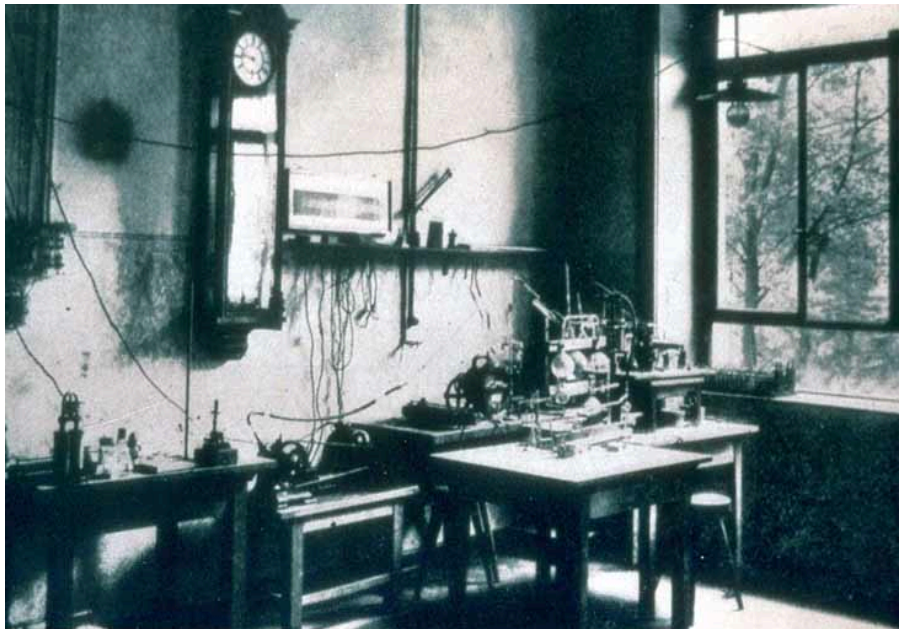
Frascati, INSIS, July 2012

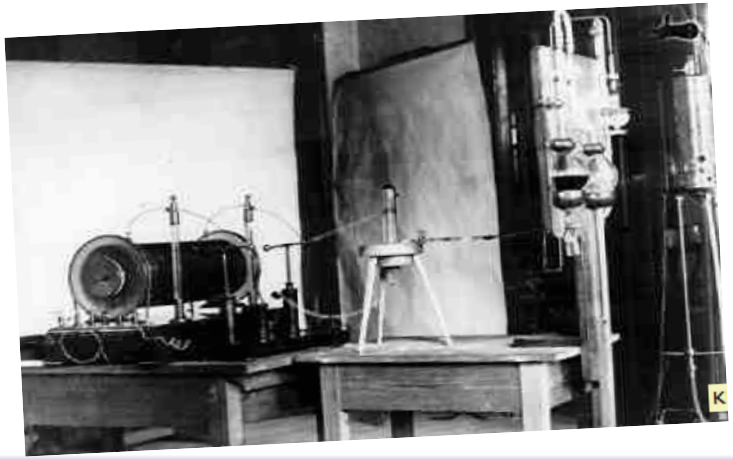
Neutron Imaging:

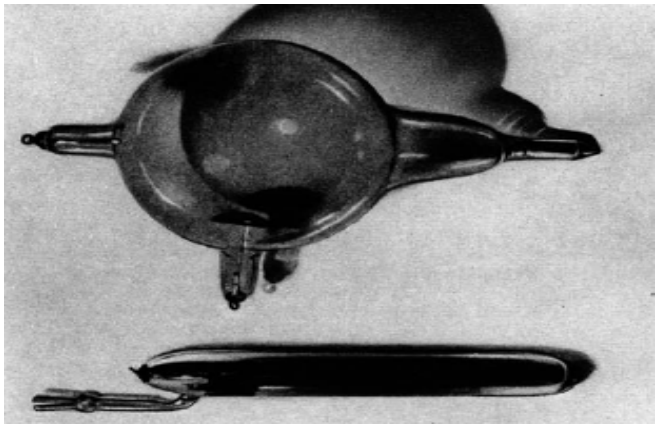
Basics, Tomography, Coherence

Franz Pfeiffer
Department of Physics
Technical University Munich

Historical Context







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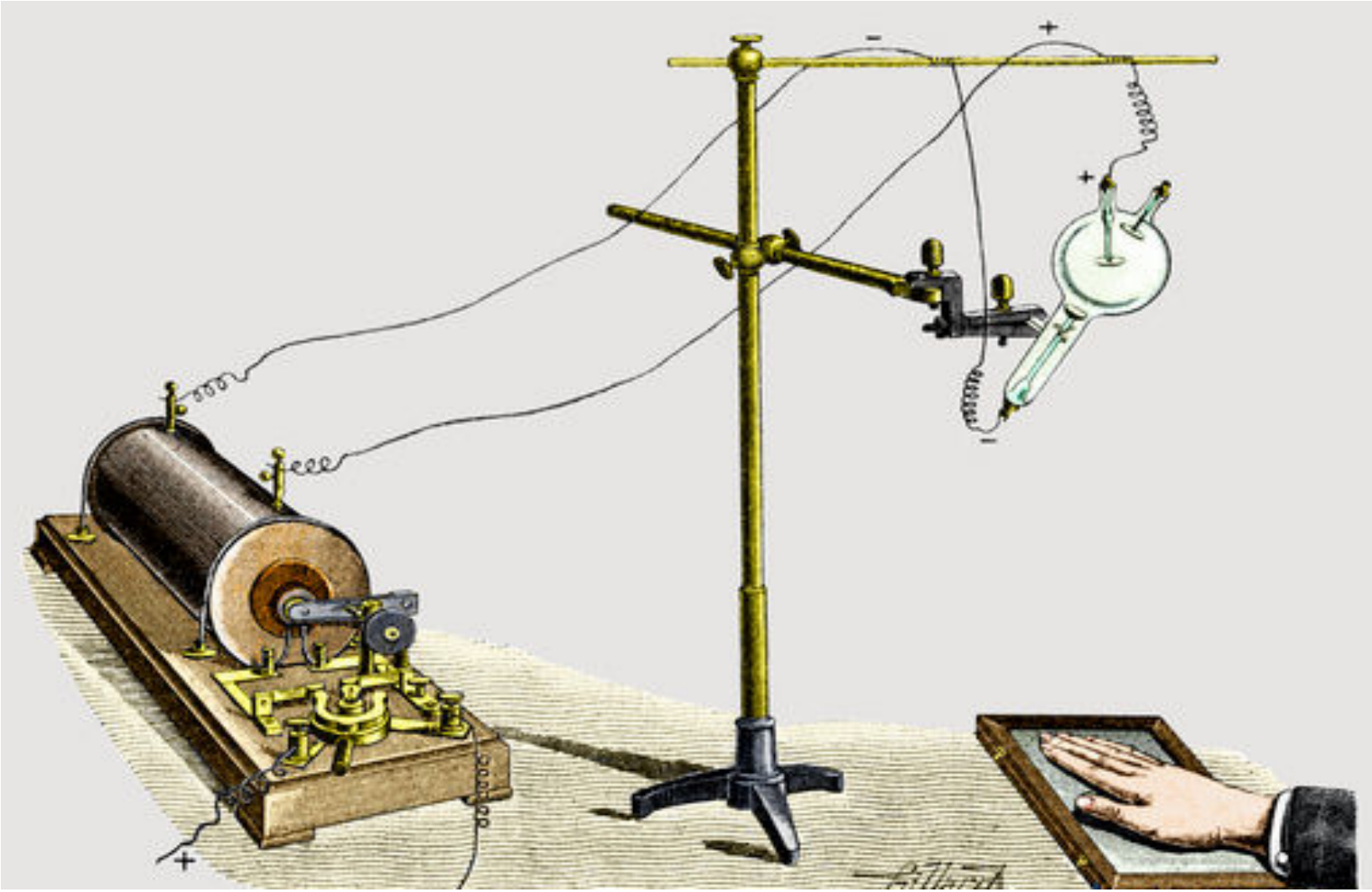
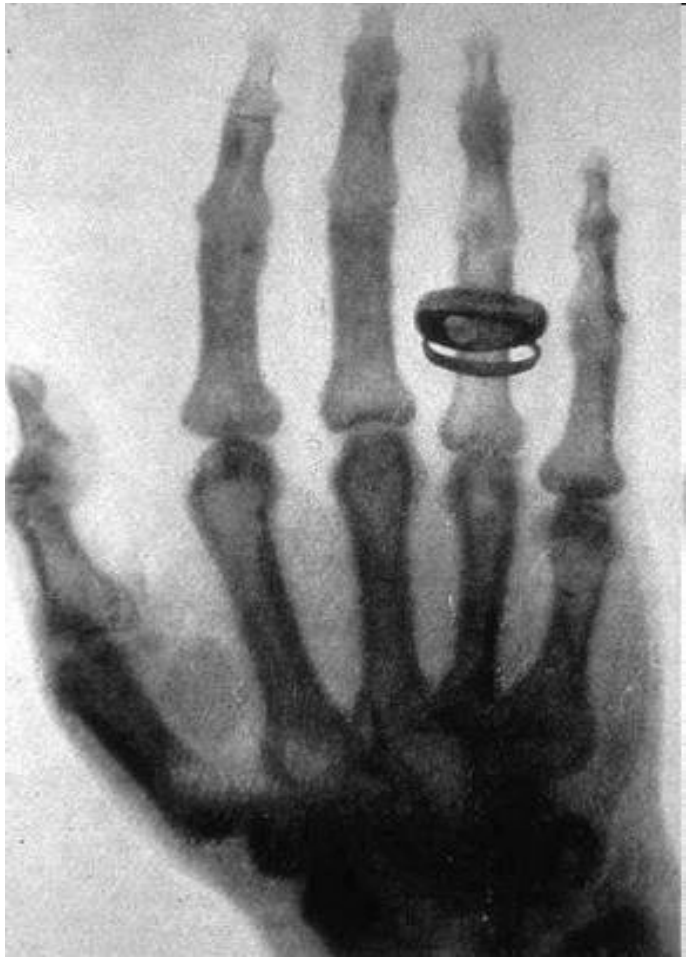


