Image processing:
Neutron Tomography

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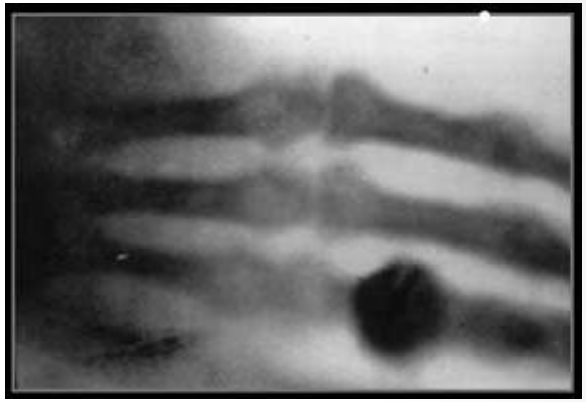
History of tomography

- **1895:** Roentgen discovered X-Rays and performed the first radiography

- **1917:** Radon developed the original mathematical framework

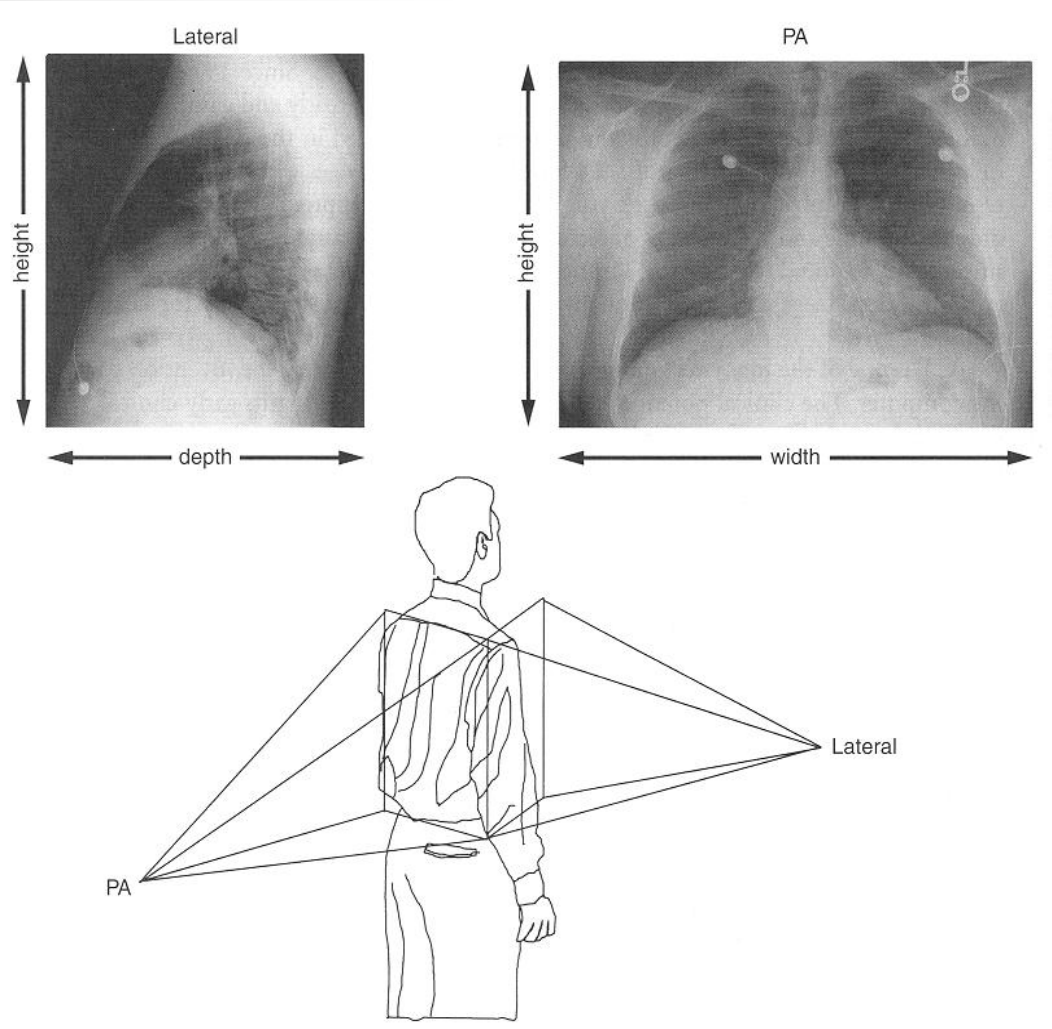
- **1972:** Hounsfield developed the first X-ray Computerised Tomography (CT) scanner



From radiography to tomography

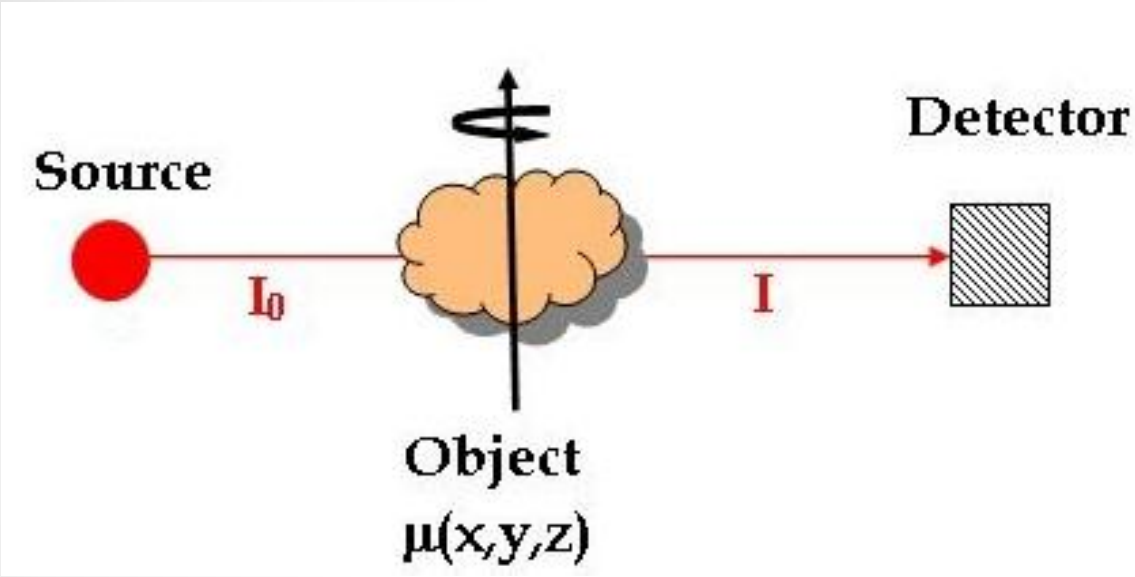
- With a conventional radiograph, information related to the dimension parallel to the beam is lost.
- This limitation can be overcome, to some degree, by acquiring two perpendicular images.
- For objects that can be identified in both images, location information can be identified.

From radiography to tomography



With an infinite number of radiographies at different angles the whole volume could be reconstructed

Transmission tomography set-up



$$dI(x) = -I(x) \cdot n \cdot \sigma \cdot dx$$

integrating ➡ $I = I_0 \cdot e^{n \sigma x}$

dI = change in intensity

I_0= initial intensity

n = the number of atoms/cm^3

σ = proportionality constant (total probability of a radiation being scattered or absorbed)

dx = incremental thickness of material traversed

x = thickness of material traversed

Attenuation coefficient

$$I = I_0 \cdot e^{n \sigma x}$$

$$n\sigma = \mu$$

$$I = I_0 \cdot e^{\mu x}$$

μ: fraction of a beam absorbed or scattered per unit thickness of the absorber

Number of atoms in a cubic cm volume of material

Probability interaction between matter and radiation

I = transmitted intensity

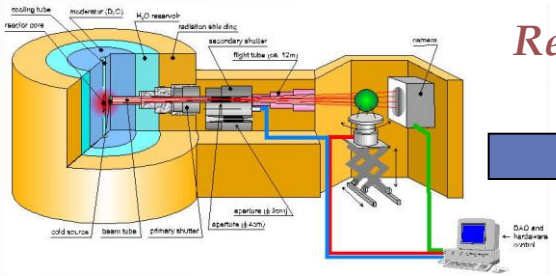
I_0 = initial intensity

μ = linear attenuation coefficient

x = distance travelled

Set-up and data reconstruction

Example: OCTOPUS software

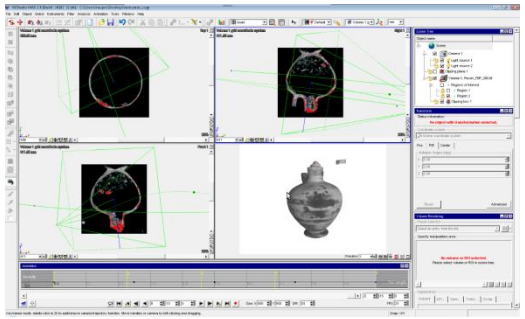


Reconstruction algorithm

Image stack

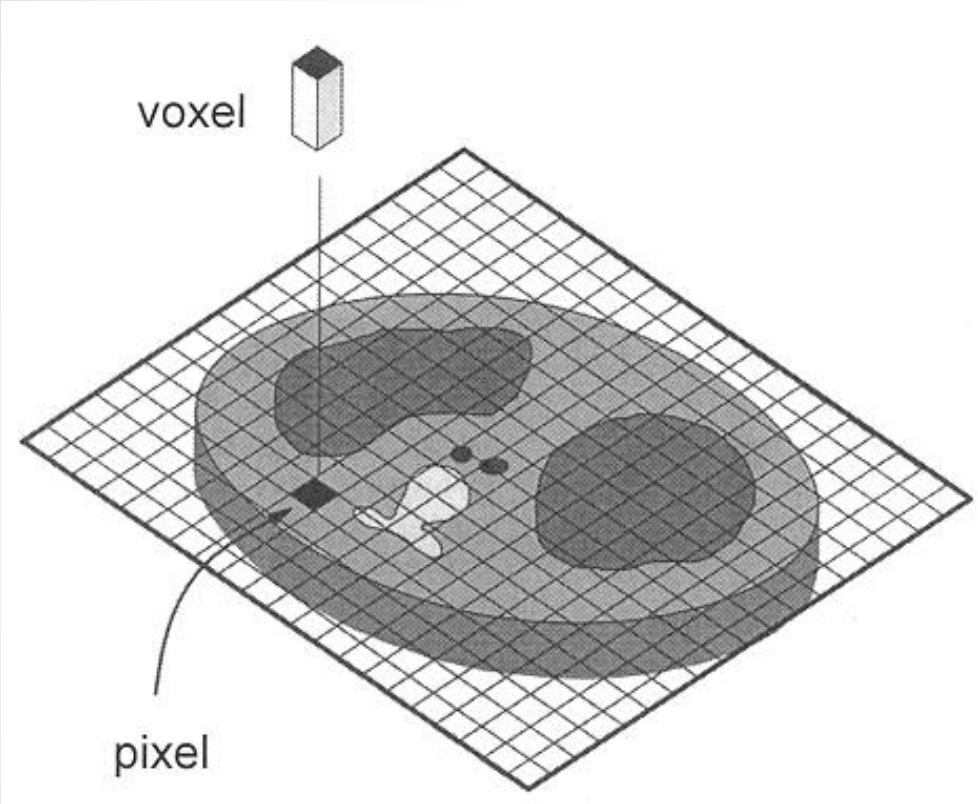
Tomography instrument output = radiographies

Example: VGStudio Max software



3D processing

Tomography principle



Reconstruction of the attenuation coefficient in a voxel (3D pixel) through a reconstruction algorithm (back-projection)

Display of the numerical values of 3D attenuation contribution

Reconstruction from projections
(non-diffracting source)

