Proceedings of the 4th International Topical Meeting on High Temperature Reactor Technology
HTR2008
September 28-October 1, 2008, Washington, DC USA

HTR2008-58215

PRELIMINARY DEVELOPMENT OF A UNIFIED VISCOPLASTIC CONSTITUTIVE MODEL FOR ALLOY 617 WITH SPECIAL REFERENCE TO LONG TERM CREEP BEHAVIOR*

T.-L. (Sam) Sham
Oak Ridge National Laboratory
Oak Ridge, Tennessee, USA
Email: shamt@ORNL.gov

Kevin P. Walker
Engineering Science Software, Inc.
Smithfield, Rhode Island, USA

ABSTRACT

The expected service life of the Next Generation Nuclear Plant is 60 years. Structural analyses of the Intermediate Heat Exchanger (IHX) will require the development of unified viscoplastic constitutive models that address the material behavior of Alloy 617, a construction material of choice, over a wide range of strain rates.

Many unified constitutive models employ a yield stress state variable which is used to account for cyclic hardening and softening of the material. For low stress values below the yield stress state variable these constitutive models predict that no inelastic deformation takes place which is contrary to experimental results. The ability to model creep deformation at low stresses for the IHX application is very important as the IHX operational stresses are restricted to very small values due to the low creep strengths at elevated temperatures and long design lifetime.

This paper presents some preliminary work in modeling the unified viscoplastic constitutive behavior of Alloy 617 which accounts for the long term, low stress, creep behavior and the hysteretic behavior of the material at elevated temperatures. The preliminary model is presented in one-dimensional form for ease of understanding, but the intent of the present work is to produce a three-dimensional model suitable for inclusion in the user subroutines UMAT and USERPL of the ABAQUS and ANSYS nonlinear finite element codes.

Further experiments and constitutive modeling efforts are planned to model the material behavior of Alloy 617 in more detail.

* Notice: This submission was sponsored by a contractor of the United States Government under contract DE-AC05-00OR22725 with the United States Department of Energy. The United States Government retains, and the publisher, by accepting this submission for publication, acknowledges that the United States Government retains, a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this submission, or allow others to do so, for United States Government purposes.

1. Introduction

The High Temperature Gas-cooled Reactor (HTGR) design has been selected by the U.S. Department of Energy for the Next Generation Nuclear Plant (NGNP) Project. The NGNP will demonstrate the use of nuclear power for electricity and hydrogen production. It will have an outlet gas temperature in the range of 900°C and a plant design service life of 60 years. Alloy 617 is the material of choice for the Intermediate Heat Exchanger (IHX), a primary system component.

An ASME Boiler and Pressure Vessel (B&PV) Code, Section III draft Code Case for Alloy 617 was developed in the late 1980's. It provides rules of construction for HTGRs with reactor outlet temperatures of about 950°C. Due to the inelastic behavior exhibited by Alloy 617 that includes: (1) the lack of a clear distinction between time-independent and time-dependent behavior; (2) the high

 Copyright © 2008 by ASME

dependence of flow stress on strain rate; and (3) softening with time, temperature, and strain, Corum and Blass (1991) recommended that inelastic design analyses must be based on unified constitutive equations, which do not distinguish between time-independent plasticity and time-dependent creep.

Several unified viscoplastic constitutive models for Alloy 617 were developed in the 1990's. Chang and Pugh (1991) and Corum and Blass (1991) developed a unified viscoplastic constitutive model originated by Robinson (1984). Corum and Blass (1991) tabulated the material constants for this model of Alloy 617 at temperatures ranging from 649°C up to 982°C. At about the same time Schwertel et al. (1991) developed the constitutive models of Robinson and Chaboche to describe the constitutive behavior of Alloy 617 at 600°C and 850°C.

For one-dimensional loading, the flow rule of these viscoplastic models can be represented generally in the form

$$\dot{\varepsilon}^p = \dot{\varepsilon}_0 \Phi_1(p) \left\langle \frac{F}{D} \right\rangle^n \operatorname{sgn}(\sigma - X) \tag{1}$$

where $\dot{\varepsilon}^p$ is the viscoplastic strain rate, $\dot{\varepsilon}_0$ the reference strain rate, σ the stress, and X the back stress. The Macauley bracket is defined by $\langle x \rangle = xH(x)$, where $H(x)$ is the Heaviside function, and $\operatorname{sgn}(x)$ is the signum function. The state variable D is the drag stress and the quantity F is given as

$$Robinson: \qquad F = (\sigma - X)^2 - Y^2$$

$$Chaboche: \qquad F = |\sigma - X| - Y$$

where Y is the "yield stress state variable" that serves as a cut-off value for the magnitude of the overstress, $|\sigma - X|$, below which $\dot{\varepsilon}^p$ is zero, i.e., only elastic deformation occurs. The function $\Phi_1(p)$ is used to model tensile stress drops and tertiary creep and is discussed in section 5. Since this function is assumed to be unity at the start of deformation and increases slowly with cumulative viscoplastic deformation, it is taken as unity in the succeeding sections of the paper. It is discussed in detail in sections 5 and 8 of the paper.