First X-Ray Images/ Radiographies

1895, Hand of his wife



1896, Hand of his assistant
Alfred von Kolliker

Historical Context: Neutron Radiography

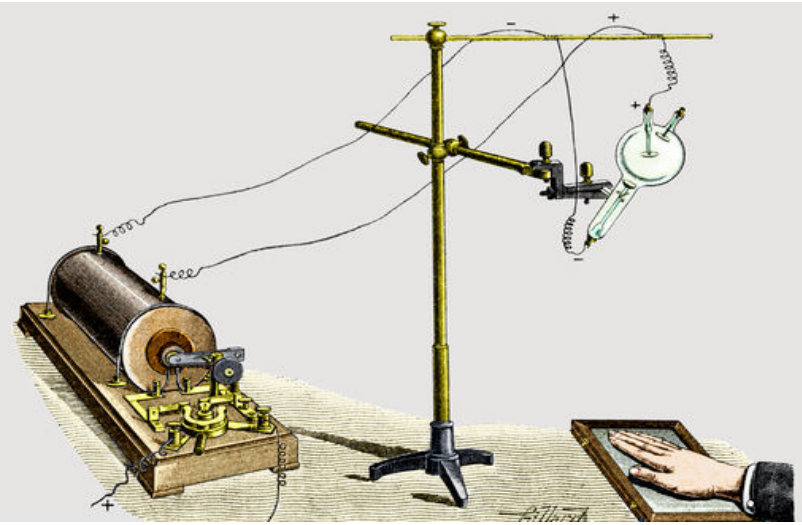
1935 First neutron radiography by Kallmann & Kuhn in Berlin

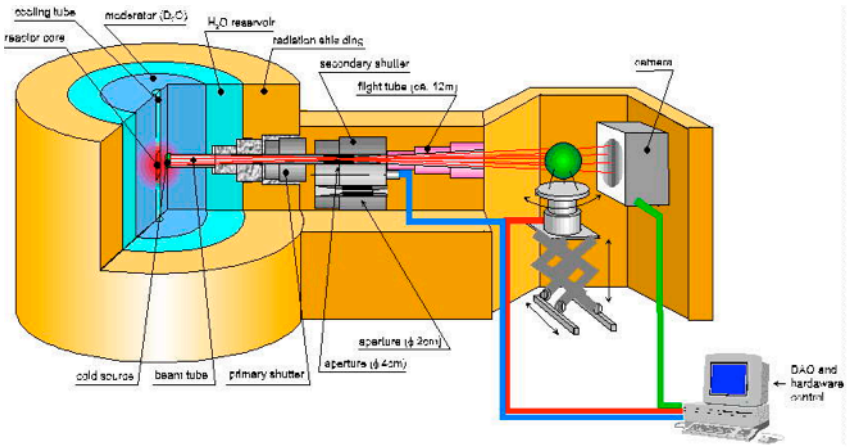
[Kallmann, Research 1, 254 (1947)]

1946 Similar efforts by Peter et al

[Peter, Naturforscher 1, 557 (1946)]

1956 First use of reactor sources by Thewlis & Derbyshire, UK Atomic Energy Research





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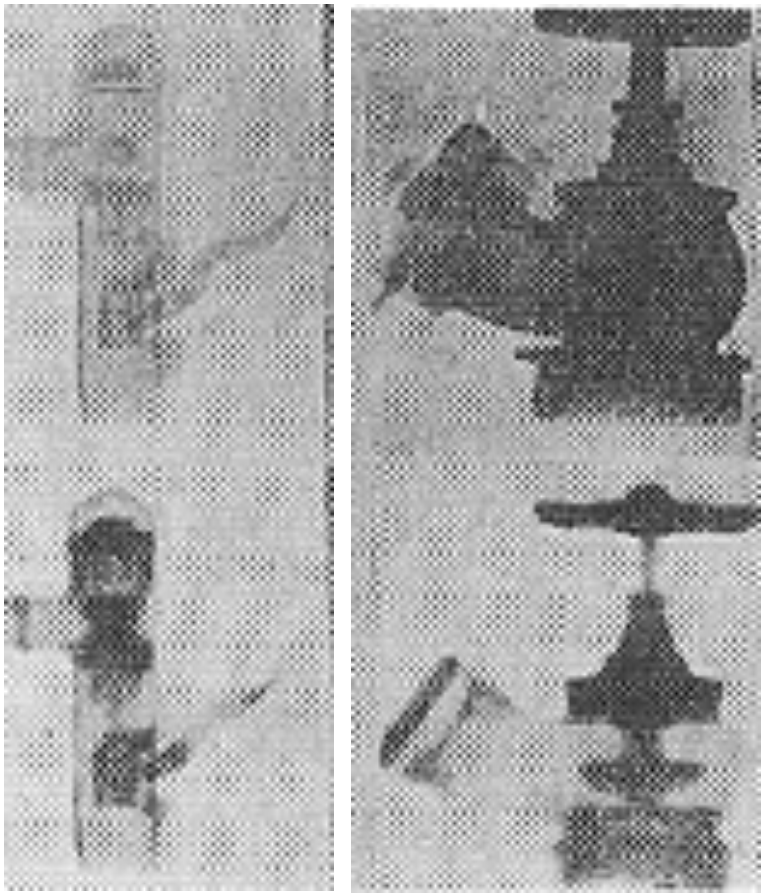
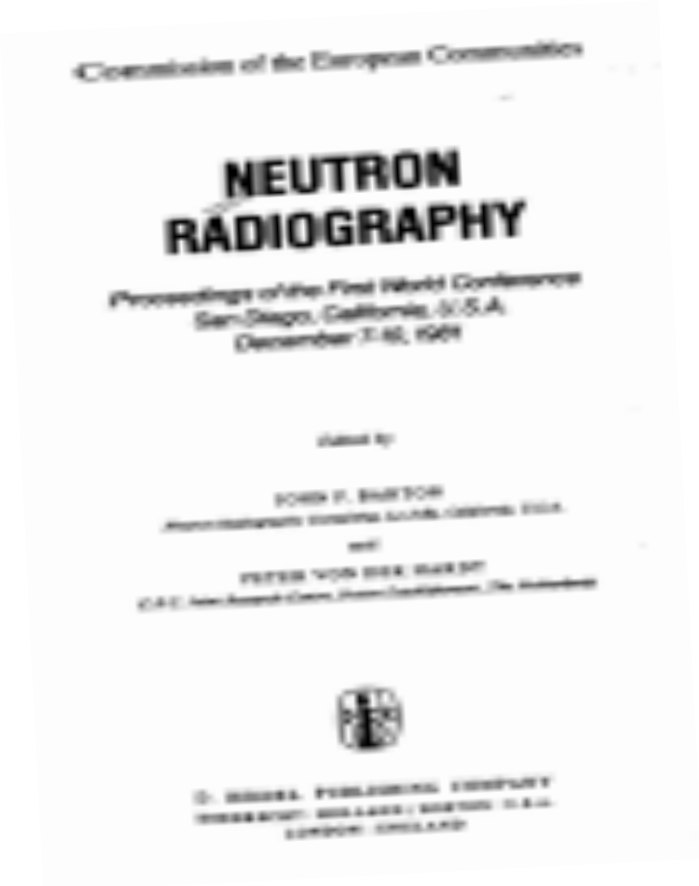
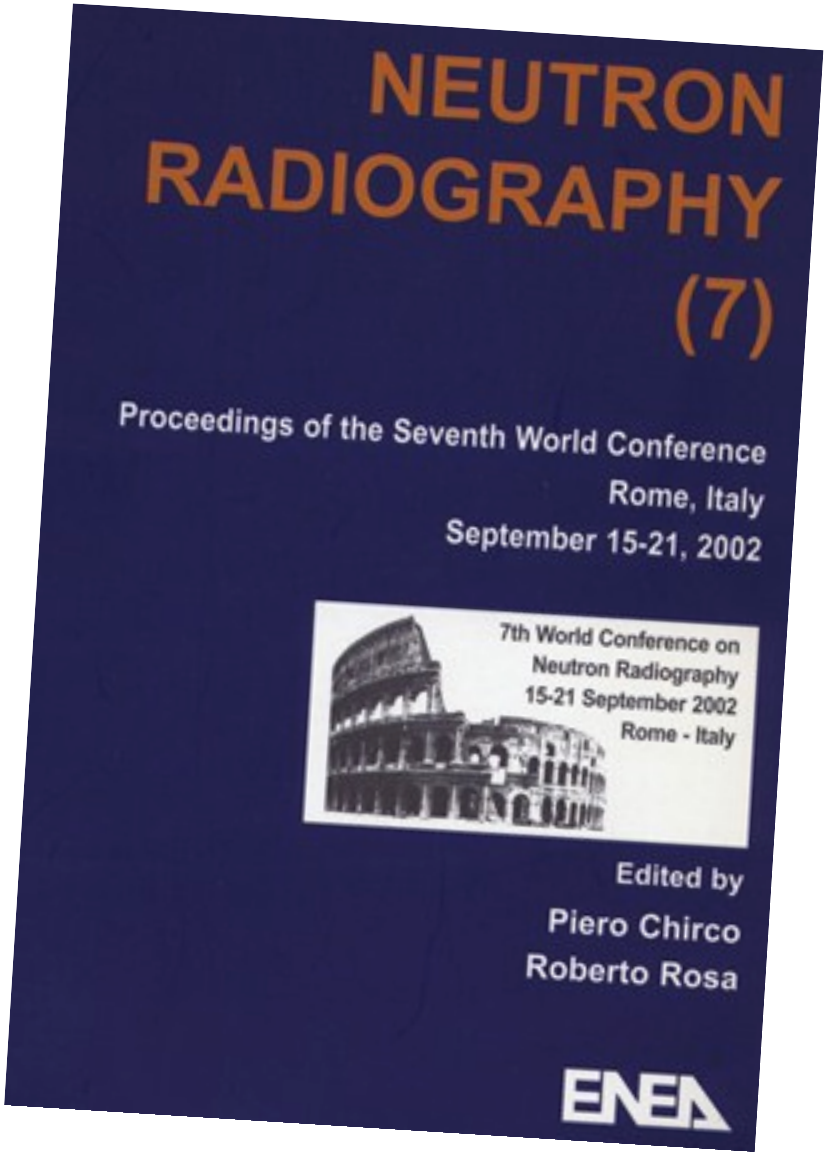