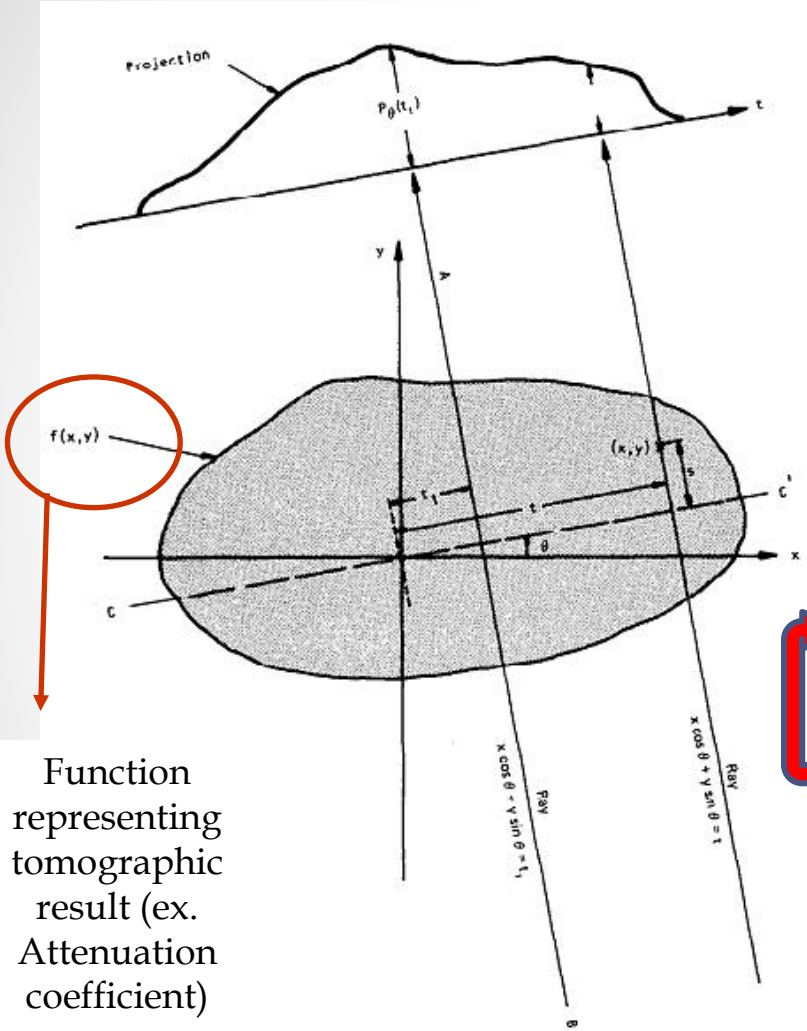
<u>Reconstruction problem:</u>

how to recover an image of the total cross section of an object from the projection data

There are many reconstruction algorithms

Filtered back-projection reconstruction is most widely used in CT scanners

Reconstruction algorithm: parallel geometry



Function representing tomographic result (ex. Attenuation coefficient)

$$P_\theta(t) = \int_{(\theta,t)\ \text{line}} f(x, y)\, ds.$$

Line integral → total attenuation suffered by a beam, along a line (the ray direction) through the object

2 Coordinate system: x,y (rettangular coordinates); θ, t (polar coordinates)

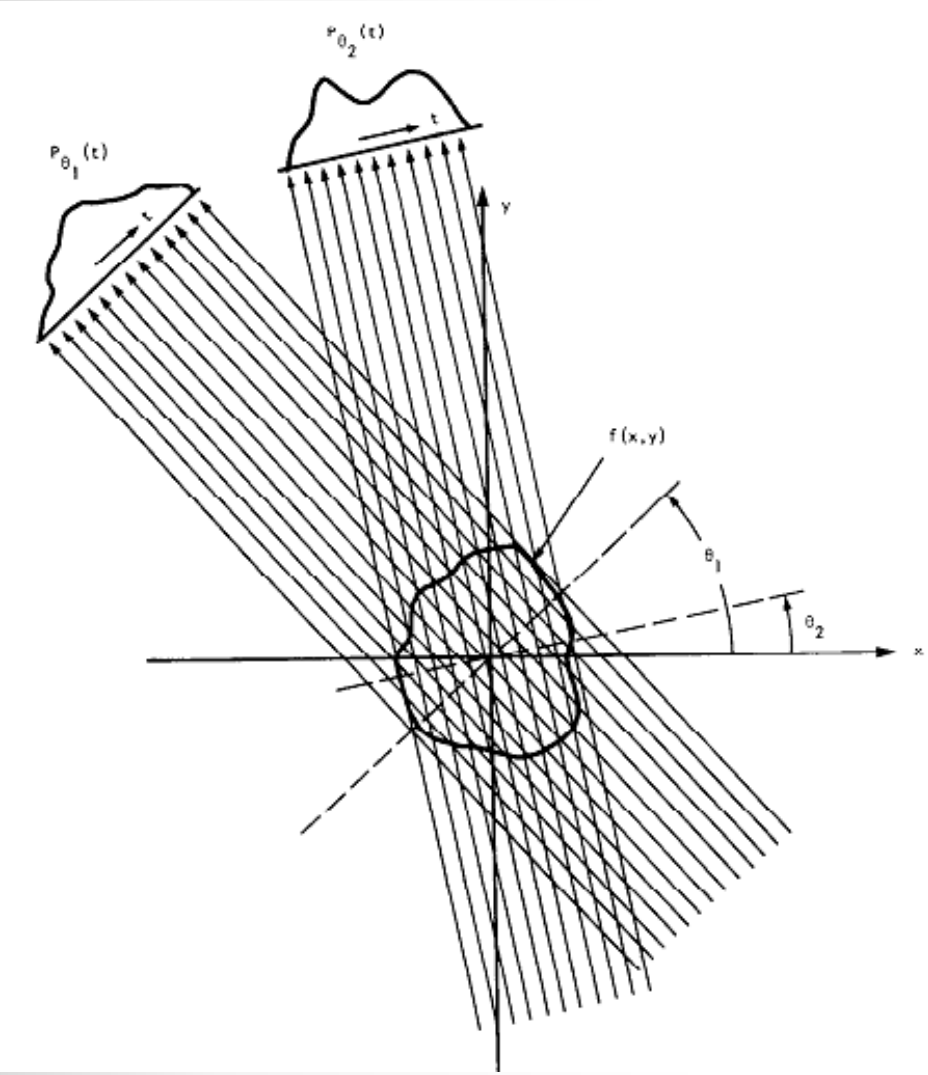
Using a delta function along the line AB, this can be rewritten as

$$P_\theta(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\delta(x\cos\theta + y\sin\theta - t)\, dx\, dy$$

collection of all projections

$P_\theta(t)$ is known as the Radon transform of the function f(x,y)

Parallel projection



A projection is formed by combining a set of integrals

The simplest projection
→ parallel ray integrals
→ $P_\theta(t)$ for a constant θ

Reconstruction algorithm: parallel geometry

$$\begin{bmatrix} t \\ s \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Taking the **one-dimensional Fourier transform of a parallel projection** we can see that is **equal to a slice of the two dimensional Fourier transform of the original object**

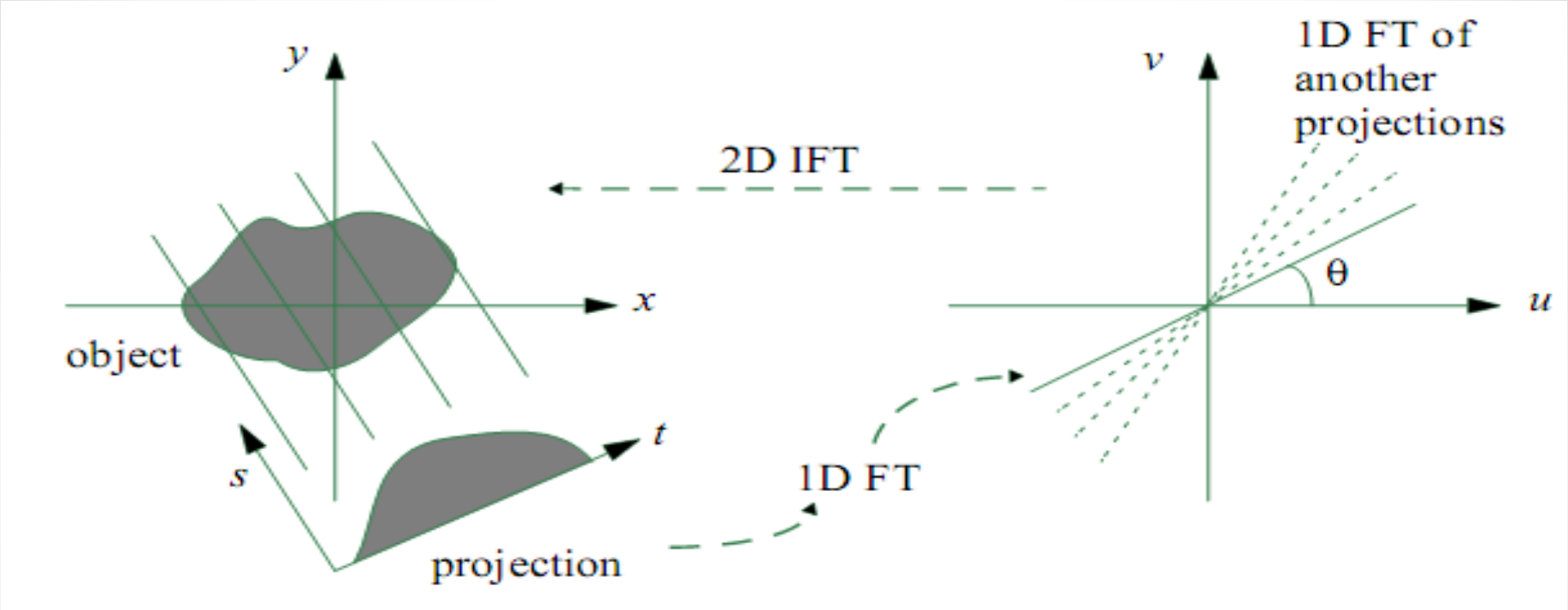
$$S_\theta(\omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y) e^{-2\pi i \omega (x\cos\theta + y sen\theta)} dxdy$$

Fourier Slice Theorem

$$\boxed{S_\theta(\omega) = F(\omega,\theta) = F(\omega\cos\theta, \omega sen\theta)}$$

Represents the 2-dimensional Fourier Transform of the f(x,y) at a spatial frequency of (u=ωcosθ, v= ωsenθ) → u,v=coordinate in the frequency plane

Fourier Slice Theorem



The 1D **Fourier Transform** (FT) of a **projection** taken at angle θ **=** the central radial slice at angle θ of the 2D **FT** of the **original object** (f(x,y))

Which means: Inverse Fourier Transform (IFT) of the $F(\omega,\theta)=S_\theta(\omega)$ give the f(x,y) → ex. Attenuation coefficient map