The material constants which Schwertel et al. (1991) used with the Chaboche model at 850°C give good correlation with the experimental hysteretic behavior of Alloy 617 at strain rates between 5×10^{-6} and 5×10^{-2} per second. At a strain amplitude of ± 0.25% the stress amplitude varies from 150 to 300 MPa. For this temperature the yield stress state variable is taken as 25.2 MPa, so that creep cannot take place below a stress threshold of 25.2 MPa. Similarly, the model developed by Chang and Pugh (1991) and Corum and Blass (1991) has a yield stress state variable of 16.5 MPa at 850°C, so that no creep is possible at stresses below 16.5 MPa.

Clearly, in order to model creep at low stress levels, the value of the yield stress state variable must either be zero or very small. Many models use the yield stress state variable as a repository for cyclic strain hardening and softening, so that making the yield stress state variable zero or small is not a desirable option in the constitutive model.

Another option does exist. Freed and Walker (1995) proposed making the asymptotic values of the state variables in unified viscoplastic constitutive models depend either linearly or quadratically on the saturated stress. This procedure seems to have originated with Miller (1976). In a recent publication Michel, Assire and Cambefort (2001) also used this procedure in modeling the constitutive behavior of an austenitic stainless steel. The advantages of this procedure in simplifying the material constant determination are well known (Freed, and Walker (1995)). It now appears to be not only advantageous, but also necessary, if the yield stress state variable is retained as a repository for cyclic hardening and softening behavior, and the viscoplastic model is still required to model low stress creep behavior.

The framework for such a unified viscoplastic constitutive model is discussed in the following sections.

2. Viscoplastic Model Requirements

Many tests on a wide range of materials have shown that the limiting form of the back stress at saturation (well into the plastic region and at high strain rates) ranges from $0.4\,\sigma$ to $0.6\,\sigma$, (see Figure 4 of Qian and Fan, 1991) where σ is the saturated stress in the plastic region.

In a creep test at high stress levels (> 70 MPa for Alloy 617), the back stress will grow from $X \approx 0$ at the beginning of the creep test up to its maximum value of $a\,\sigma$ $(0.4 < a < 0.6)$. As X grows, the value of the overstress $\sigma - X$ decreases in the primary creep region, after which $\sigma - X = \sigma - a\,\sigma = (1-a)\,\sigma = \text{constant}$ and the creep rate will remain constant as the material enters the secondary creep regime whose rate is governed by the value of $1 - a$.

Copyright © 2008 by ASME

Larger amounts of primary creep are expected when the value of the back stress X at the beginning of the creep test is small and the value of a is relatively large ($a < 1$), while smaller amounts are expected if the initial value of X is large and a is small. The constant a can thus be used to vary the amount of primary creep exhibited by the material before it enters the secondary, or steady state, creep regime. It also governs the secondary creep rate.

At large stress values the amount of primary creep is large and the secondary creep rate is also large. This fits in with the observed values ($0.4 < a < 0.6$) exhibited by many materials. At small stress values the amount of primary creep is small or non-existent and the secondary creep rate is also small until tertiary creep sets in. This seems to indicate a small value of $(1 - a)$, implying that $0.95 < a < 0.99$.

This requires an evolutionary equation for the back stress in which the limiting value at saturation, where $\dot{X} \approx 0$, is given by $X \approx a\,\sigma$ ($0.4 < a < 0.6$) at large strain rates and by $X \approx b\,\sigma$ ($0.95 < b < 0.99$) at small strain rates.

Since a yield stress state variable will be included to model the evolution of cyclic hardening, the preceding restrictions apply to the saturation value of the internal stress, which is the sum of the back stress and yield stress state variables.

3. Flow Rule

Introduce a binary switch function, ξ, by means of the relation:

$$\xi = \frac{|X|}{\mathcal{L}(\dot{p})} \tag{2}$$

where $\mathcal{L}(\dot{p})$ is the saturation value of the back stress X when $\dot{X} = 0$, including thermal recovery, and $\dot{p}$ is the equivalent viscoplastic strain rate.

The flow rule takes the form

$$\dot{\varepsilon}^p = \dot{\varepsilon}_0 \left\langle \frac{|\sigma - X| - Y}{D} \right\rangle^n \operatorname{sgn}(\sigma - X) \tag{3}$$

where a new yield stress state variable is defined as

$$Y = (k + R)\,\xi^m |X| \tag{4}$$

in which k and R are non-dimensional state variables and $m \geq 0$. The state variable k is constant but R can evolve in time.