Historical Context: Neutron Radiography

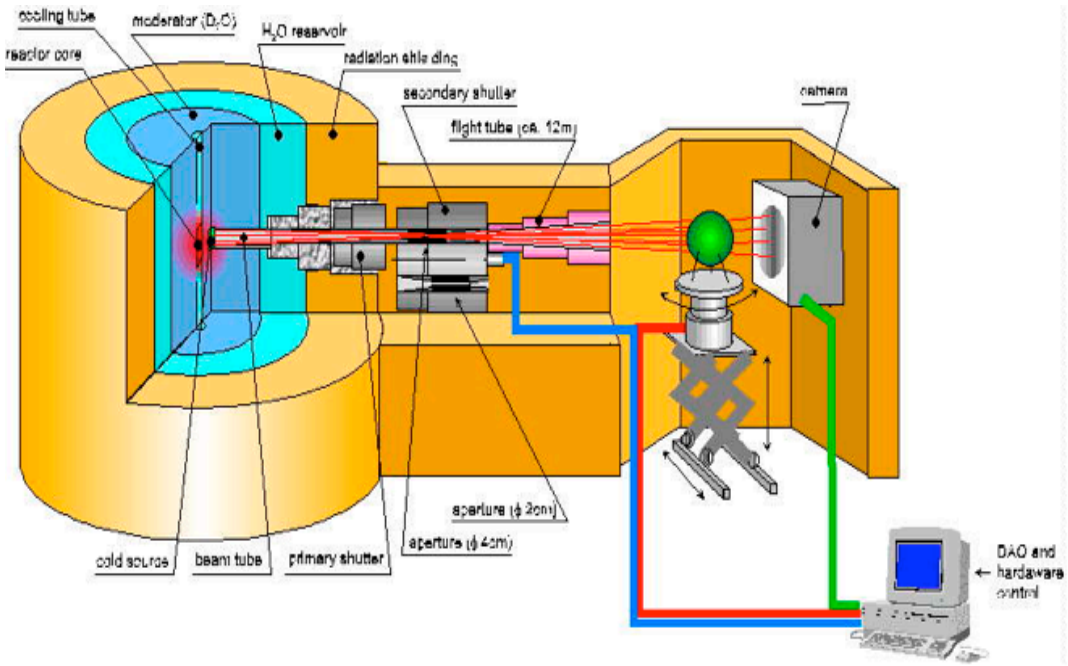
First World Conference on Neutron Radiography:

San Diego, 1981





Technical Improvements



Better neutron sources:

- more flux
- less band-width
- smaller source size

Better neutron detectors:

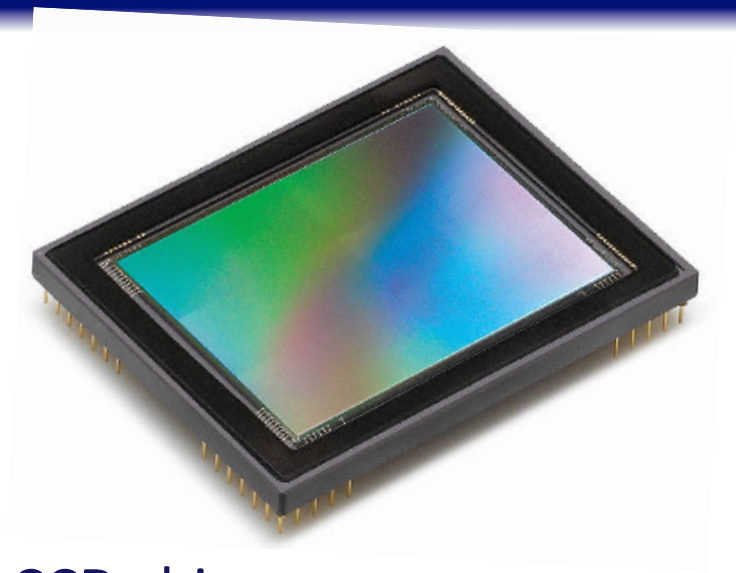
- higher efficiency and resolution
- from analog to digital

Image processing:

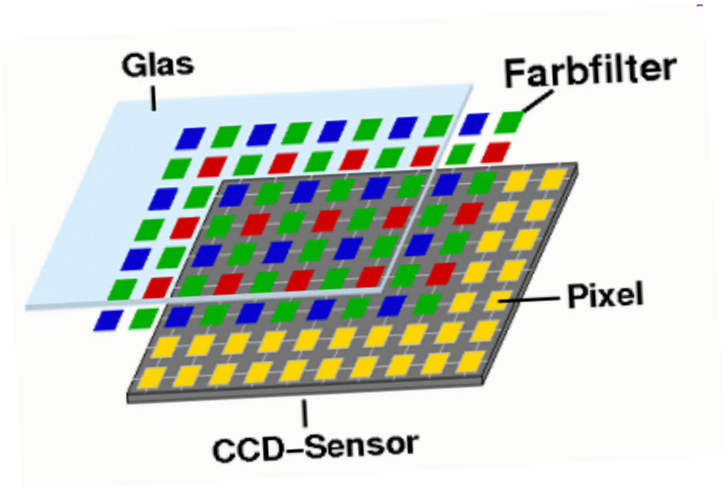
- CT reconstruction
- Image deconvolution