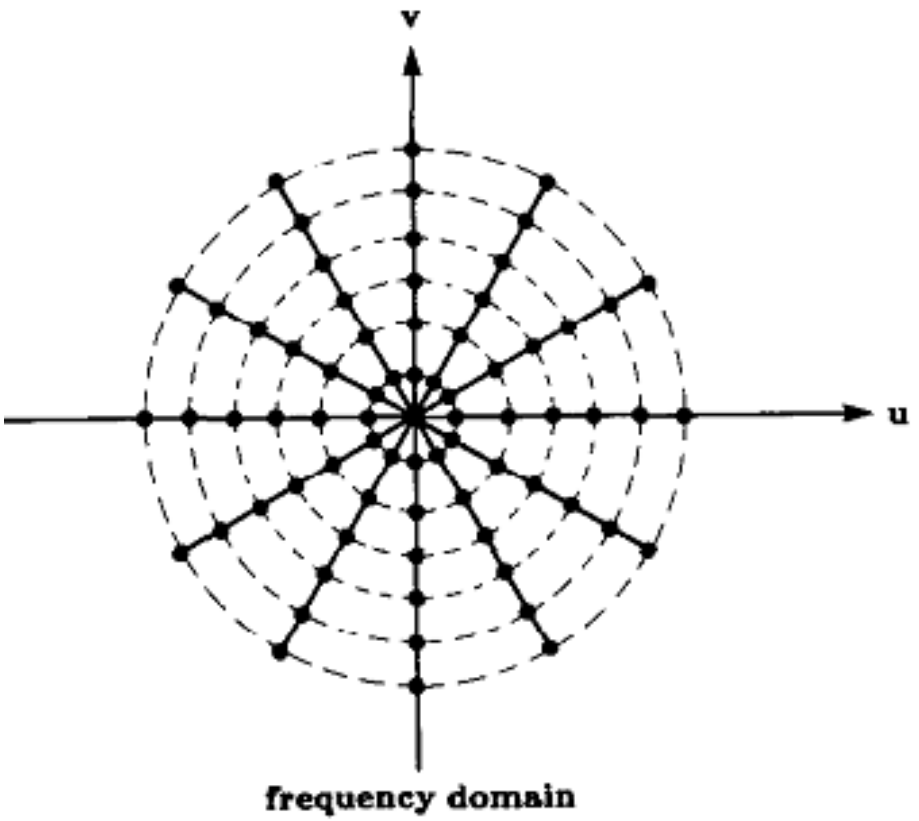
Reconstruction from projections

Experimentally the Radon Transform is collected for

- a finite set of directions (angles) that corresponds to the lines that pass through the origin

- a finite set of points in a defined direction (points on the lines)

Each line is a projection



Back-projection algorithm (for parallel beam)

Recalling the Inverse FT

$$f(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,v)e^{2\pi i(ux+vy)}dudv$$

Exchanging the coordinate system in the frequency domain

$$u = \omega\cos\theta$$
$$v = \omega sen\theta$$

$$\Rightarrow \quad f(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\omega,\theta)e^{2\pi i\omega(x\cos\theta+y\sin\theta)}\omega d\omega d\theta$$

Integral can be split into intervals (0,π) and (π, 2 π)

$$f(x,y) = \int_0^{\pi}\int_0^{\infty} F(\omega,\theta)e^{2\pi i\omega(x\cos\theta+y\sin\theta)}\omega d\omega d\theta + \int_0^{\pi}\int_0^{\infty} F(\omega,\theta+\pi)e^{2\pi i\omega[x\cos(\theta+\pi)+y\sin(\theta+\pi)]}\omega d\omega d\theta$$

Back-projection algorithm (for parallel beam)

Using the property $F(\omega,\theta+\pi)=F(-\omega,\theta)$

but $t = x\cos\theta + y\sin\theta$ and f(x,y) may be writter as:

$$f(x,y)=\int_0^{\pi}\left[\int_{-\infty}^{+\infty}F(\omega,\theta)|\omega|e^{2\pi i\omega t}d\omega\right]d\theta$$

For the **Fourier Slice Theorem** the $F(\omega,\theta)=S_\theta(\omega)$

$$f(x,y)=\int_0^{\pi}\left[\int_{-\infty}^{+\infty}S_\theta(\omega)|\omega|e^{2\pi i\omega t}d\omega\right]d\theta$$

Filtered back-projection = $Q_\theta(t)$

Theory of the sampling

Theory is developed in continuum

Measurements are discrete

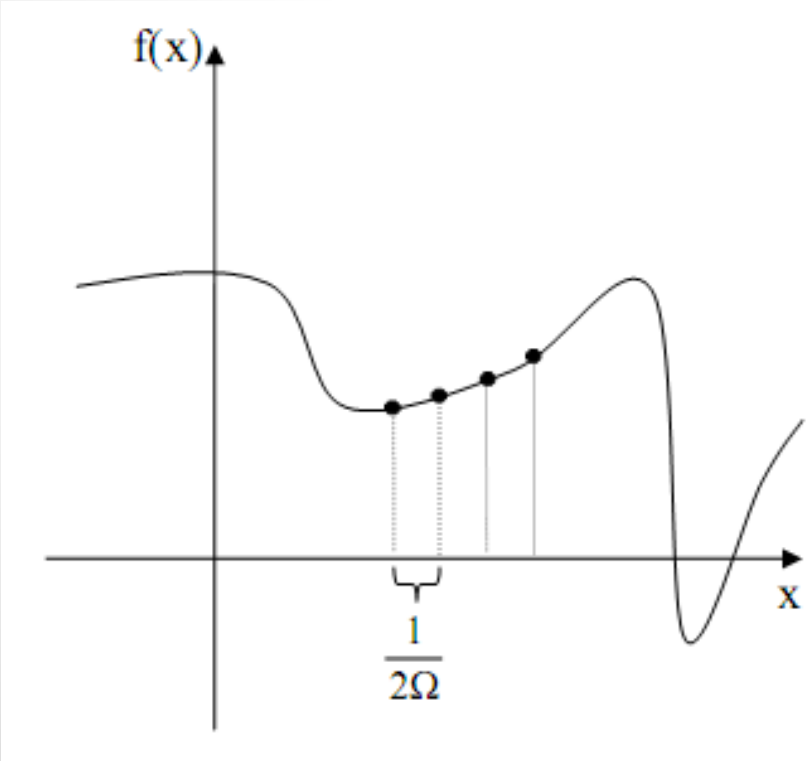
(radiographies for finite set of angles)

We need a passage from the continuum to the discrete formalism and vice versa

THEORY OF SAMPLING

Theory of the sampling

Sampling is the process of **converting a signal** (ex. function of continuous space) **into a numeric sequence** (a function of discrete space) → signal quantization



$$\tau = \frac{1}{2\Omega}$$ **sampling frequency**

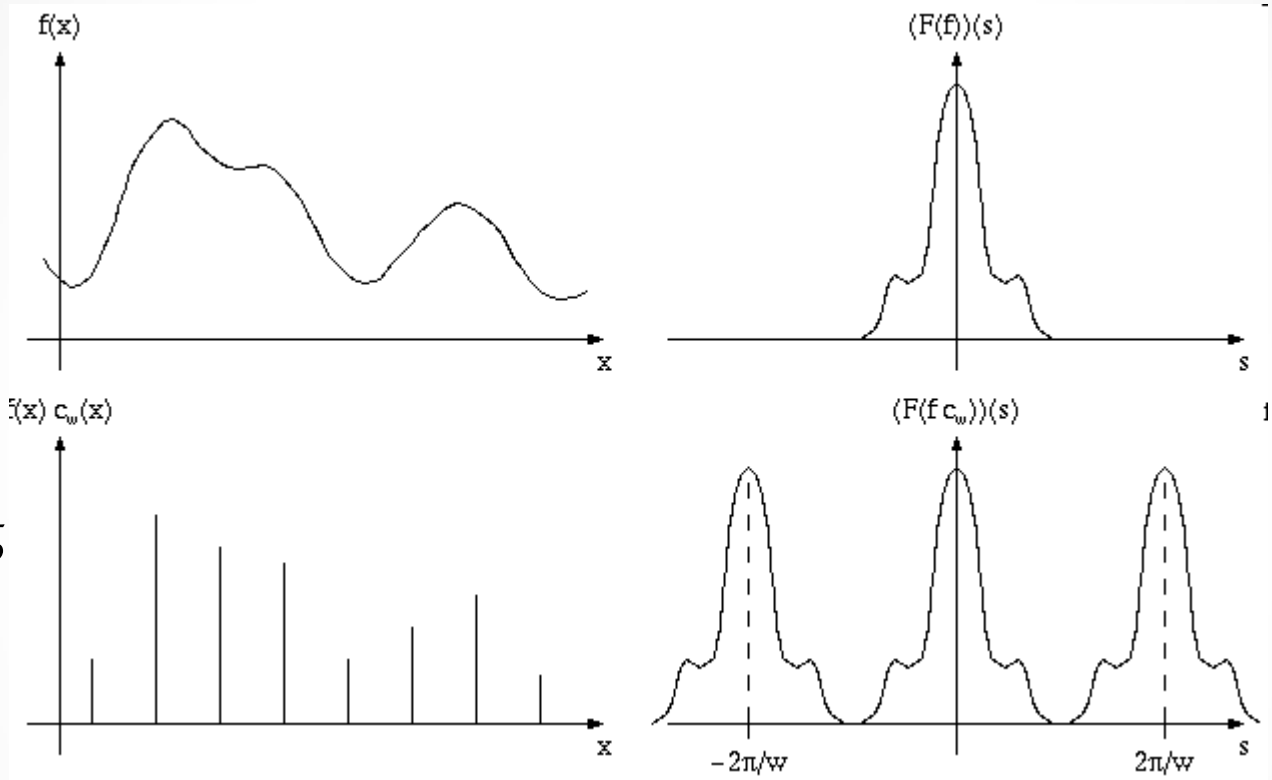
Sampling theorem: in order to recover the signal function f(x) exactly, it is necessary to sample f(x) at a rate greater than twice its highest frequency component (s_0)

($2s_0$ = Nyquist rate)

Theory of the sampling

Function

Fourier Transform

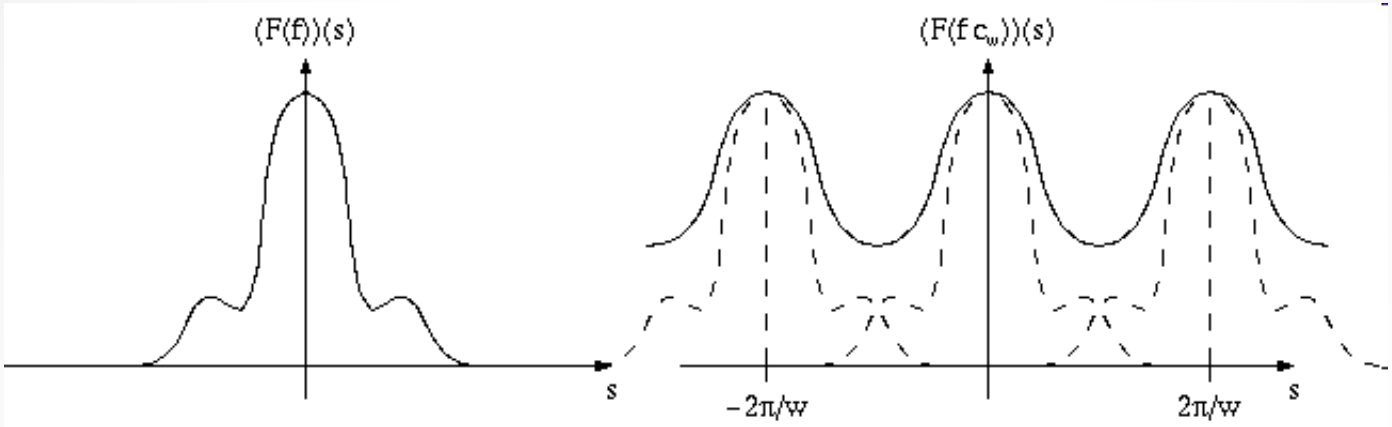


Sampling

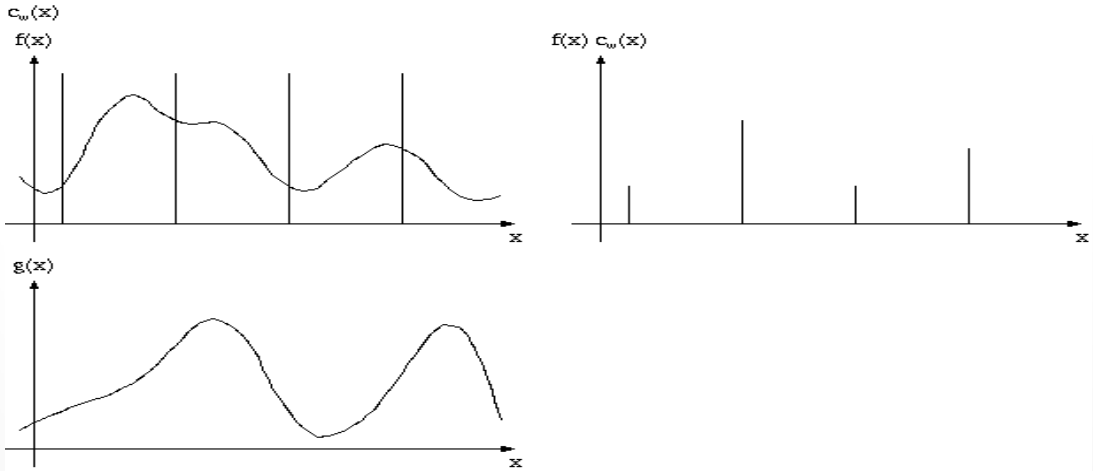
Sampling Fourier Transform

After sampling the spectrum F(f) is replicated infinitely often in both direction