The binary switch function ξ is unity in the plastic region for large stresses and strain rates when the back stress X saturates to its limit value $\mathcal{L}(\dot{p})$, and is essentially zero in the elastic regions and at small stresses and strain rates where X is small. Both ξ and X are responsible for reducing the yield stress state variable Y to zero at small stress values, with the power exponent in the expression ξ^m governing how quickly the yield stress state variable Y approaches zero as the stress is reduced. Note that these features for the yield stress state variable are what distinguish this new flow rule from others in the Chaboche type models.

Under tensile loading conditions, the flow rule in equation (3) may be written as

$$\dot{\varepsilon}^p = \dot{\varepsilon}_0 \left(\frac{\sigma - X(1 + (k + R)\xi^m)}{D} \right)^n = \dot{\varepsilon}_0 \left(\frac{\sigma - Z}{D} \right)^n \tag{5}$$

in which $Z = X + Y$ is the internal stress. Under steady state conditions when the back stress X and binary variable ξ (but not necessarily the slowly evolving state variable R) have reached their saturated values, the preceding discussion indicates that at saturation the internal stress, Z, takes on the values

$$Z = X + Y = a\,\sigma \tag{6}$$

with $0.4 < a < 0.6$ at large strain rates, and

$$Z = X + Y = b\,\sigma \tag{7}$$

with $0.95 < b < 0.99$ at small strain rates.

If the amount of internal stress at saturation, $a\sigma$ and $b\sigma$, respectively, is partitioned into the back stress X and yield stress state variable Y in the form

$$X = f\,a\,\sigma\text{, and} \tag{8}$$

$$Y = (1 - f)\,a\,\sigma \tag{9}$$

equation (6) is recovered at high strain rates, and by replacing a with b in equations (8) and (9), equation (7) is recovered at small strain rates. With the use of equations (4), (8) and (9), and taking $\xi = 1$ at saturation, the additional relations

$$f = \frac{1}{1 + (k + R)} \text{ and} \tag{10}$$

Copyright © 2008 by ASME

$$1 - f = \frac{(k+R)}{1+(k+R)} \tag{11}$$

are obtained.

Thus at saturation, where $\dot{X} = 0$, the back stress X is required to have the following properties:

$$X = fa\,\sigma \text{ at large strain rates, and} \tag{12}$$

$$X = fb\,\sigma \text{ at small strain rates.} \tag{13}$$

Under tensile loading conditions, the flow rule in equation (5) can be used to obtain

$$D\left(\frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0}\right)^{1/n} = \sigma - Z \tag{14}$$

At the saturation of the internal stress Z, equations (6) and (7) can be used, together with $|\dot{\varepsilon}^p| = \dot{p}$, in equation (14) to arrive at the limiting conditions

$$\sigma = \frac{D}{1-a}\left(\frac{\dot{p}}{\dot{\varepsilon}_0}\right)^{1/n} \text{ at large strain rates} \tag{15}$$

$$\sigma = \frac{D}{1-b}\left(\frac{\dot{p}}{\dot{\varepsilon}_0}\right)^{1/n} \text{ at small strain rates} \tag{16}$$

4. Back Stress Evolutionary Equation

The back stress evolutionary equation takes the special form

$$\dot{X} = c\,\dot{\varepsilon}^p - c\,\frac{X}{L(\dot{p})}\,\dot{p} - c\,\Lambda\,\dot{\varepsilon}_0\frac{|X|^{n-1}X}{D^n} \tag{17}$$

where c is a constant and $L(\dot{p})$ in the dynamic recovery term is given by the relation

$$L(\dot{p}) = \frac{fa}{1-a}D\left(\frac{\dot{p}}{\dot{\varepsilon}_0}\right)^{1/n} + \zeta \tag{18}$$

with ζ being a constant. The term Λ in the thermal recovery term of equation (17) is given by

$$\Lambda = \left(\frac{1-b}{fb}\right)^n \tag{19}$$

At large strain rates, the first term in equation (18) is dominant, while the second term dominates at slow strain rates. The magnitude of the constant ζ determines the strain rate at which the first term becomes large or insignificant compared with the second term. Thus

$$L(\dot{p}) = \frac{fa}{1-a}D\left(\frac{\dot{p}}{\dot{\varepsilon}_0}\right)^{1/n}, \text{ for large } \dot{p} \tag{20}$$

$$L(\dot{p}) = \zeta\text{ , for small } \dot{p} \tag{21}$$

On setting $\dot{X} = 0$ in equation (17) the limiting back stress is determined by either of the implicit relations

$$\mathcal{L}(\dot{p}) = L(\dot{p})\left[1 + \Lambda\,\frac{\dot{\varepsilon}_0}{\dot{p}}\,\frac{L(\dot{p})\,\mathcal{L}^{n-1}(\dot{p})}{D^n}\right]^{-1} \tag{22}$$

or

$$\mathcal{L}(\dot{p}) = D\left(\frac{(L(\dot{p}) - \mathcal{L}(\dot{p}))\,\dot{p}}{\Lambda L(\dot{p})\dot{\varepsilon}_0}\right)^{1/n} \tag{23}$$

where the first equation is used when thermal recovery is small in comparison with dynamic recovery, while the second is used in the opposite case. These equations are solved by Newton-Raphson iteration.