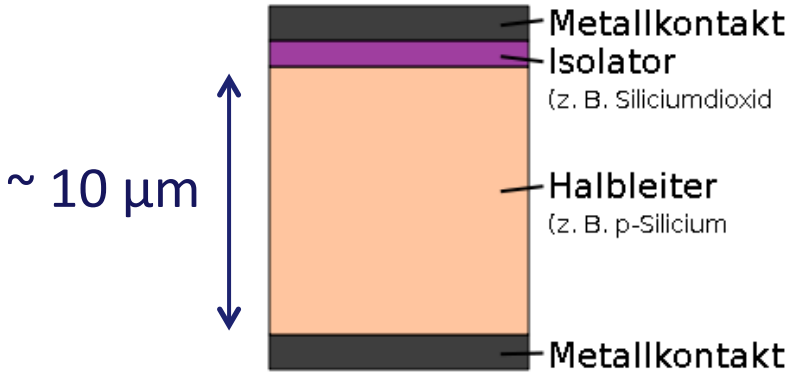
From Analogue to Digital

From Analogue to Digital



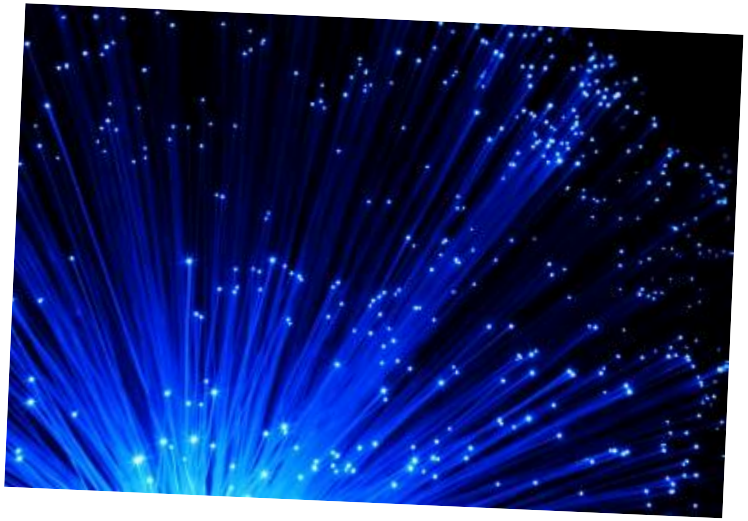
CCD chip

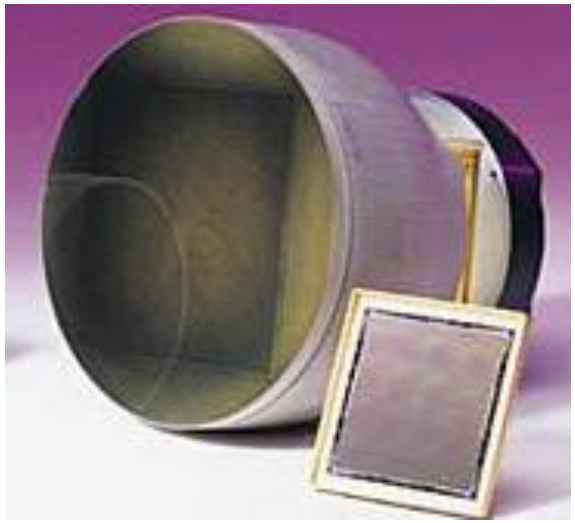




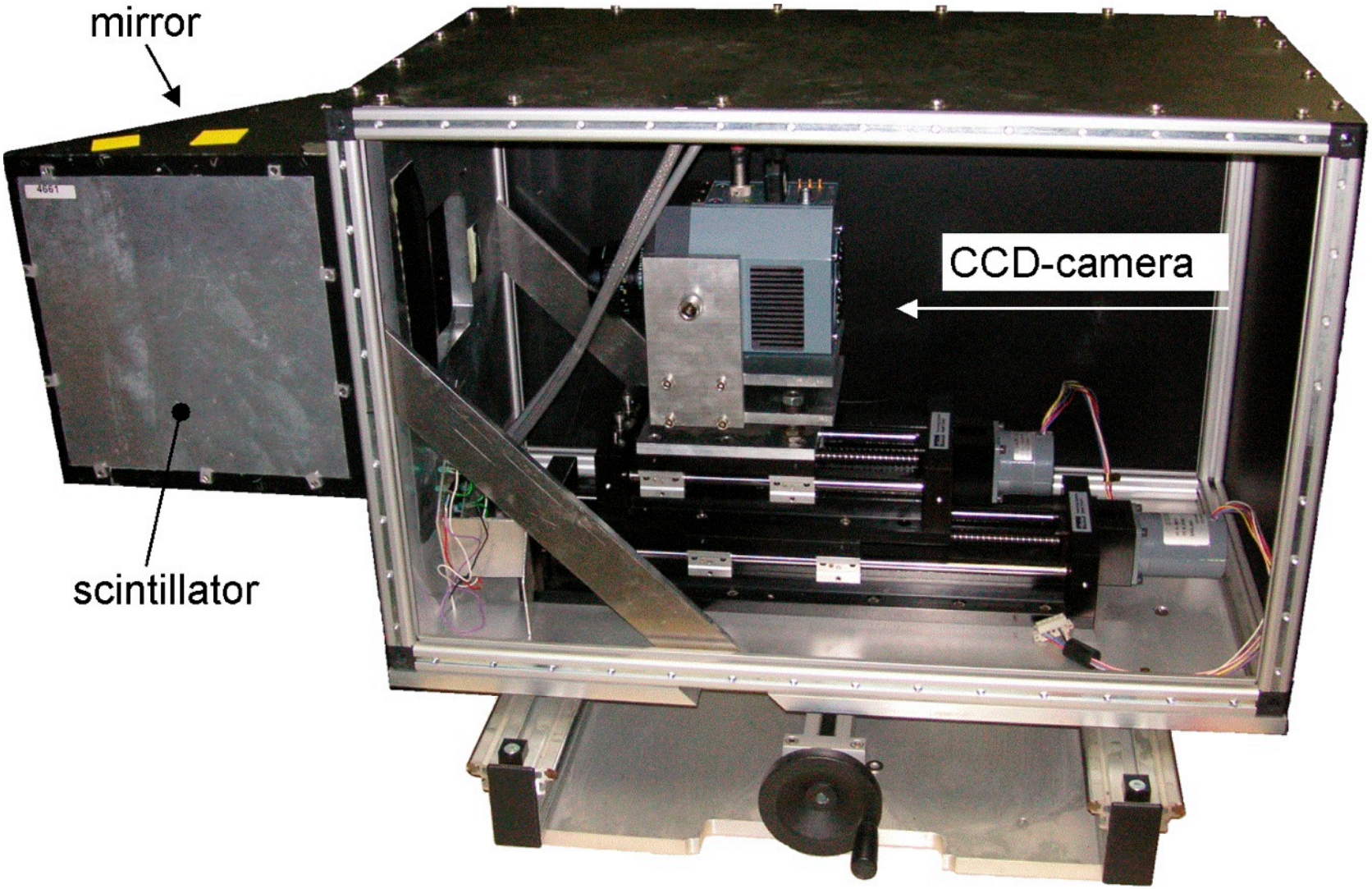
CCDs for Neutron Imaging

szintillator





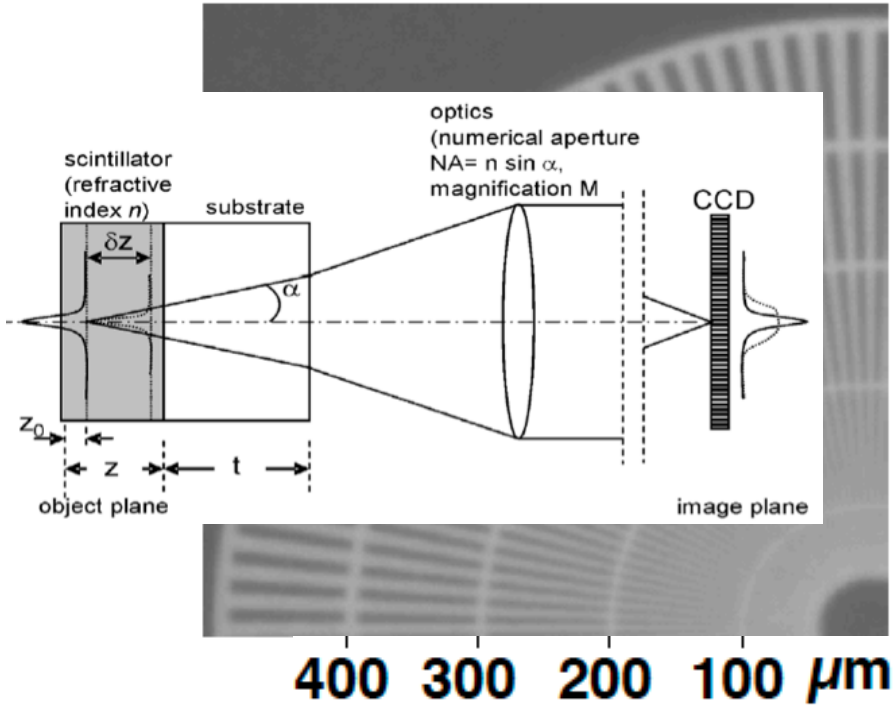
Neutron Imaging System

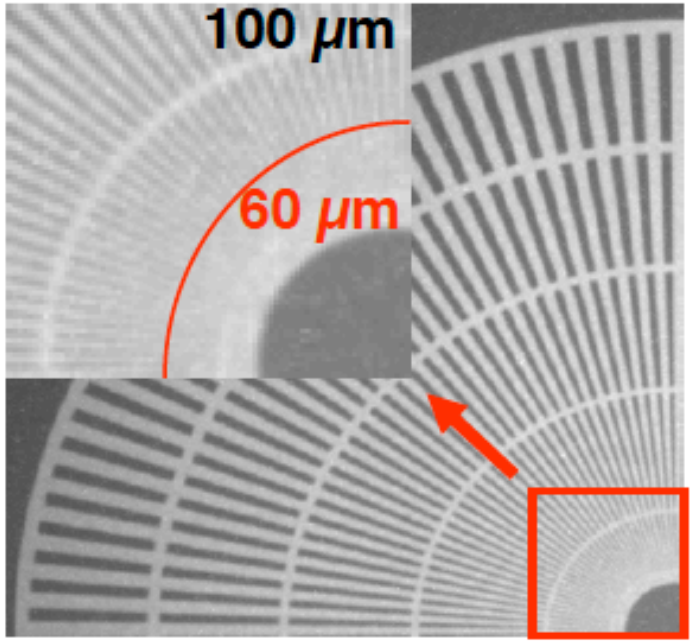


Neutron Imaging System

Scintillator: 200 μm ^{6}LiF
Lens system: 105 mm
Pixel size: 30 μm
Exposure time: 20 s

Scintillator: 5 μm Gadox
Lens system: 105 mm
Pixel size: 30 μm
Exposure time: 120 s

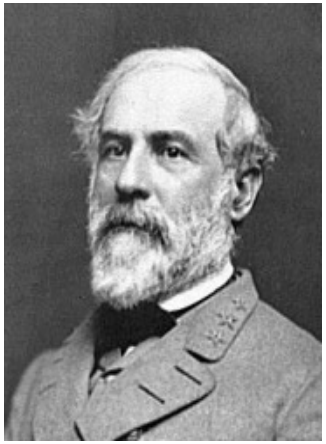




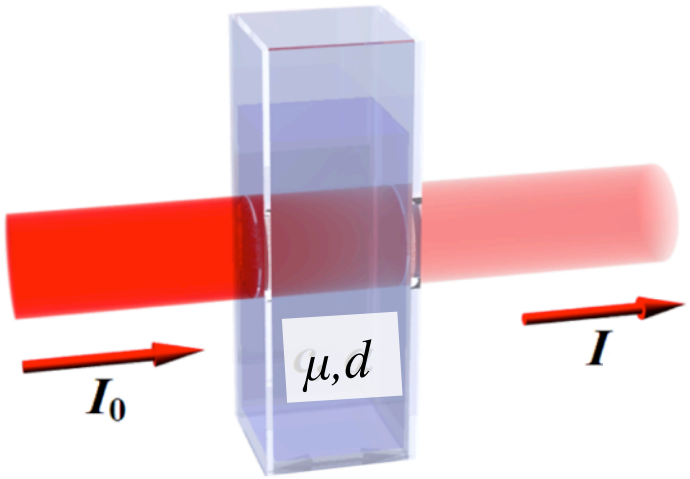
What are we actually measuring ?