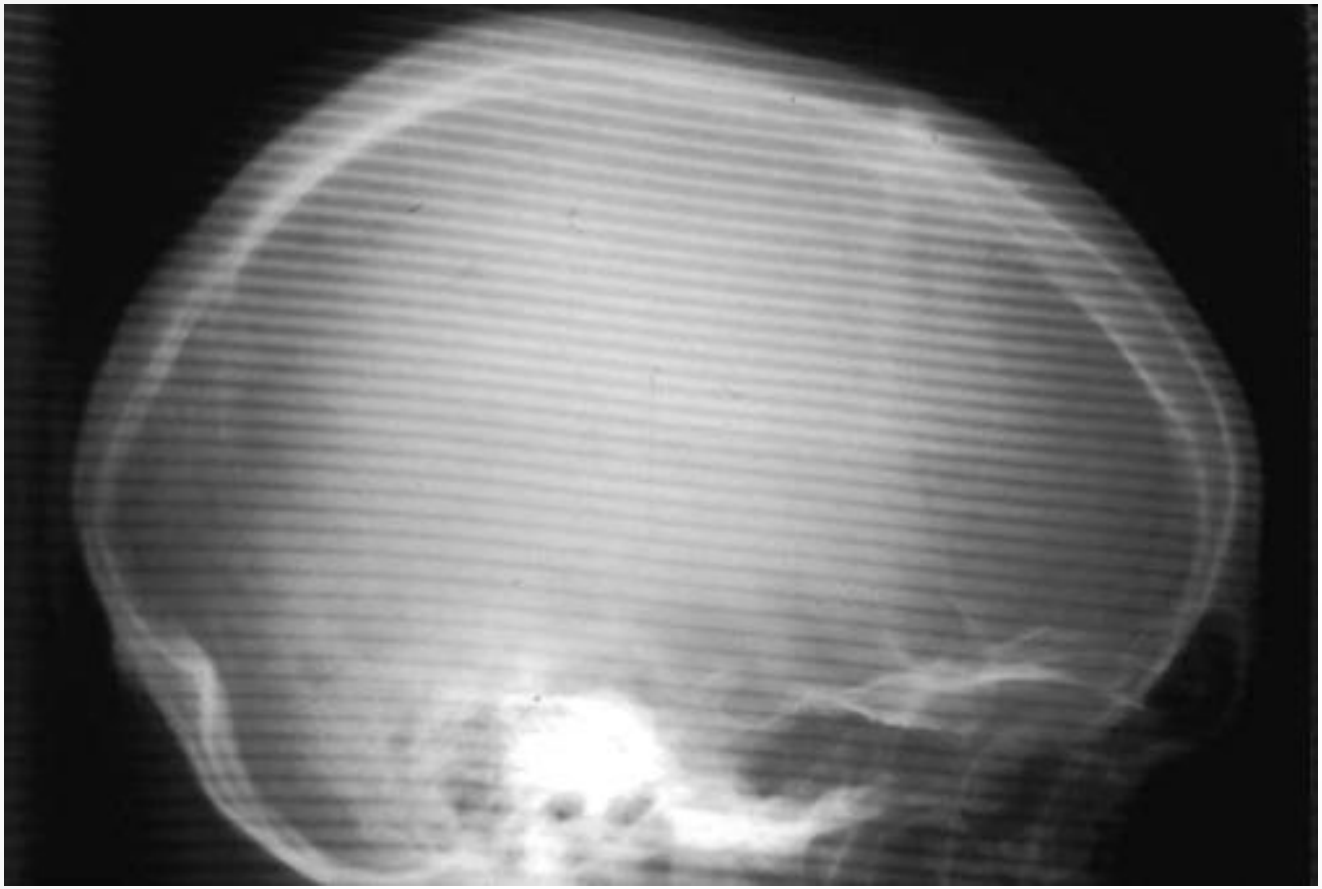
Errors of the sampling- Aliasing



The consequences of sampling a signal at a rate below its highest frequency component results in a phenomenon known as **aliasing**

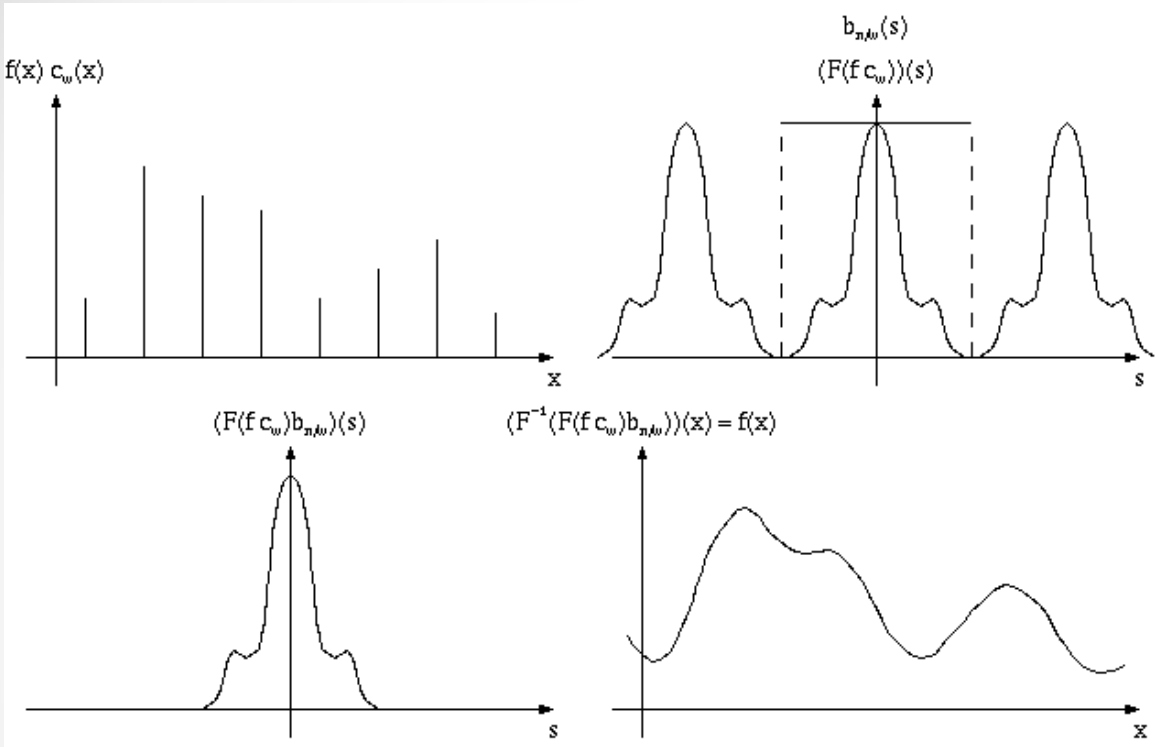


Errors of the sampling- Aliasing



Aliasing artifact on a lateral skull X-Ray image

Optimal reconstruction

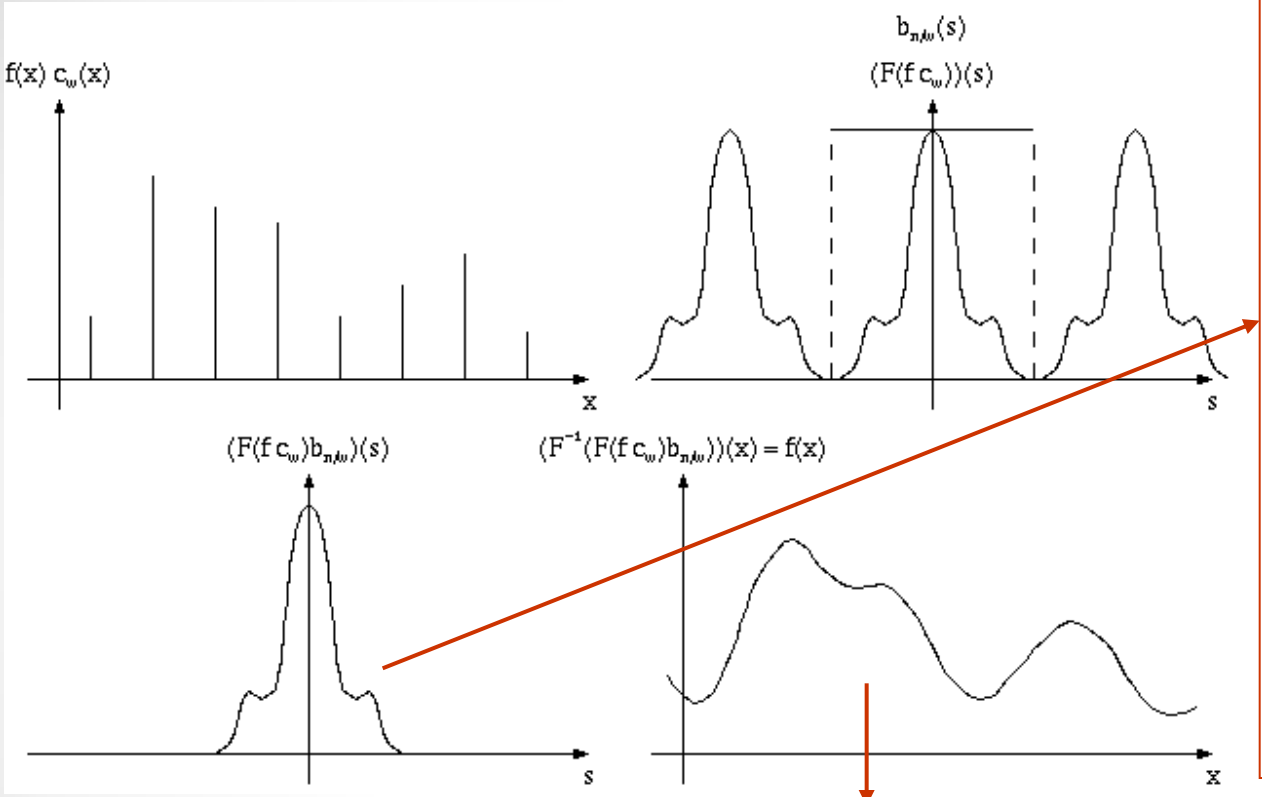


The only difference between *f* and its sampled version is that *f*'s spectrum has been replicated by the sampling process

Solution: multiply the Fourier transform of the sampled version by a **box filter** (function that is 1 in the interval [-ω,ω]