The preceding relations show that the limiting value of the back stress becomes smaller as Λ becomes larger and D and n become smaller.

For large $\dot{p}$, equation (22) gives

$$\mathcal{L}(\dot{p}) = L(\dot{p}) = \frac{fa}{1-a}D\left(\frac{\dot{p}}{\dot{\varepsilon}_0}\right)^{1/n} \tag{24}$$

where equation (20) was used in the second equality. Use of equation (15) in equation (24) gives

$$\mathcal{L}(\dot{p}) = fa\sigma \text{ at large strain rates} \tag{25}$$

Similarly, equation (22) admits the limit

$$\mathcal{L}(\dot{p}) = \frac{fb}{1-b}D\left(\frac{\dot{p}}{\dot{\varepsilon}_0}\right)^{1/n} \tag{26}$$

for small $\dot{p}$, and upon using equation (16), it becomes

$$\mathcal{L}(\dot{p}) = fb\sigma \text{ at small strain rates} \tag{27}$$

Hence, the back stress evolutionary equation (17) leads to results that satisfy the requirements given in equations (12) and (13). Later refinements of the viscoplastic model will require the use of a number of back stress components in order to more accurately capture the shape of the stress-strain curves for Alloy 617 under tensile and hysteretic loading.

5. Tertiary Creep Modeling

As the strains of interest are less than 5%, no distinction between stress and load control is necessary in interpreting creep test data in that strain range. As a first effort at modeling tertiary creep, ways that various

 Copyright © 2008 by ASME

authors have used in modeling tertiary creep without the explicit introduction of a new damage state variable were explored.

Consider the following form of the flow rule for tensile loading of a creep specimen

$$\dot{\varepsilon}^p = \dot{\varepsilon}_0 \left(\frac{|\sigma - X| - (k+R)\,\xi^m |X|}{D} \right)^n \tag{28}$$

In order for $\dot{\varepsilon}^p$ to increase during the tertiary creep process when the stress σ is held constant, it is necessary that one or more of the following takes place:

1) The back stress X decreases so that $\sigma - X$ increases and the yield stress state variable $Y = (k+R)\xi^m|X|$ decreases, both of which increase the numerator in (28);
2) The drag stress state variable D decreases in the denominator of (28), so that the bracketed term increases;
3) The power law exponent n decreases.

If one or more of the processes in 1) to 3) are stronger than the others, then it may be permissible for the other processes to operate in a manner opposite to that (if they alone operated) required for tertiary creep. All that is required is that the net influence of all three processes is such as to produce an increase in the right hand side of equation (28). The introduction of a new damage state variable would cause the drag stress to be multiplied by this variable, producing a new effective drag stress that would essentially decrease the denominator of (28) during creep.

Tachibana and Krempl (1998) have proposed an evolutionary equation for the back stress limit at large strain rates, corresponding to $L(\dot{p})$ in equation (17), which decreases during the creep process. They achieve this decrease by postulating that the evolutionary equation governing the limit back stress is give by

$$\dot{L} = -\beta\, p\, (L - L_{\min}) \tag{29}$$

where p is the cumulative viscoplastic strain

$$p = \int_0^t |\dot{\varepsilon}^p|\, \mathrm{d}\tau \tag{30}$$

The saturation back stress is assumed to decrease to a limiting value of $L_{\min}$ from its initial value of L_1 at a rate depending on the value of βp. Since p is the cumulative inelastic deformation, this rate increases with cumulative deformation. There is a provision in their model for thermal recovery to reduce the limiting back stress to zero, using a formulation similar to equation (22).