Beer-Lambert's Law:

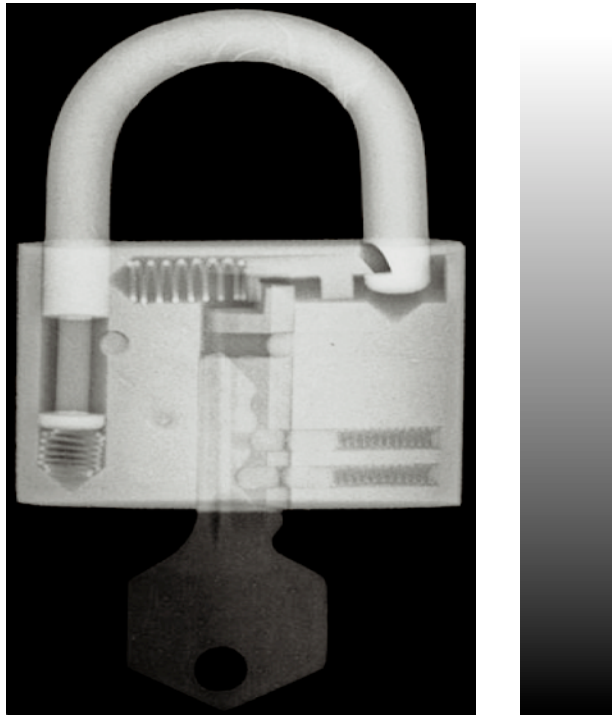
$$I = I_0 \, e^{-\mu d}$$



I_0	incident intensity
I	exiting intensity
μ	attenuation coefficient
d	thickness of material

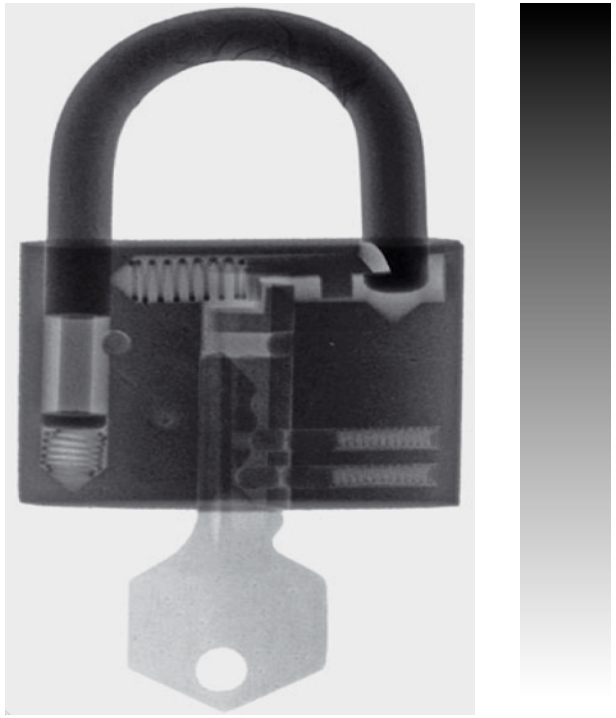


Quantitative Display of Images



μd

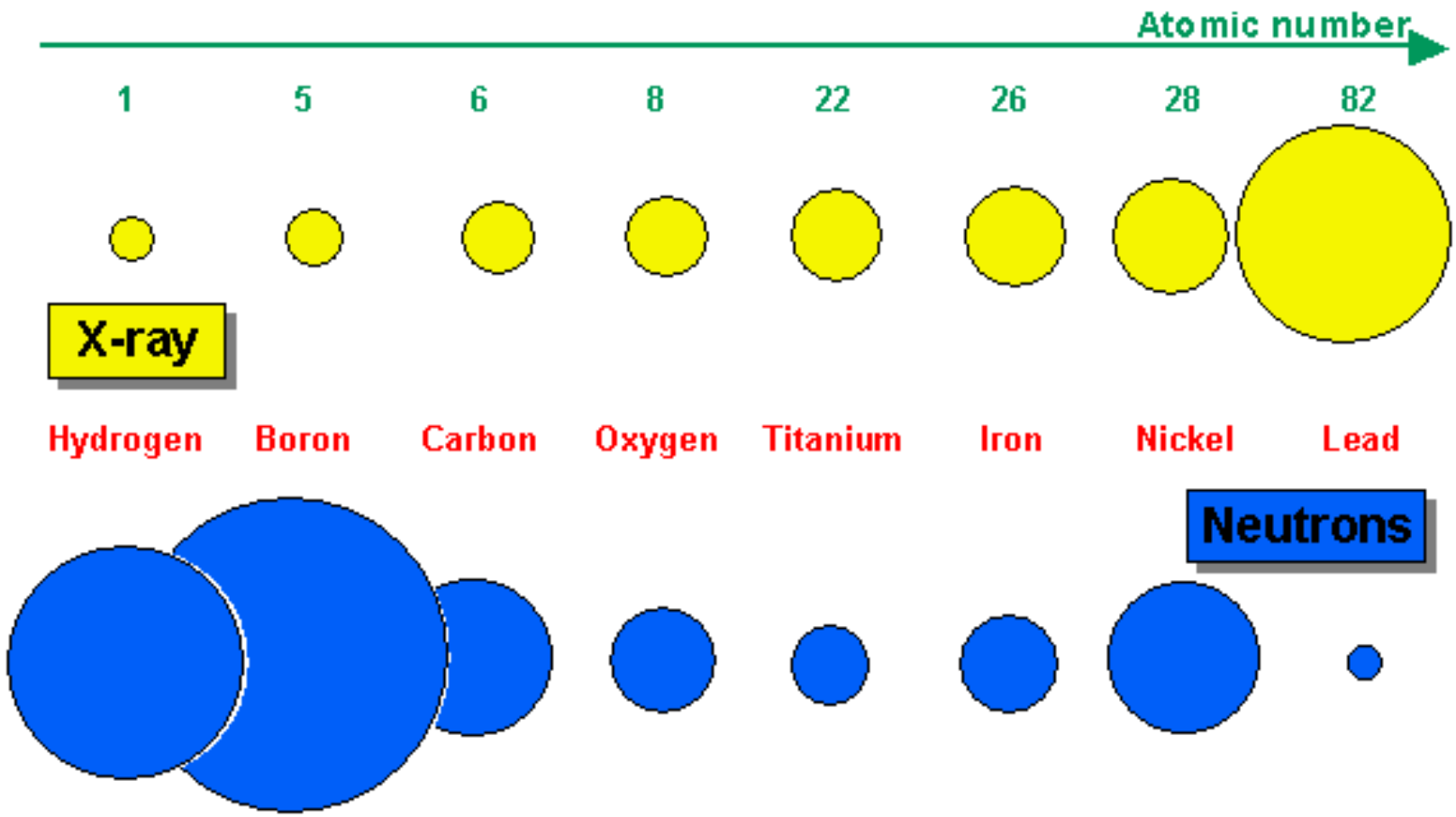
0



μd

0

Complementarity of X-Rays and Neutrons



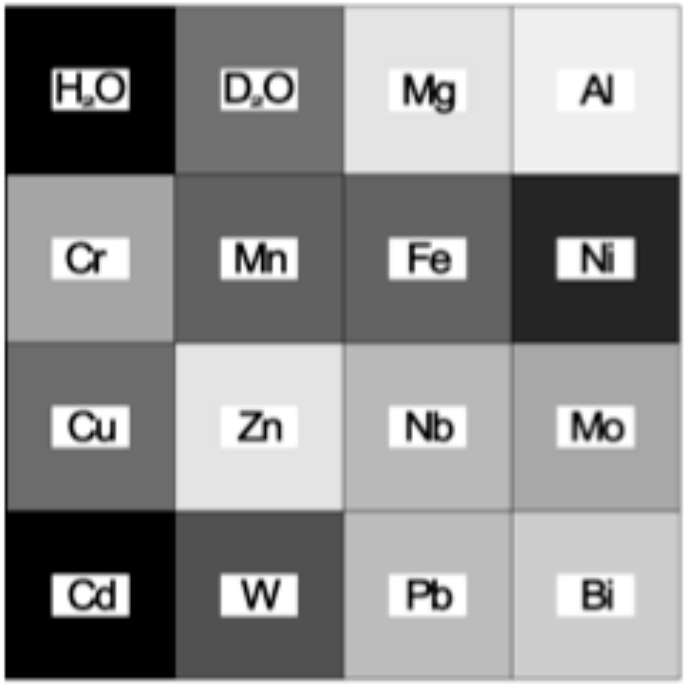
diameter of circles = attenuation coefficient μ