Window Method

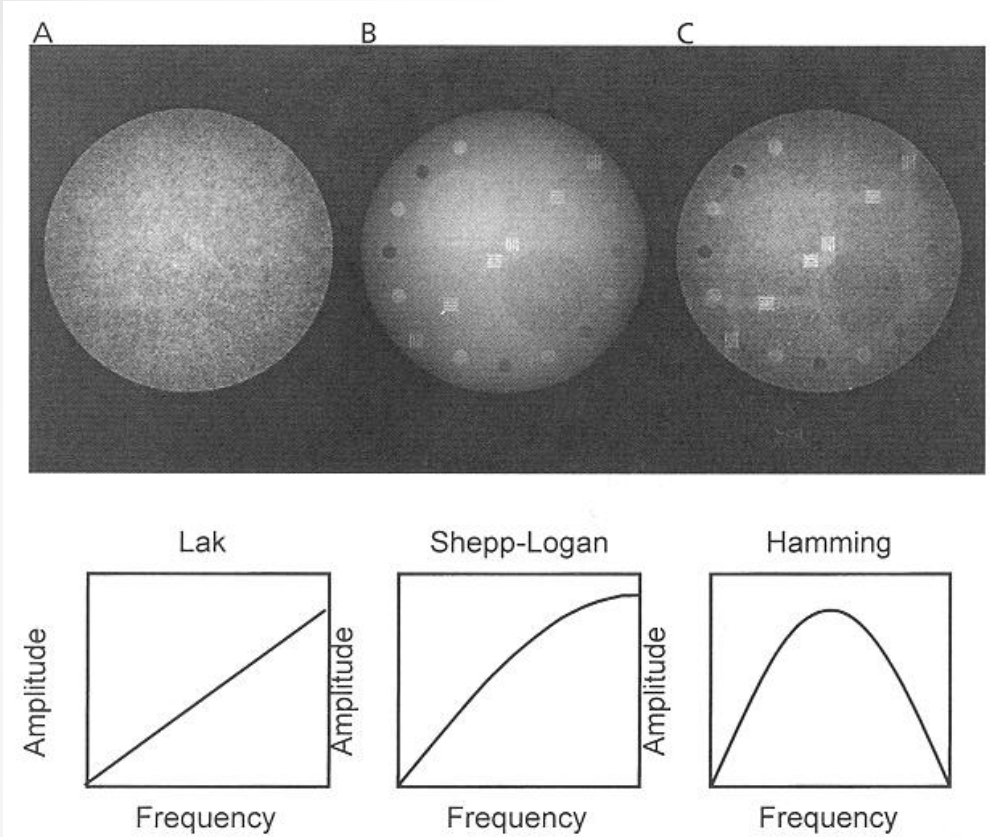
Box windows



Multiplying the sampled spectrum point-by-point with the box filter will extract the original spectrum from the sampled version

calculate the inverse Fourier transform of the box-filtered spectrum and (if we have frequency above the Nyquist rate) this will exactly reconstruct *f*

Filters



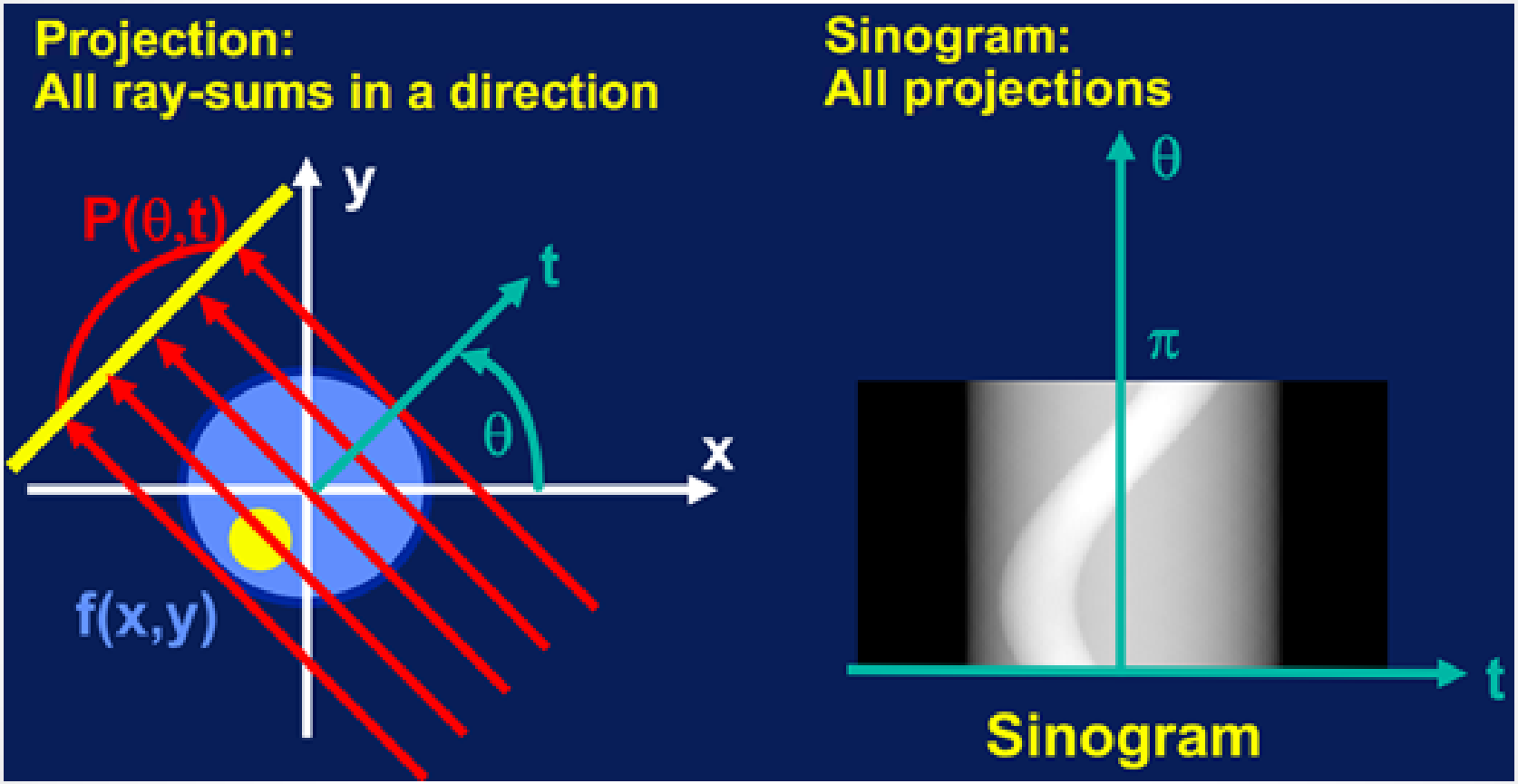
Different filters can be used to emphasize different characteristics in the CT image. Several filters, shown in the frequency domain, are illustrated, along with the reconstructed CT images they produced.

increases amplitude linearly as a function of frequency - *ramp filter*

roll-off at higher frequencies → reducing high-frequency noise in the final CT image

Even more pronounced high-frequency *roll-off*, with better high-frequency noise suppression.

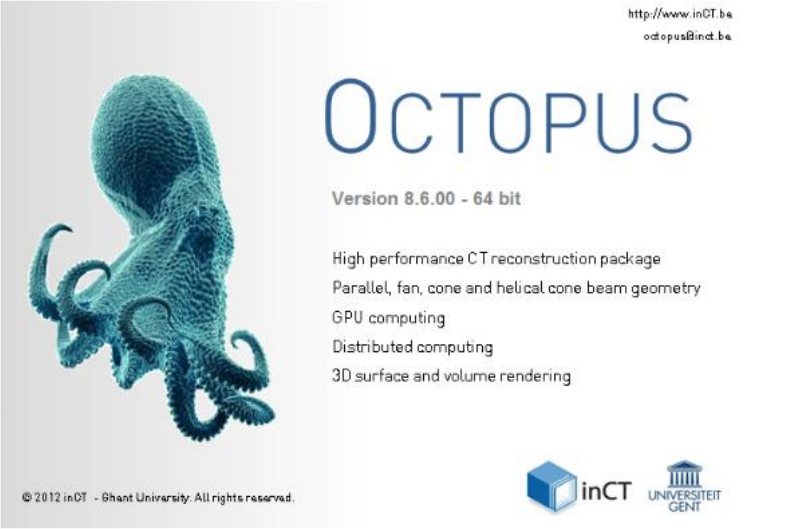
Data reconstruction

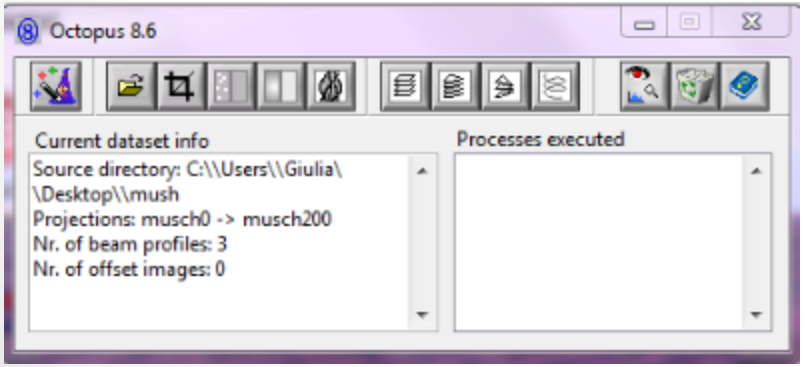


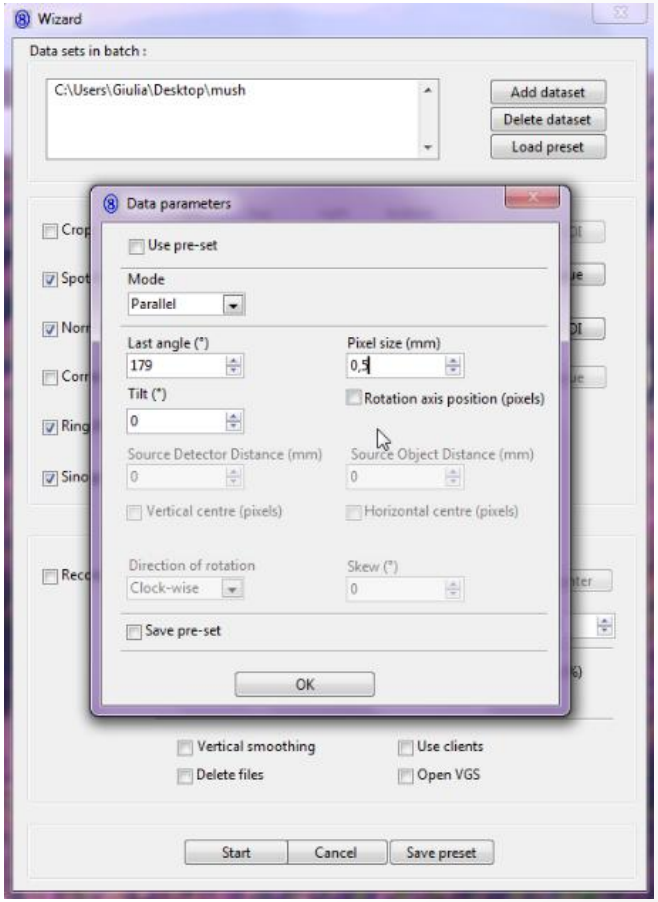
Step by step procedure to obtain results from set of neutron radiographies

Step by step procedure

Step 1: from radiographies to sinograms (Output → sinograms)



