A Cut-Cell Approach for 2D Cartesian Meshes that Preserves Orthogonal Grid Sweep Ordering

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INTRODUCTION

In this paper, we present a cut-cell methodology for solving the two-dimensional neutral-particle transport equation on an orthogonal Cartesian grid. We allow the rectangular cell to be subdivided into two polygonal subcells. We ensure that this division (or cut) conserves the volumes of the materials in the subcells and we utilize a step-characteristics (SC) slice balance approach (SBA) to calculate the angular fluxes exiting the cell as well as the average scalar fluxes in each subcell.

Solving the discrete ordinates transport equation on an arbitrary mesh has historically been difficult to parallelize while maintaining good parallel efficiency. However on Cartesian meshes, the KBA algorithm maintains good parallel efficiency using a direct solve [1]. The ability to preserve this algorithm was a driving factor in the development of our cut-cell method. This method also provides a more accurate depiction of a material interface in a cell, which leads to more accurate solutions downstream of this cell. As a result, fewer spatial cells can be utilized, resulting in reduced memory requirements.

We apply this approach in the 2D/3D discrete ordinates neutral-particle transport code Denovo [2], where we analyze a 2D 3×3 lattice of pincells. We show that, for eigenvalue problems, a significant increase in accuracy for a given mesh size is gained by utilizing the cut-cell, SC equations instead of the standard homogenized-cell, SC equations.

BACKGROUND

To date, Denovo has focused on 2D/3D transport solutions on orthogonal Cartesian meshes [3, 4]. Generally, the cross section in a cell is determined by volume weighting the material cross sections in the cell. This approach smears the material uniformly in the cell and can lead to significant inaccuracies if the cell is large relative to a neutron's mean free path (mfp), or if one of the materials has a significant absorption or fission cross section. We refer to this method as the homogenized-cell method.

To facilitate reactor analysis that features non-Cartesian geometries, such as fuel pins and reactor cores, subcell solution techniques were explored. These methods will conserve the parallelizable KBA algorithm and also increase the accuracy of the solution in a cell containing multiple materials.

CUT-CELL APPROACH

In the cut-cell approach, the first step is to "cut" each cell into appropriately sized subcells. In this paper, we consider pincell/reactor problems with circles and rectangles as the only geometric figures and restrict the maximum number of subcells to two. This is an obvious first restriction that can be eased in the future. On each subcell, we implement an SBA [5, 6].

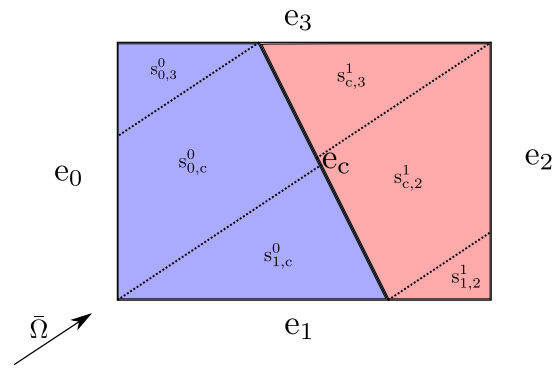

Fig. 1. SBA utilized on a standard rectangular mesh cell with a single cut, where e_c is the cut edge, e_i is the edge of side i ($i \in [0, 3]$), and $s^m_{ii,jj}$ is the slice in subcell m from edge ii to edge jj.

The SBA uses an angle-dependent spatial decomposition to divide the 2D cells into slices for each discrete ordinate direction. It uses a multiple balance approach that consists of exact spatial moments balance equations on cells and slices along with auxiliary relations on slices. This decomposition allows the transport equation to be solved on each slice in a cell (or subcell) and then summed to get cell-averaged and face-averaged values. In the simplified subcell case, the rectangular cell is subdivided into at most two subcells. The two subcells are then sliced using the SBA geometric decomposition. An example of this subdivision and associated SBA are shown in Figure 1.

Step Characteristics (SC) SBA was implemented in Denovo and compared with the homogenized-cell SC method. The SC approach assumes that incoming angular fluxes and volumetric sources are constant in a subcell [7]. Exact analytic solutions to the characteristics equation are used to exactly preserve the average of the outgoing edge angular fluxes. A subcell balance is used to exactly preserve the average angular flux in a subcell.

