of unknowns is reduced to 3, compared to 5 in the primary loop scaling. The reason is that one unknown, $\left(\dot{Q}\right)_R$, denoting the heat removed by the secondary loop, is the same as that removed by the primary loop, assuming that the heat loss is negligible. In addition, the primary and secondary loops are coupled through the DRACS Heat Exchanger (DHX), as shown in Fig. 1. The time ratio between the two loops will be the same if the thickness of the tubes of the DHX is considerably small. As a result, the loop height ratio is the only assumed parameter for the secondary loop scaling. The flow chart for the primary loop scaling can then be applied to the secondary loop as well.

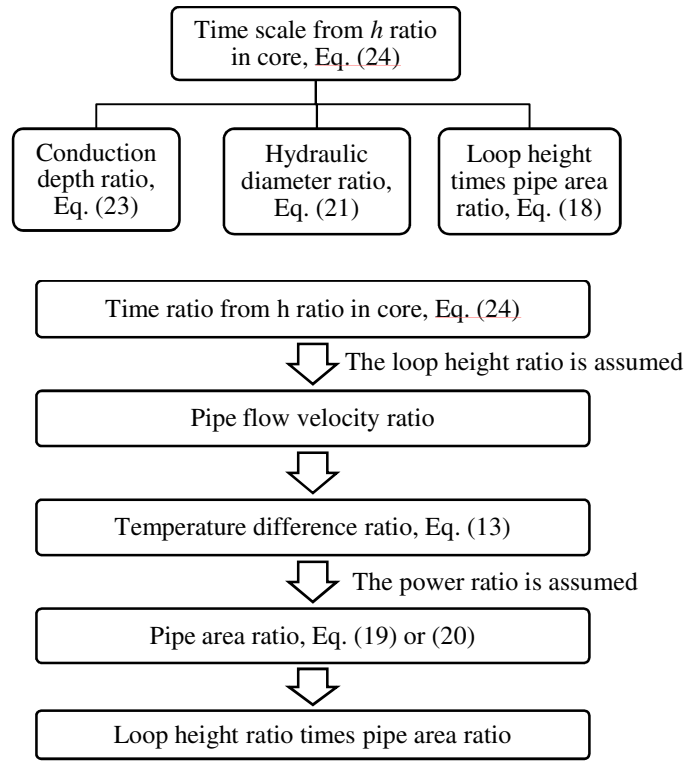

Fig. 2. Flow charts for the core and primary loop scaling

As the heat transfer and pressure losses inside the DHX and NDHX (Natural Draft Heat Exchanger) are important, Eqs. (21)-(23), as well as the requirement of the friction number, are applied to DHX and NDHX, respectively. The fluidic diode in the primary loop exhibits negligible heat loss, but experiences significant pressure drop, which indicates that only the requirement of the friction number needs to be enforced on the fluidic diode. Upon the completion of the component scaling, Eqs. (11) and (12) are examined for the primary and secondary loops, respectively. If these requirements are not met, some compromise will need to be made in the scaling of some components.

SUMMARY

In the present work, the similarity laws are derived from the non-dimensional conservation equations characterizing the DRACS system. A methodology is subsequently developed in applying these laws to specific components in the DRACS, as well as the primary and

secondary loops. Based on this methodology, a reduced-scale experimental model has been designed for DRACS performance testing, which will be discussed in a companion paper [4].

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