Complementarity of X-Rays and Neutrons

Thickness of materials: 1 cm

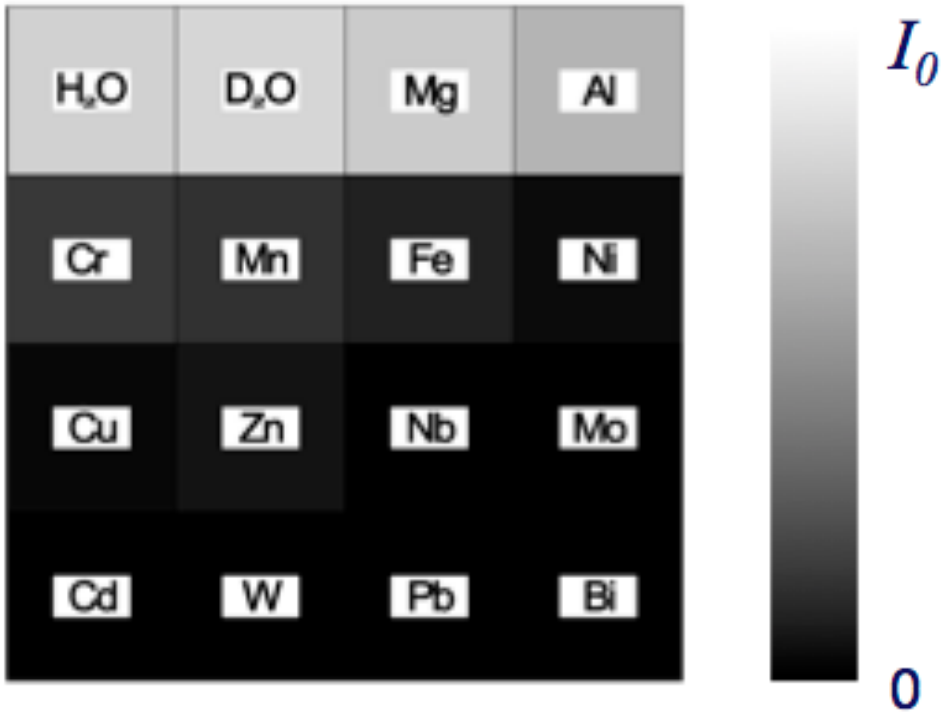
Neutrons

thermal neutrons (E = 25 meV)

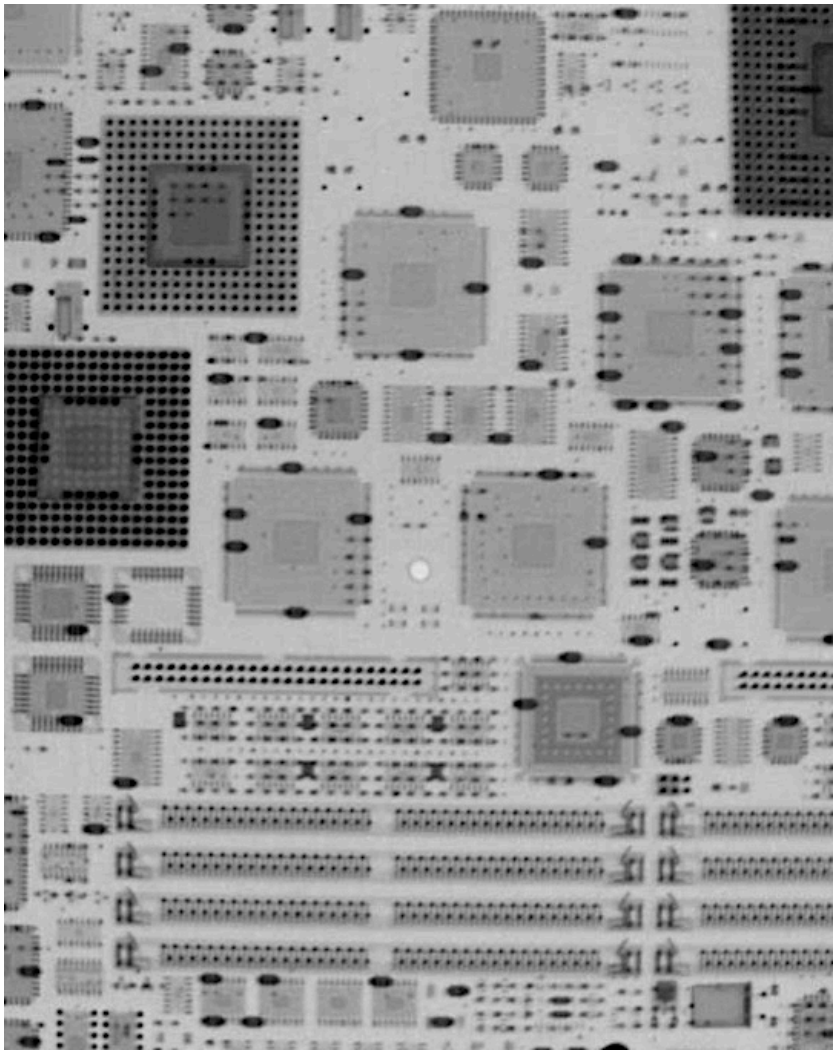


X-rays and gamma-rays

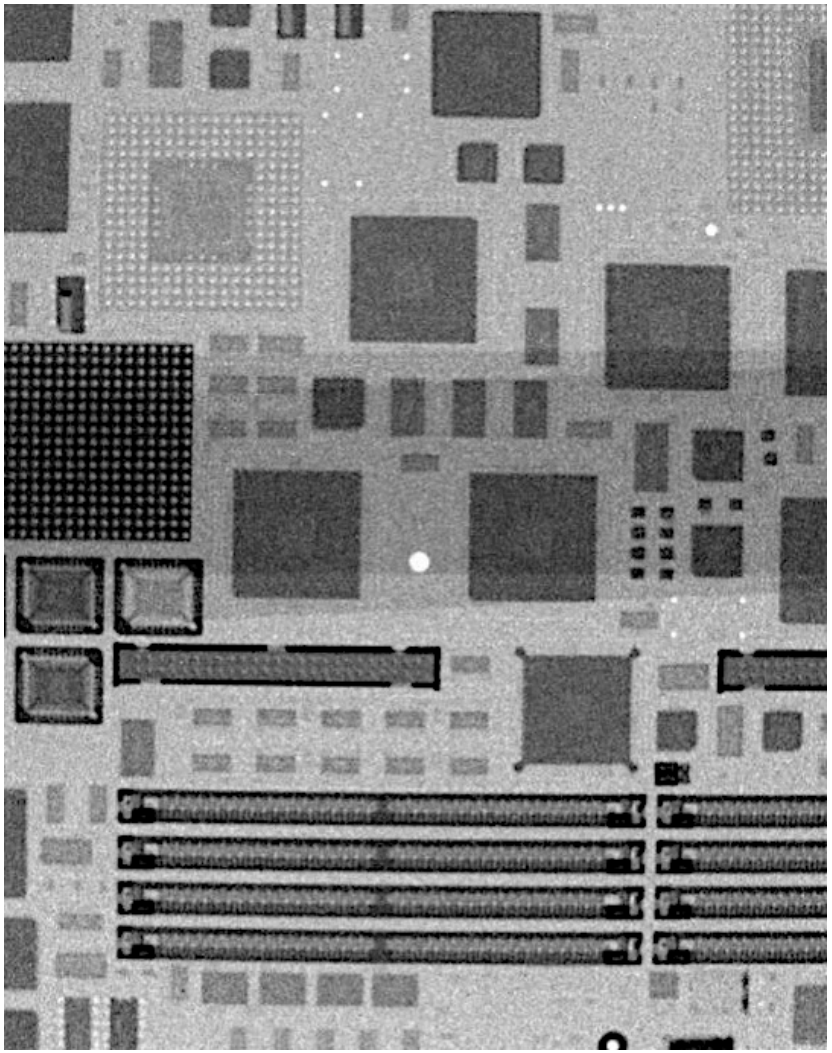
X-rays (E = 120 keV)