RESULTS

3×3 Simplified Fuel Pin Lattice

The model problem consists of a 3×3 array of pincells. Each cell consists of a circular fuel-clad region and a moderator

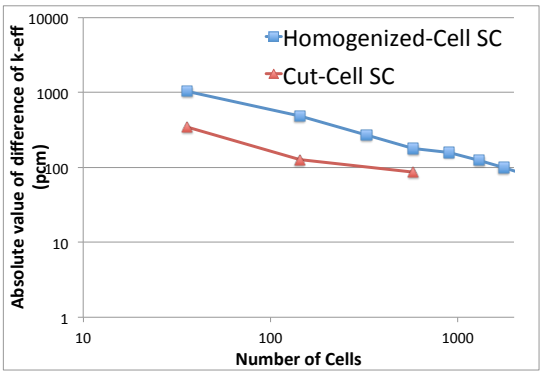

Fig. 2. Absolute value of the difference in the calculated eigenvalue in pcm for homogenized-cell SC and cut-cell SC methods versus number of cells in the seven-group, 3×3 lattice problem.

region. The lattice layout is

1	2	1
2	3	2
1	2	1

,

where 1 is a UO_2 pincell, 2 is a 4.3% MOX pincell, and 3 is a guide tube pincell. The MOX and the UO_2 pins have a radius of 0.48 cm and the guide tube has a radius of 0.54 cm. The seven-group material definitions are described in detail in the C5G7 benchmark problem [8].

The reference solution is a Monte Carlo solution using 100 inactive cycles, 500 active cycles, and 100,000 particles per cycle for a total of 50 million active particles. We ran the homogenized-cell method and the cut-cell method with SC with a triangular Quadruple Range LS_{28}-like quadrature set (210 angles per octant) [9, 10] for different numbers of cells. The absolute value of the difference in the calculated eigenvalue in pcm versus the number of cells in the problem is shown in Figure 2. The cut-cell method consistently yields more accurate solutions than the homogenized-cell method.

CONCLUSION

For the model lattice problem, the cut-cell solution is more accurate than the standard homogenized-cell SC method for an equivalent number of cells. The cut-cell solution using 576 cells was as accurate as the homogenized-cell SC solution using ~2115 cells.

We note that this decrease in the number of cells (memory) is offset by an increase in run-time. For a given cut-cell cell, generally there will be four exponential evaluations whereas a homogenized-cell cell requires only one. These evaluations along with the necessity to determine which subcell to solve first ensures that each cut-cell will take a minimum of four times longer to solve and possibly longer. However, because not all cells are cut-cells, the overall increase in run time may be significantly smaller.

It is not clear that, for all problems, the reduction in memory will always be compensated by an increase in run time. There may be many cases where time (not memory) is the limiting resource, which would make the standard homogenized-cell solution scheme more desirable.

In conclusion, we note that this cut-cell, SC implementation shows that subcell methods can be more accurate than standard, orthogonal, homogenized-cell solution schemes for the same mesh size. New subcell schemes with improved accuracy and efficiency are currently begin investigated.

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REFERENCES

1. K. KOCH, R. BAKER, and R. ALCOUFFE, "A Parallel Algorithm for 3D S_n Transport Sweeps," Tech. rep., LA-CP-92-406, Los Alamos National Laboratory, 1992.
2. T. EVANS and G. DAVIDSON, "Denovo Methods Manual," Tech. rep., Oak Ridge National Laboratory (2007).
3. G. DAVIDSON, T. EVANS, R. SLAYBAUGH, and C. BAKER, "Massively Parallel Solutions to the k-Eigenvalue Problem," *Trans. Am. Nucl. Soc.*, **81**, 318–320 (2010).
4. T. EVANS, A. STAFFORD, R. SLAYBAUGH, and K. CLARNO, "DENOVO: A new three-dimensional parallel discrete ordinates code in SCALE," *Nuclear Technology*, **171**, 171–200 (2010).
5. R. GROVE, *A Characteristic-Based Multiple Balance Approach for Solving the S_n Equations on Arbitrary Polygonal Meshes*, Ph.D. thesis, University of Michigan, Ann Arbor, MI (1996).
6. R. GROVE, "The Slice Balance Approach (SBA): A Characteristic-Based, Multiple Balance Sn Approach on Unstructured Polyhedral Meshes," Tech. Rep. LM-05K094, Knolls Atomic Power Laboratory, Niskayuna, NY (2005).
7. K. LATHROP, "Spatial differencing of the transport equation: positivity vs. accuracy," *Journal of computational physics*, **4**, 4, 475–498 (1969).

8. M. SMITH, E. LEWIS, and B. NA, "Benchmark on Deterministic Transport Calculations Without Spatial Homogenization-A 2-D/3-D MOX Fuel Assembly Benchmark," Tech. Rep. ISBN 92-64-0219-6, NEA/NSC/DOC. 2003.
9. I. ABU-SHUMAYS, "Angular Quadratures for Improved Transport Computations," *Transport Theory and Statistical Physics*, **30(2&3)**, 169–204 (2001).
10. J. JARRELL, M. ADAMS, and J. RISNER, "Application of Quadruple Range Quadratures to Three-Dimensional Model Shielding Problems," *Nuclear Technology*, **168(2)**, 424 (2009).