Chang and Pugh (1991) modified the saturated back stress in their constitutive model, which included only thermal recovery and no dynamic recovery, by taking its limiting form as

$$\text{Limit back stress} = \frac{L}{\Phi} = L\Phi^{-1} \tag{31}$$

where the function Φ is given by $\Phi = 1 + \alpha p^{\lambda}$, or if λ is small enough, by $\Phi = \exp(\alpha p^{\lambda})$. The inverse of this function, $\Phi^{-1} = \exp(-\alpha p^{\lambda})$, goes to zero as $p \to \infty$, resulting in a limiting saturation stress which eventually goes to zero. As deformation proceeds, the saturated back stress decreases, causing an increase in the viscoplastic flow rate which enables tertiary creep to be modeled. The variation of the saturated limit stress with cumulative deformation in equation (31) was first used with unified viscoplastic models by Marquis (1979) to model cyclic hardening and softening of the back stress dynamic recovery term, and is used in the Chaboche models at ONERA in France.

In their model, Chang and Pugh (1991) also multiply the flow rule by the function Φ, which, being an increasing function of the cumulative deformation, also enhances the tertiary creep flow rate. This also has another effect besides increasing the tertiary creep flow rate, which is discussed below.

Chang and Pugh (1991) use the Orowan equation $\dot{\varepsilon}^p = N\Phi b v$, where $N\Phi$ is the mobile dislocation density, b is the Burgers vector, and v is the average dislocation velocity to relate their unified viscoplastic model to dislocation dynamics. In particular, the velocity v is related to the power of the overstress (i.e. the numerator in equation (28)), while the mobile dislocation density, $N\Phi$, is related to the drag stress $(1/D)^n$ in the denominator of a power law model.

In a tensile test conducted at a constant strain rate, the condition $\dot{\varepsilon} \approx \dot{\varepsilon}^p$ holds in the saturated portion of the stress-strain curve. If the mobile dislocation density $N\Phi$ increases as the plastic region is entered, due to the increase in Φ, then the average mobile dislocation velocity v must decrease in order to maintain the total strain rate, $\dot{\varepsilon}$, constant. Since the dislocation velocity v depends on the stress or overstress raised to a power, the stress must also decrease. This stress drop on entering the

Copyright © 2008 by ASME

plastic region was observed (see Chang and Pugh, 1991) in tensile tests on Alloy 617 at a temperature of 950°C.

Two functions, $\Phi_1(p)$ and $\Phi_2(p)$, will be used in a preliminary effort at modeling the stress drop and tertiary creep in the flow rule. The first function directly multiplies the flow rule, while the inverse of the second function directly multiplies the back stress and the yield stress in the flow rule. Chang and Pugh (1991) used the same function in the flow rule and in the saturated back stress. The Φ functions are presented in section 8.

6. Assumption of Constant Drag Stress

The flow rule in equations (5) to (9) can be rearranged, and for tensile loading, can be written as in equation (15) at saturation for large strain rates. Cyclic hardening or softening during strain amplitude controlled fatigue tests can be modeled by the evolutionary equations for the back stress X or by the state variable R.

Consider a monotonic tensile test loaded under constant strain rate $\dot{\varepsilon}_1$ until saturation is well established. At time t_1 in this saturation regime, the strain rate is increased by a decade to $\dot{\varepsilon}_2$. The straining continues under this new strain rate, and after some transient period, saturation is re-established at time t_2 at a different level. The change in the stresses at these two respective saturation levels is given by, assuming both $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ are high,

$$(1-a)\left[\sigma(t_2)-\sigma(t_1)\right]=A \tag{32}$$

where

$$A \equiv D(t_2)\left(\frac{\dot{\varepsilon}^p(t_2)}{\dot{\varepsilon}_0}\right)^{1/n} - D(t_1)\left(\frac{\dot{\varepsilon}^p(t_1)}{\dot{\varepsilon}_0}\right)^{1/n} \tag{33}$$

Thus the change in the saturated stresses depends only on the magnitudes of the viscoplastic strain rates if the drag stress D is constant. If the drag stress hardens/softens during cyclic deformation, the change in the saturated stresses will increase/decrease after the specimen has undergone fatigue cycling. Many metals exhibit approximately the same change in the saturated stresses before and after fatigue testing (Krempl, 1987), and this rules out incorporating secular cyclic hardening or softening over large numbers of cycles into the drag stress state variable.

7. Cyclic Hardening and Softening

Because the change in the saturated stresses due to change in strain rates after fatigue cycling do not change in magnitude, the repository for cyclic hardening must reside in either the yield stress state variable or the back stress. The evolutionary equation for the growth of the dimensionless isotropic state variable R is taken to be a standard growth law of the form

$$\dot{R} = g\,(R_\infty - R)\dot{p} + \ell\,|R_q - R|^{\rho-1}(R_q - R) \tag{34}$$

where $R(t=0)=0$.

Below a threshold value of $R = R_q$, the static recovery term is positive and enhances the hardening. When the state variable R exceeds this threshold value, the thermal recovery becomes negative and the rate of hardening towards the final value of $R = R_\infty$ is decreased. This decrease in the hardening rate is enhanced at slow rates of straining. At very small strain rates, the term in $\dot{p}$ in (34) becomes much smaller than the thermal recovery term, and R hardens (or softens) to the final value of R_q depending on whether R_q is positive (or negative). Alloy 617 exhibits this type of cyclic hardening behavior in fatigue tests conducted at constant strain amplitude at elevated temperatures.