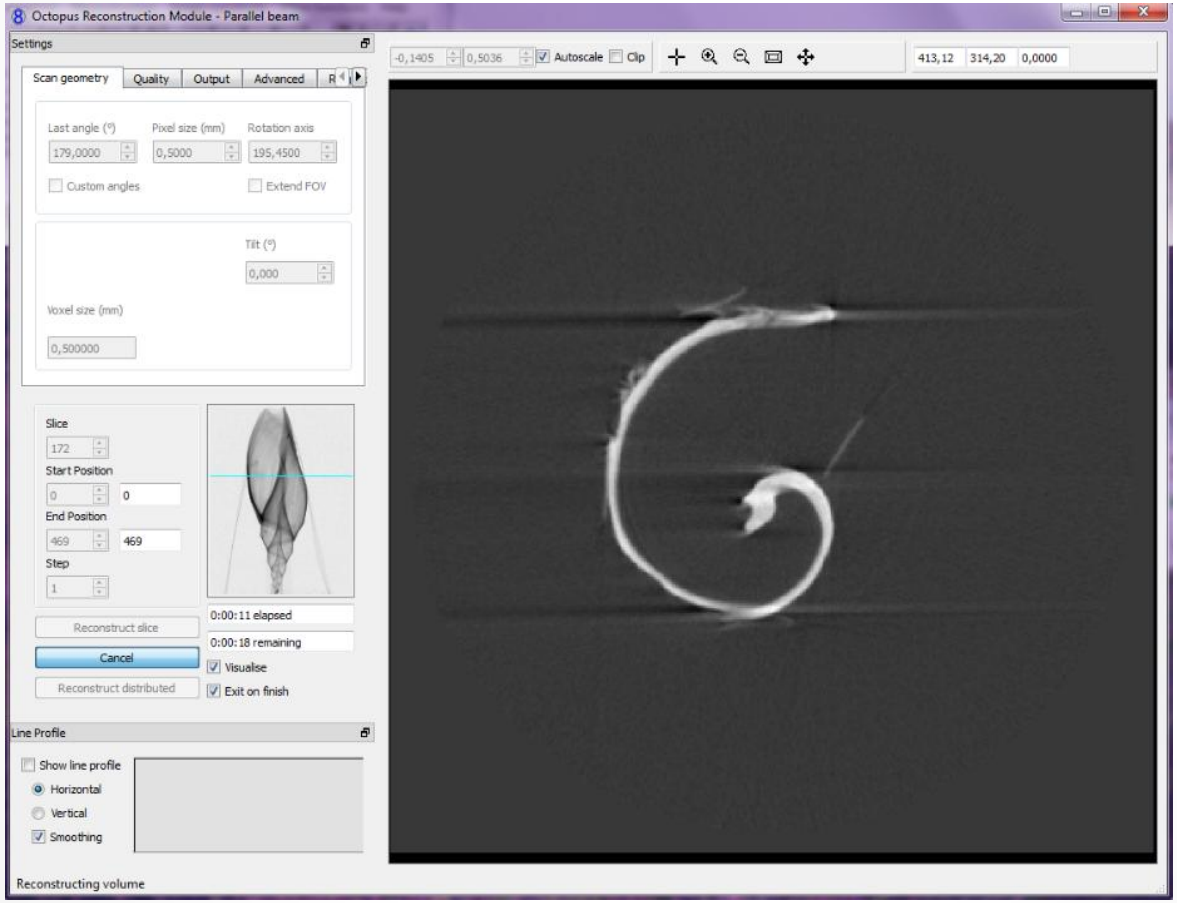




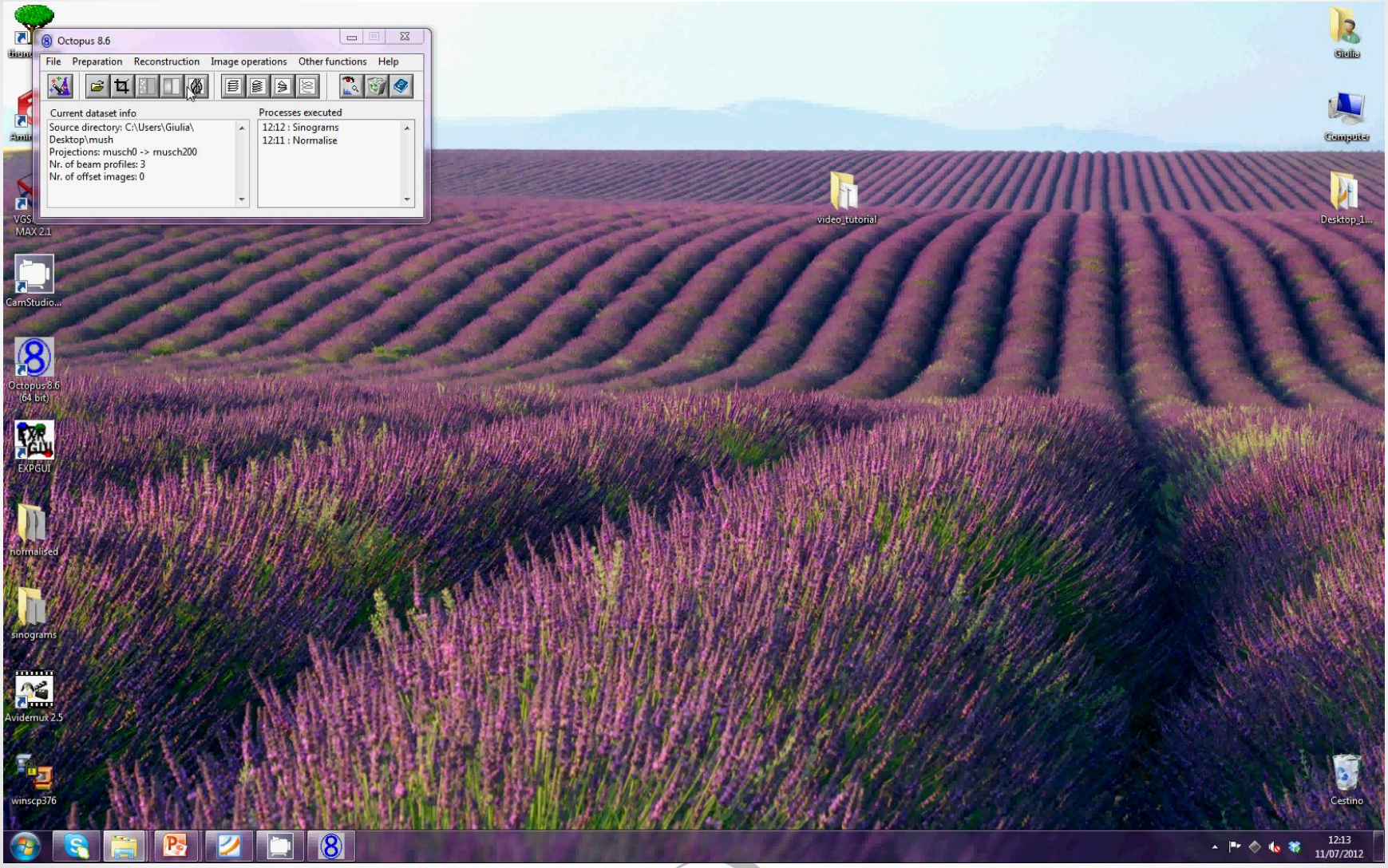
Step 1: from radiographies to sinograms

Step by step procedure

Step 2: reconstruction algorithm (Output → slices, image stack)