Complementarity of X-Rays and Neutrons



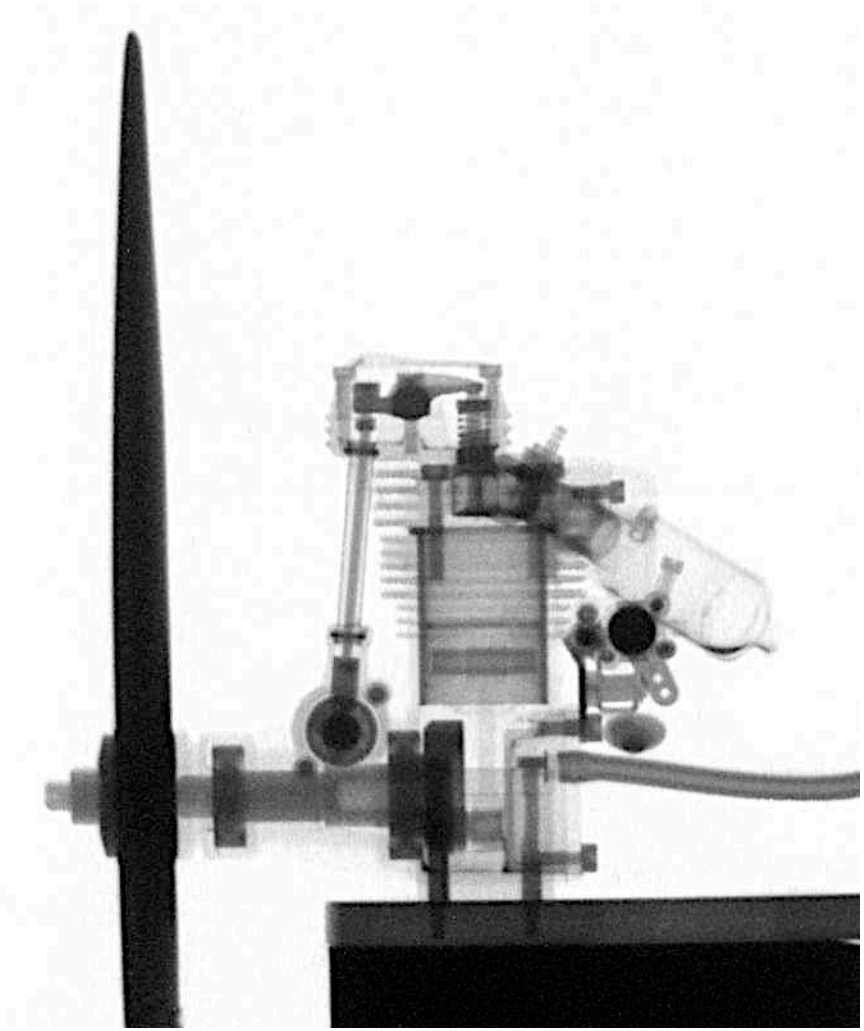
X-rays



neutrons

Complementarity of X-Rays and Neutrons

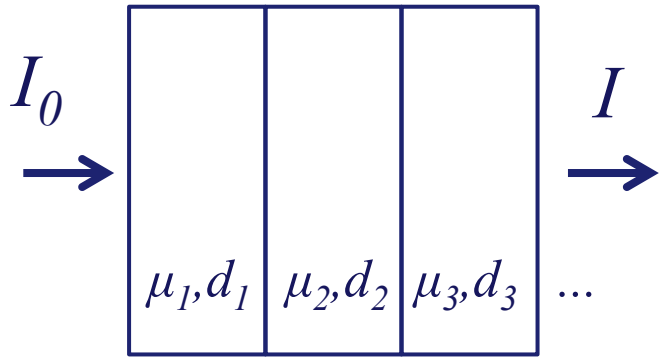
X-rays



neutrons

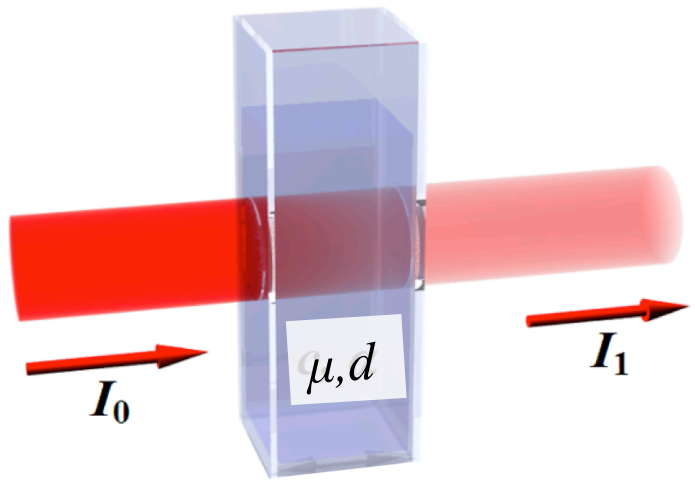
What if we have several layers ?

Beer-Lambert's Law:

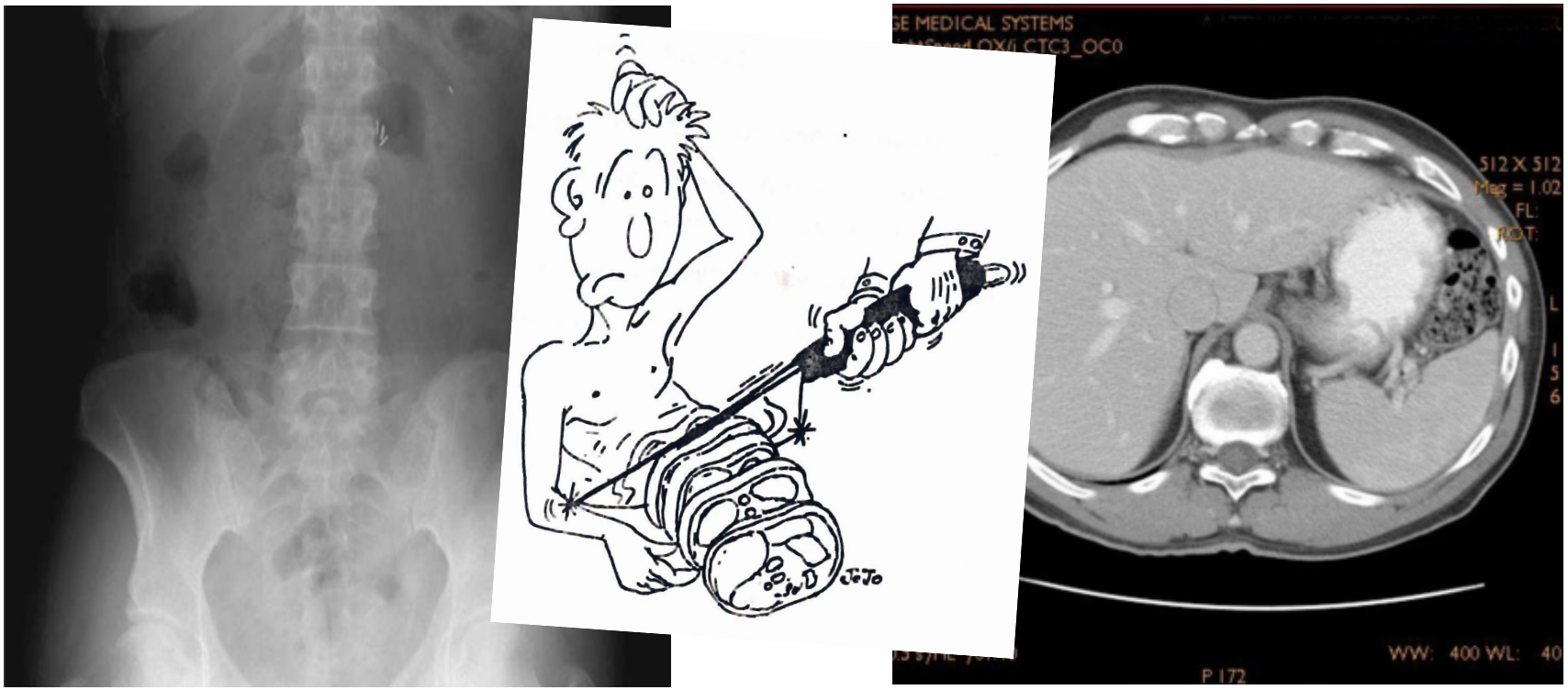


$$I = I_0 \left(e^{-\mu_1 d_1} e^{-\mu_2 d_2} e^{-\mu_3 d_3} \ldots\right)$$

I_0	incident intensity
I	exiting intensity
μ	attenuation coefficient
d	thickness of material