8. Unified Viscoplastic Constitutive Model

The equations proposed for use in an initial modeling effort to characterize the deformation behavior of Alloy 617 between room temperature and temperatures up to 950°C are collected below.

$$\dot{\varepsilon}^p = \dot{\varepsilon}_0\,\Phi_1(p)\left\langle \frac{F}{D} \right\rangle^n \operatorname{sgn}\left(\sigma - X\Phi_2^{-1}(p)\right)$$

$$F = \left|\sigma - X\Phi_2^{-1}(p)\right| - Y\Phi_2^{-1}(p)$$

$$Y = (k+R)\,\xi^m|X|$$

$$\Phi_1(p) = 1 + \alpha_1\,p^{\lambda_1}$$

$$\Phi_2(p) = 1 + \alpha_2\,p^{\lambda_2}$$

$$\dot{p} = |\dot{\varepsilon}^p| \quad \text{and} \quad p = \int_0^t |\,\dot{\varepsilon}^p(\tau)|\,\mathrm{d}\tau$$

$$\xi = |X|/\mathcal{L}(\dot{p})$$

$$\mathcal{L}(\dot{p}) = L(\dot{p})\left[1 + \Lambda\,\frac{\dot{\varepsilon}_0}{\dot{p}}\,\frac{L(\dot{p})\,\mathcal{L}^{n-1}(\dot{p})}{D^n}\right]^{-1}$$

Copyright © 2008 by ASME

$$L(\dot{p}) = \frac{fa}{1-a} D \left(\frac{\dot{p}}{\dot{\varepsilon}_0} \right)^{1/n} + \zeta$$

$$\Lambda = \left(\frac{1-b}{fb} \right)^n$$

$$f = [1 + (k + R)]^{-1}$$

$$\dot{X} = c\,\dot{\varepsilon}^p - c\,\frac{X}{L(\dot{p})}\,\dot{p} - c\,\Lambda\,\dot{\varepsilon}_0\,\frac{|X|^{n-1}X}{D^n}$$

$$\dot{R} = g\,(R_\infty - R)\dot{p} + \ell\,|R_q - R|^{\rho-1}(R_q - R)\,,$$

$$\text{with } R(t=0) = 0$$

in which the parameters n, D, m, α_1, α_2, ζ, λ_1, λ_2, k, a, b, c, g, R_∞, R_q, ℓ, and ρ are temperature dependent material constants, and $\dot{\varepsilon}_0$ is a constant reference strain rate.

9. Numerical Example

A numerical example on strain-controlled cycling together with a 60 year creep test is presented in this section using the unified viscoplastic constitutive model developed in this paper.

Table 1. Preliminary material constants at 850ºC for hysteresis loops simulation

Material Constants (850ºC)	
E	146,700 MPa
n	5.6
D	300 MPa
k	0.14
λ_1	1
λ_2	0.35
g	0
ℓ	0
R_∞	0
R_q	0
c	400,000 MPa
m	0.75
α_1	0
α_2	3.0
a	0.45
b	0.65
$\dot{\varepsilon}_0$	1/s
ρ	1
ζ	10 MPa

Using the material constants at 850ºC from Schwertel et al. (1991) as a guide, the material constants were identified to give the expected behavior over ten decades of strain rate. They are given in Table 1. These material constants have not been optimized; they are merely approximate values chosen to demonstrate proof of concept for the preliminary constitutive model.

Computed hysteresis loops for three strain rates spanning ten decades are shown in Figure 1, with the two highest strain rates compared against the experimental data of Schwertel et al. (1991). No creep or hysteresis data are available at the lowest strain rate. The flow rule