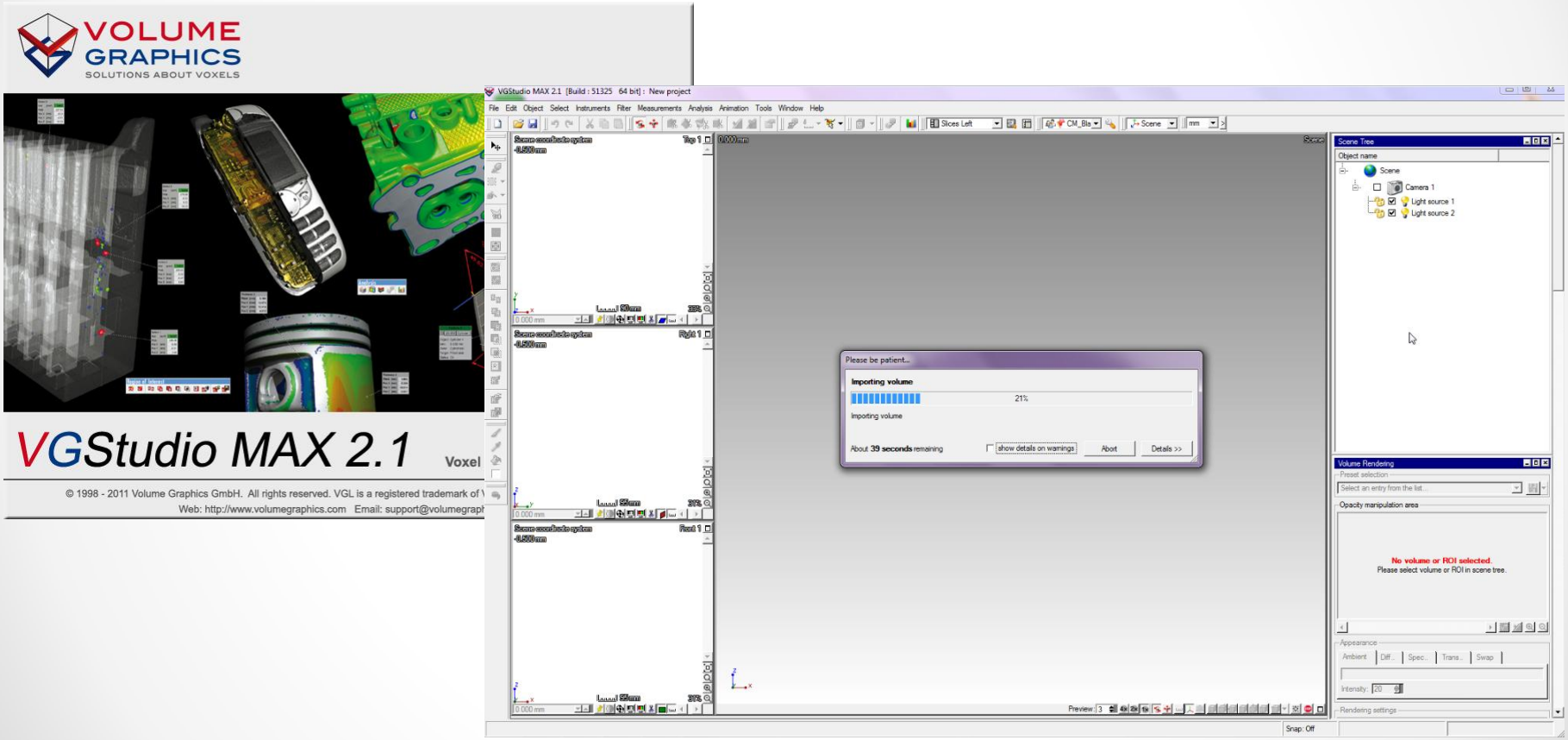


Step 2: reconstruction algorithm

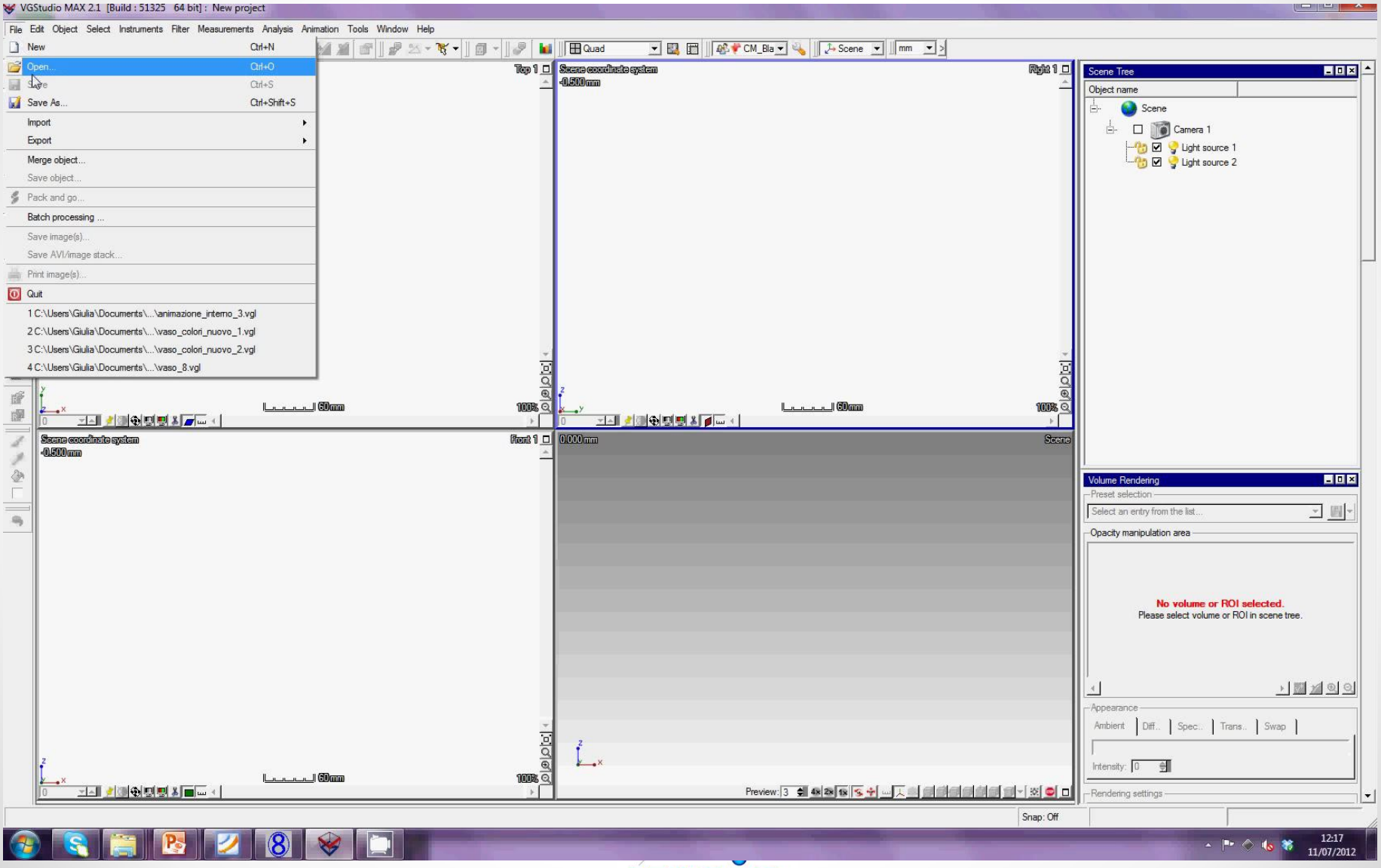


Step by step procedure

Step 3: import image stack in VG Studio MAX (3D rendering)