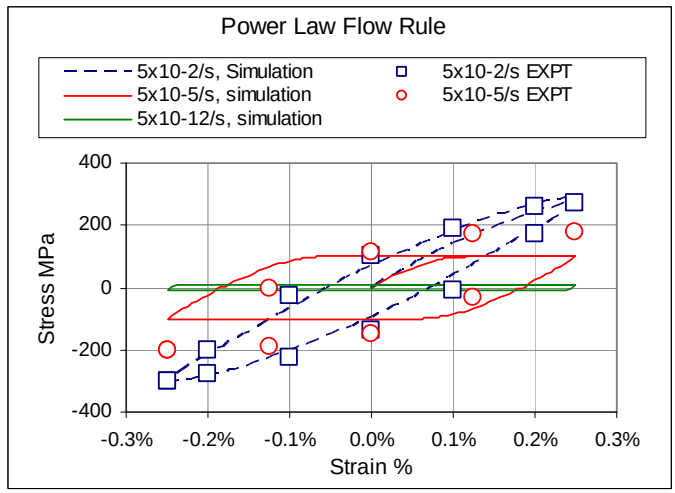

Figure 1. Computed hysteresis loops at 850ºC with a power law flow rule

exponent $n = 5.6$ was chosen to give a relatively small saturated stress of 7.5 MPa at a strain rate of 5×10^{-12} per second, using the material constants in Table 1, and it is evident that the calculated stress amplitude at the strain rate of 5×10^{-5} per second is too small for this low value of the power law exponent. Even with this low value of the exponent (Schwertel et al. (1991) use a value of - $n = 11$) a large amount of thermal recovery is required to bring the saturated stress level down to a value of 7.5 MPa. The value of $b = 0.65$ in Table 1 gives a sufficient amount of thermal recovery at low strain rates, but what is required is a value around $b = 0.95$. If the value of b is made larger than 0.65, the amount of thermal recovery will be too small and this precludes the limiting back stress from reaching a small value at very small strain rates. This is clear evidence of the failure of the power law flow rule which is valid for about five decades of strain rate, and indicates the need for a hyperbolic sine

Copyright © 2008 by ASME

flow rule to model the behavior over ten strain rate decades.

As shown in Figure 2, a hyperbolic sine flow rule of the form

$$\dot{\varepsilon}^p = \dot{\varepsilon}_0 \, A \, \Phi_1(p) \left\langle \sinh \frac{F}{D} \right\rangle^n \operatorname{sgn}\left(\sigma - X\Phi_2^{-1}(p)\right) \quad (35)$$

in which

$$F = \left|\sigma - X\Phi_2^{-1}(p)\right| - Y\Phi_2^{-1}(p) \quad (36)$$

with $A = 7 \times 10^{-6}$, $n = 4$, $b = 0.95$ and $D = 65$ MPa gives much better agreement with Schwertel et al's stress amplitude test values.

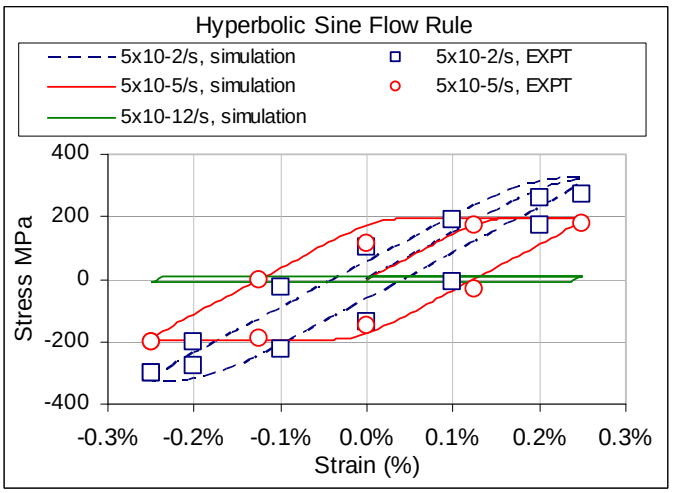

Figure 2. Computed hysteresis loops at 850ºC with a hyperbolic sine flow rule

No attempt has been made to optimize the hyperbolic material constants to fit the width of the hysteresis loops. With this flow rule the stress at the very low strain rate of 5×10^{-12} per second can be made less than 10 MPa, yet at the intermediate strain rate of 5×10^{-5} per second the stress is large enough at 200 MPa to match the experimental values. The power law shown in Figure 1 cannot achieve this behavior over ten strain rate decades.

A simulation of a very long, 60 year, creep test at a stress level of 7.5 MPa is shown in Figure 3, where the material creeps to about 1% total strain in 60 years at a constant secondary creep rate of about 5×10^{-12} per second when the tertiary creep constants $\alpha_2 = 0$ and $\lambda_2 = 0$ are set to give no tertiary creep. When the constants are chosen to have the values in Table 1, the creep deformation accelerates to about 5.5% total strain in 60 years. This stress level of 7.5 MPa for the creep test is well below the yield stress levels of 16.5 MPa in Chang and Pugh's (1991) ORNL model and 25.2 MPa in Schwertel et al's (1991) Chaboche model. These models, as presented with the accompanying material constants, would give zero creep at 7.5 MPa because the stress is below the yield threshold and only elastic behavior is predicted. The yield stress state variables in these models could be eliminated, but the repository for secular cyclic hardening would have to be taken up by either the back stress or the drag stress. Hence the need for a revision in the yield behavior at low stresses as advocated in this paper.

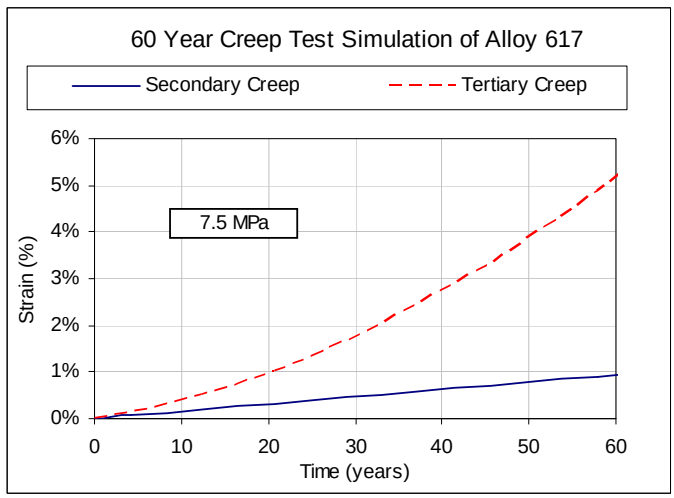

Figure 3. Computed creep curve at 850ºC with and without tertiary creep. Stress is at 7.5 MPa.

10. Summary

Some preliminary work in modeling the unified viscoplastic constitutive behavior of Alloy 617 which accounts for the long term, low stress, creep behavior and the hysteretic behavior of the material at elevated temperatures was presented in this paper. The initial numerical result shows that the approach developed is promising and further experiments and identification of the material constants are planned to model the behavior of Alloy 617 in more detail.

We have determined that a power law flow rule which operates well over five decades of strain rate is not sufficient to model the behavior of Alloy 617 over ten strain rate decades extending from 5×10^{-2} per second down to 5×10^{-12} per second (equivalent to 1% strain in 60 years) and for this requirement a hyperbolic sine rule appears promising. Preliminary simulations give encouraging results over ten decades of strain rate. A

Copyright © 2008 by ASME

modification to the yield stress state variable of unified viscoplastic models is required if both low stress creep and high stress hysteresis loop deformation behavior is to be modeled.

11. References

Chang, S.-J., and Pugh, C.F., "ORNL Unified Constitutive Equations Adapted for Strain Softening and High Strain Rate Conditions," ASME MD - Vol. 26/AMD - Vol. 121, High Temperature Constitutive Modeling – Theory and Application, pp. 351–364, 1991.

Corum, J.M., and Blass, J.J., "Rules for Design of Alloy 617 Nuclear Components to Very High Temperatures," Fatigue, Fracture and Risk, ASME PVP - Vol. 215, pp.147–153, ASME, New York, 1991.

Freed, A.D., and Walker, K.P., "Viscoplastic Model Development with an Eye Toward Characterization," Journal of Engineering Materials and Technology, Vol. 117, pp. 8–13, 1995.

Krempl, E., "Models of Viscoplasticity: Some Comments on Equilibrium (Back) Stress and Drag Stress," Acta Mechanica, Vol. 69, pp. 25–42, 1987.

Marquis, D., "Modélisation et Identification de L' écrouissage Anisotrope des Métaux," Thèse de 3ème cycle, Université Paris VI, 1979.

Michel, B., Assire, A., and Cambefort, P., "Tertiary Creep Stage in a Nonlinear Kinematic and Isotropic Hardening Elasto-Visco-Plastic Model," Transactions, SMiRT 16, 2001.

Miller, A., "An Inelastic Constitutive Model for Monotonic, Cyclic and Creep Deformation: Part I –Equations Development and Analytical Procedures," Journal of Engineering Materials and Technology, Vol. 98, pp. 97–105, 1976.

Qian, Z., and Fan, J., "Constitutive Model for Creep-Plasticity Interaction Based on Continuous Kinetics of Dislocations," ASME MD - Vol. 26/AMD - Vol. 121, High Temperature Constitutive Modeling – Theory and Application, pp. 49–63, 1991.

Robinson, D.N., "Constitutive Relationships for Anisotropic High Temperature Alloys," Nuclear Engineering and Design, Vol. 83, pp. 389–396, 1984

Schwertel, J., Merckling, G., Hornberger, K., Schinke, B., and Munz, D., "Experimental Investigations on the Ni-Base Superalloy IN617 and their Theoretical Description," ASME MD-Vol.26/AMD–Vol. 121, High Temperature Constitutive Modeling – Theory and Application, pp. 285–295, 1991.

Tachibana, Y., and Krempl, E., "Modeling of High Homologous Temperature Deformation Behavior Using the Viscoplasticity Theory Based on Overstress (VBO): Part III – A Simplified Model," Journal of Engineering Materials and Technology, Vol. 120, pp. 193–196, 1998.

Copyright © 2008 by ASME