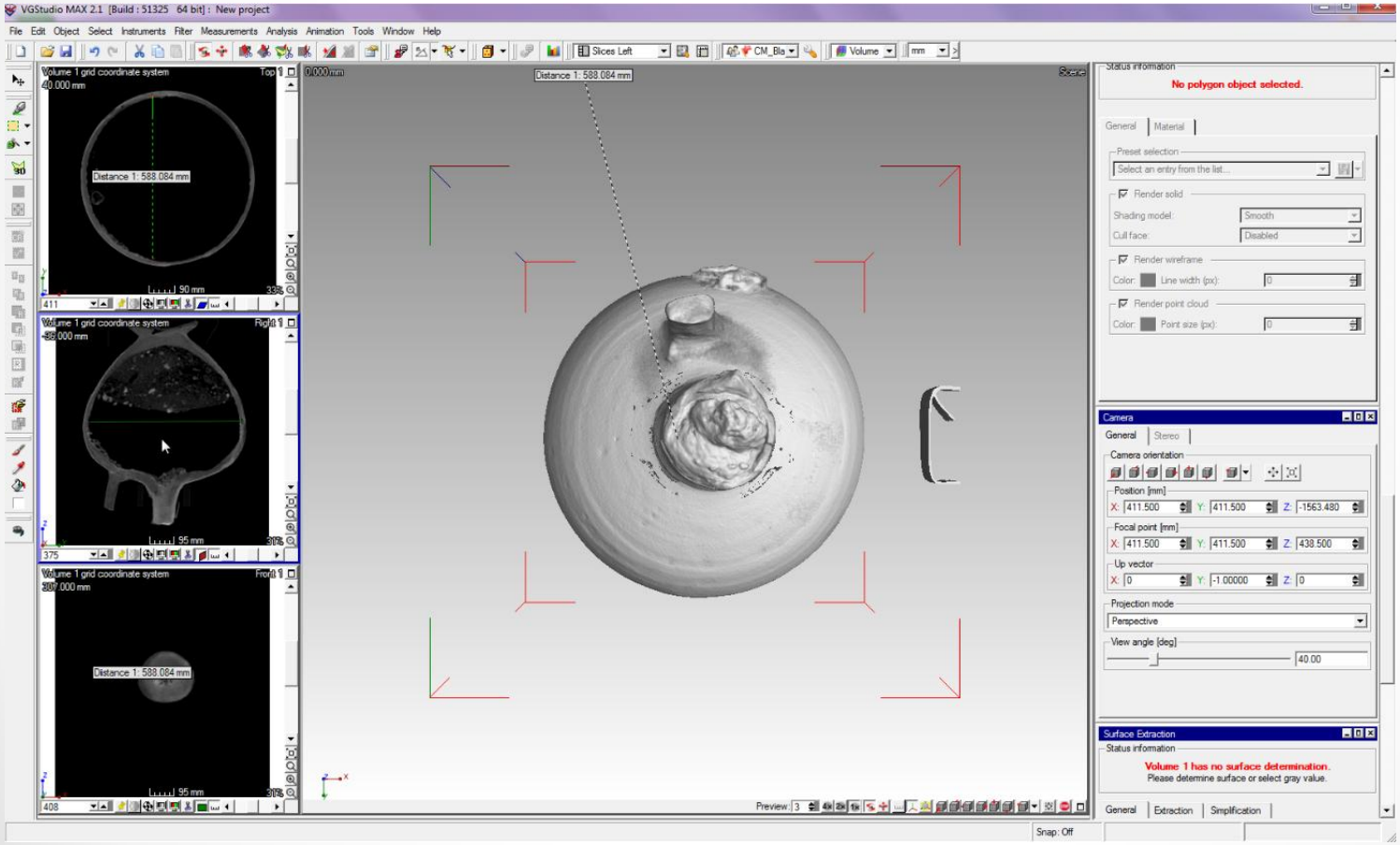


Step 3: import image stack in VG Studio MAX

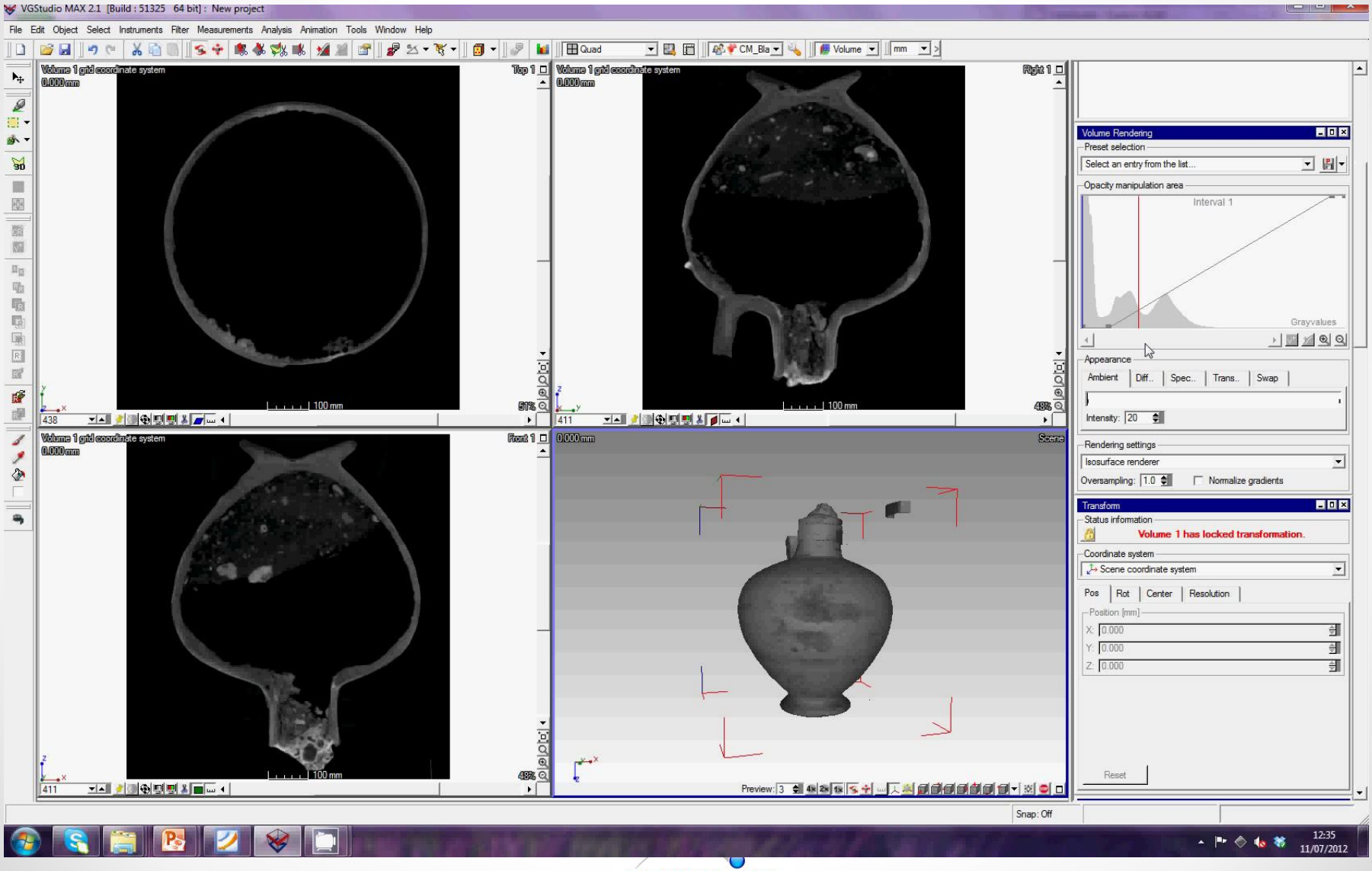


Step by step procedure

Step 4: final details with VG Studio MAX

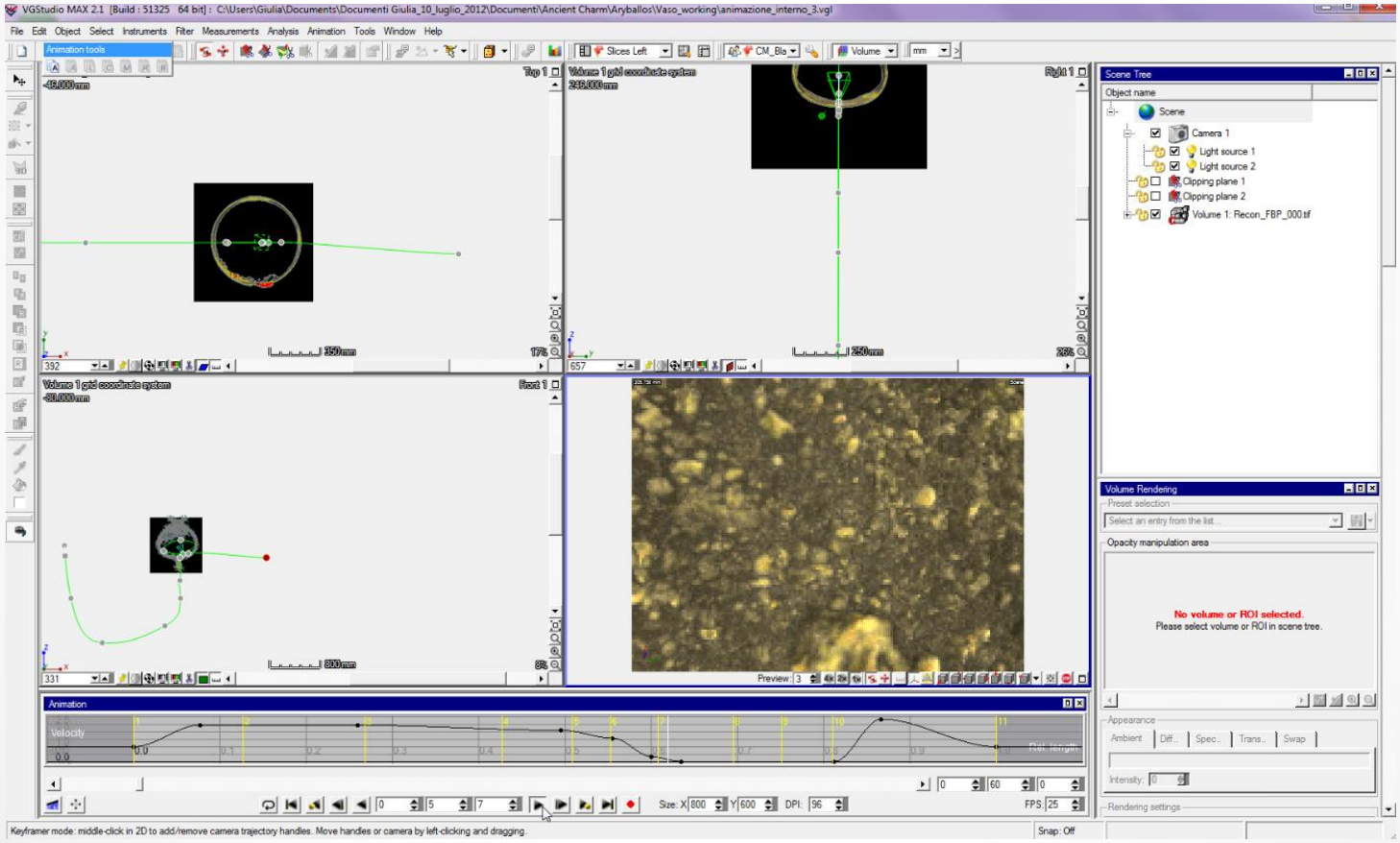


Step 4: final details with VG Studio MAX

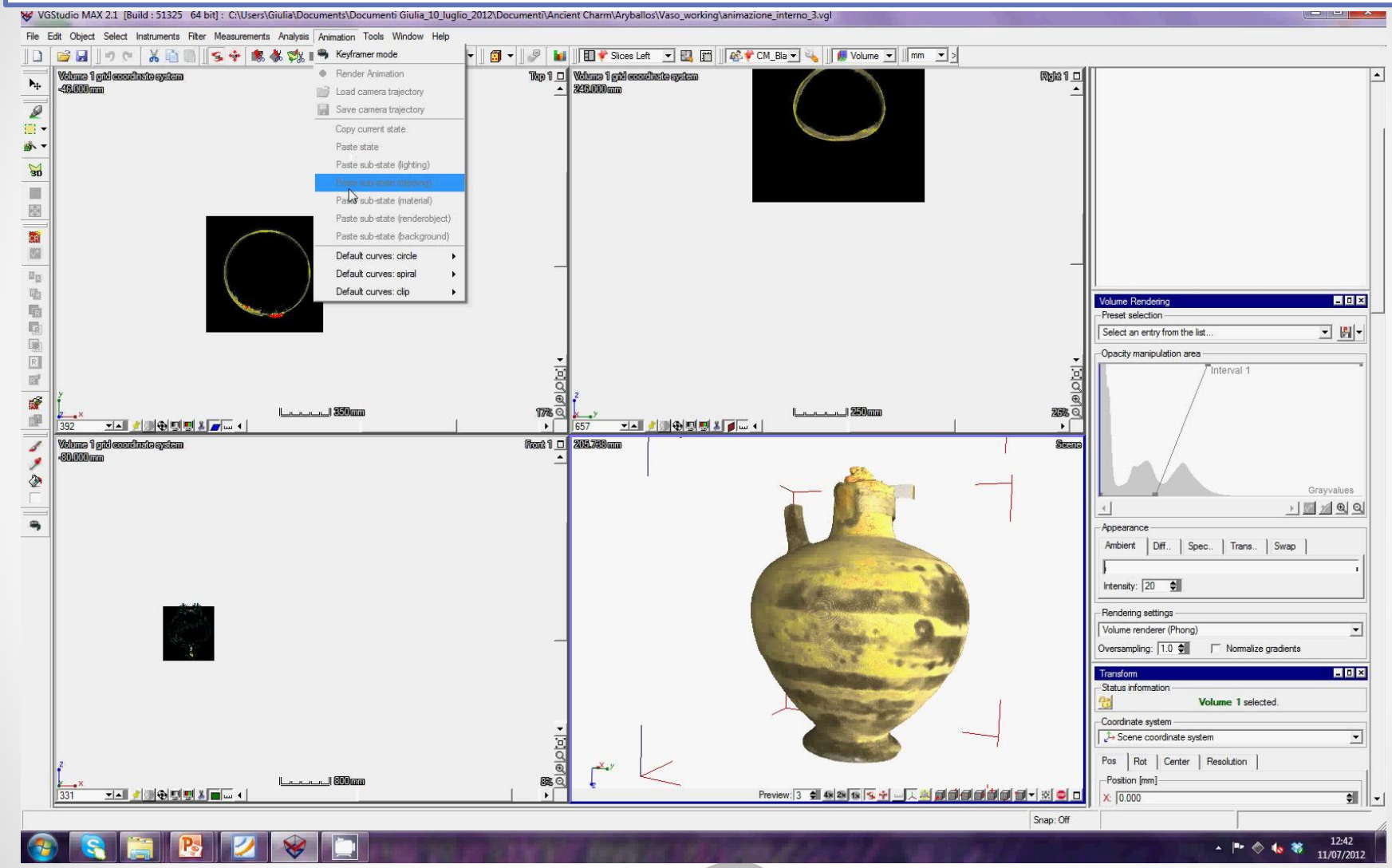


Step by step procedure

Step 5: video creation (Output → videos)



Step 5: video creation



Step by step procedure

Step 6: final video after some video editing

Protocorinthian style (VII- VI sec. A. C.)
Necropolis of the Banditaccia - Cerveteri

Aryballos

Little ceramic vase (5.9 cm height)

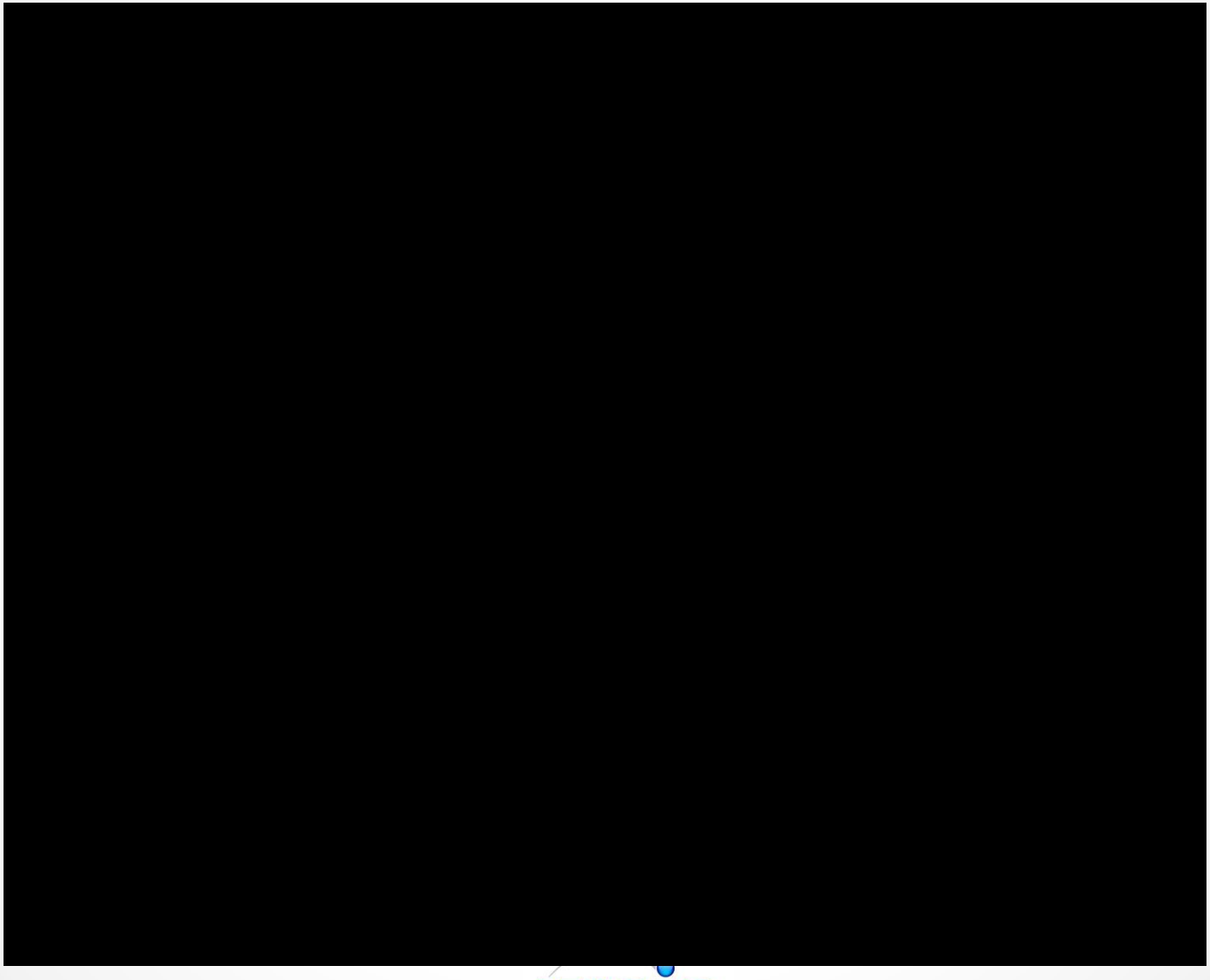
Original cap sealed and intact

From Civiche raccolte archeologiche
e numismatiche di Milano

Project: ANCIENT CHARM
Coordinator: G. Gorini (Università degli Studi di Milano- Bicocca)
Animation and video: G. Festa (Università degli Studi di Roma 'Tor Vergata')

Work performed with financial support by the European Community
"New and Emerging science and technology" contract n. 15311

Step 6: final video after some video editing



References:

[1] A. C. Kak and Malcolm Slaney, Principles of Computerized Tomographic Imaging. Society of Industrial and Applied Mathematics, 2001

[2] M.A. Picardello, Analisi di Fourier e trattamento numerico dei segnali, http://www.mat.uniroma2.it/~picard/SMC/didattica/materiali_did/Anal.Armon./LIBRO.pdf

[3] http://www.inct.be/en/software/octopus

[4] http://www.volumegraphics.com/en/

[5] http://www.psi.ch/sinq/neutra/

[6] http://einrichtungen.physik.tu-muenchen.de/antares/

[7] N. Kardjilov, A. Hilger, I. Manke, M. Strobl, M. Dawson, S. Williams, J. Banhart, Neutron tomography instrument CONRAD at HZB, Nuclear Instruments and Methods in Physics Research Section A, Volume 651, Issue 1, 21 September 2011, Pages 47–52

[8] http://nmi3.eu/news-and-media/neutron-imaging-past-present-and-future.html