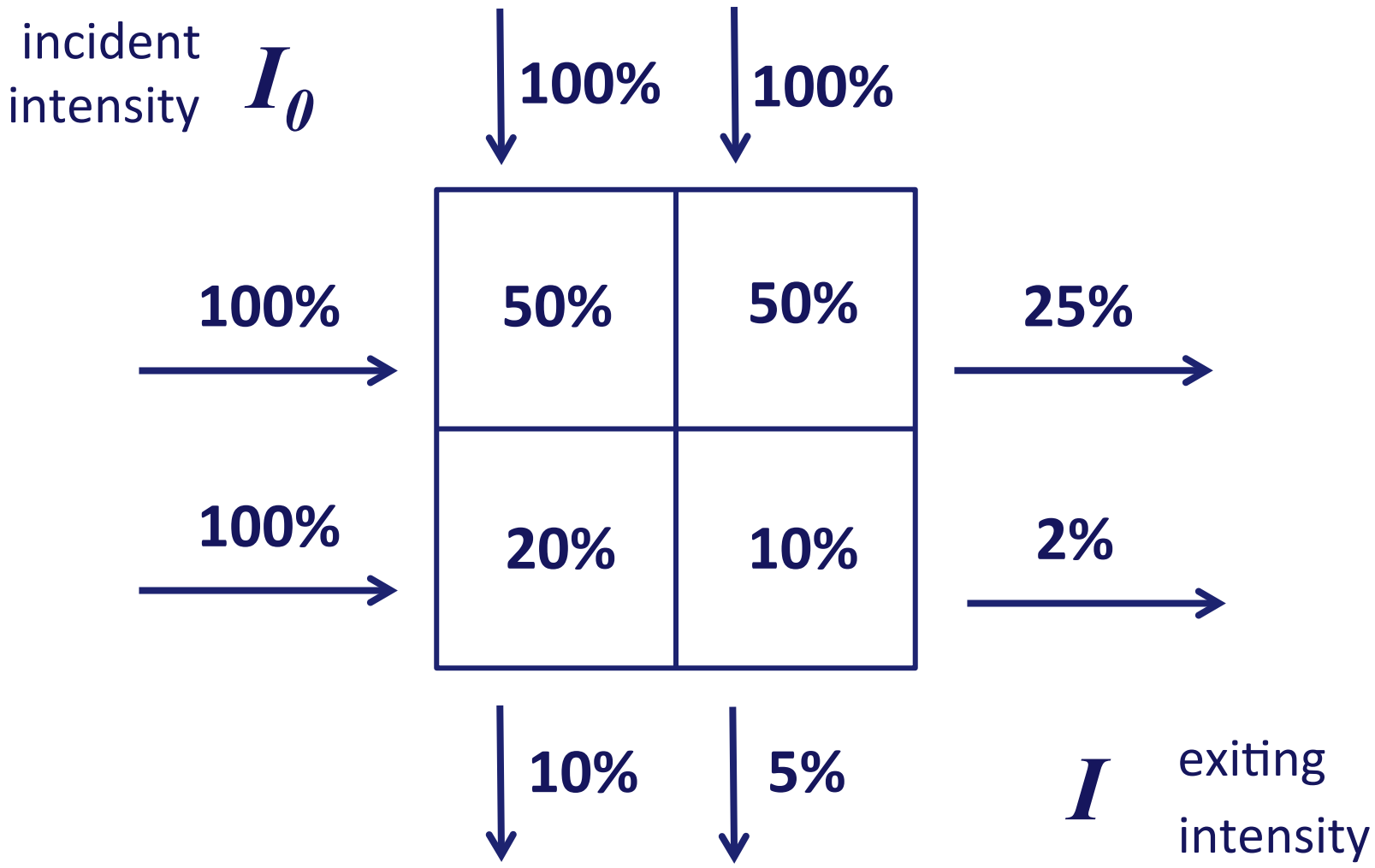
The Miracle of Computed Tomography



radiographic projections

tomographic slices

The Miracle of Computed Tomography



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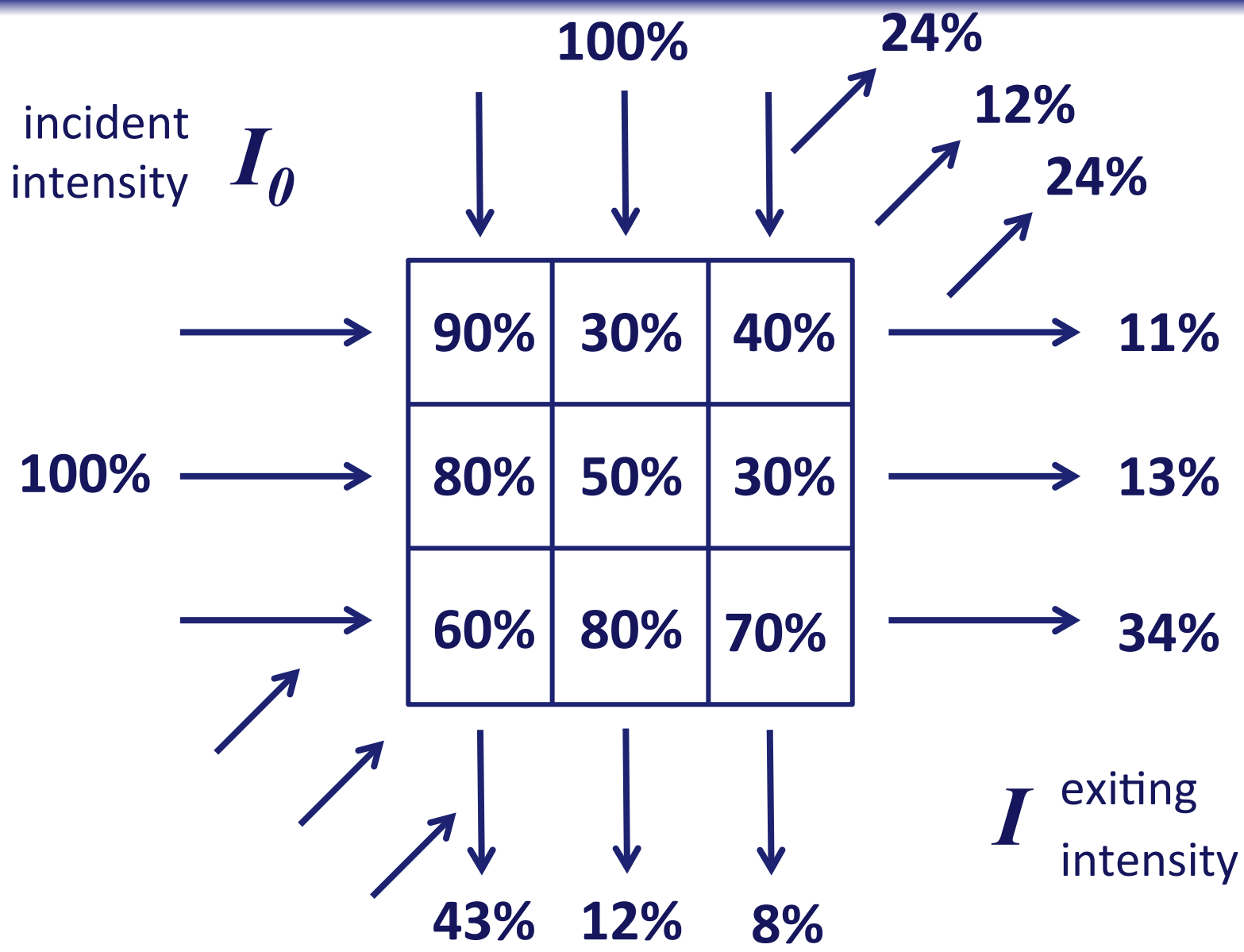
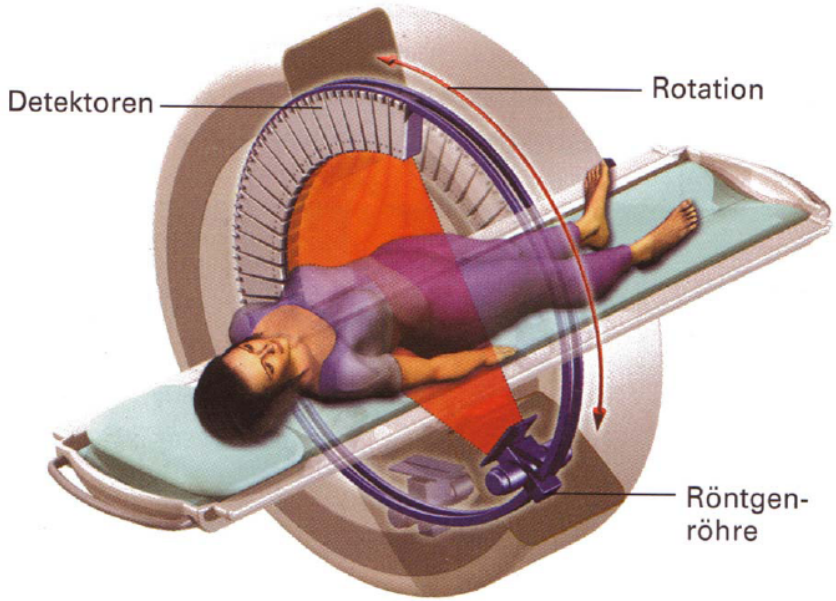


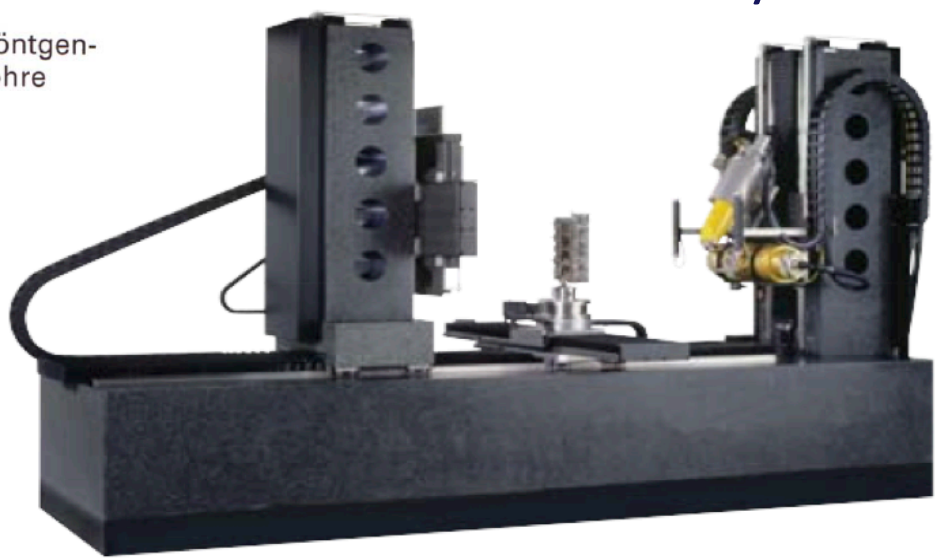
X-Ray CT Geometries

clinical X-ray CT

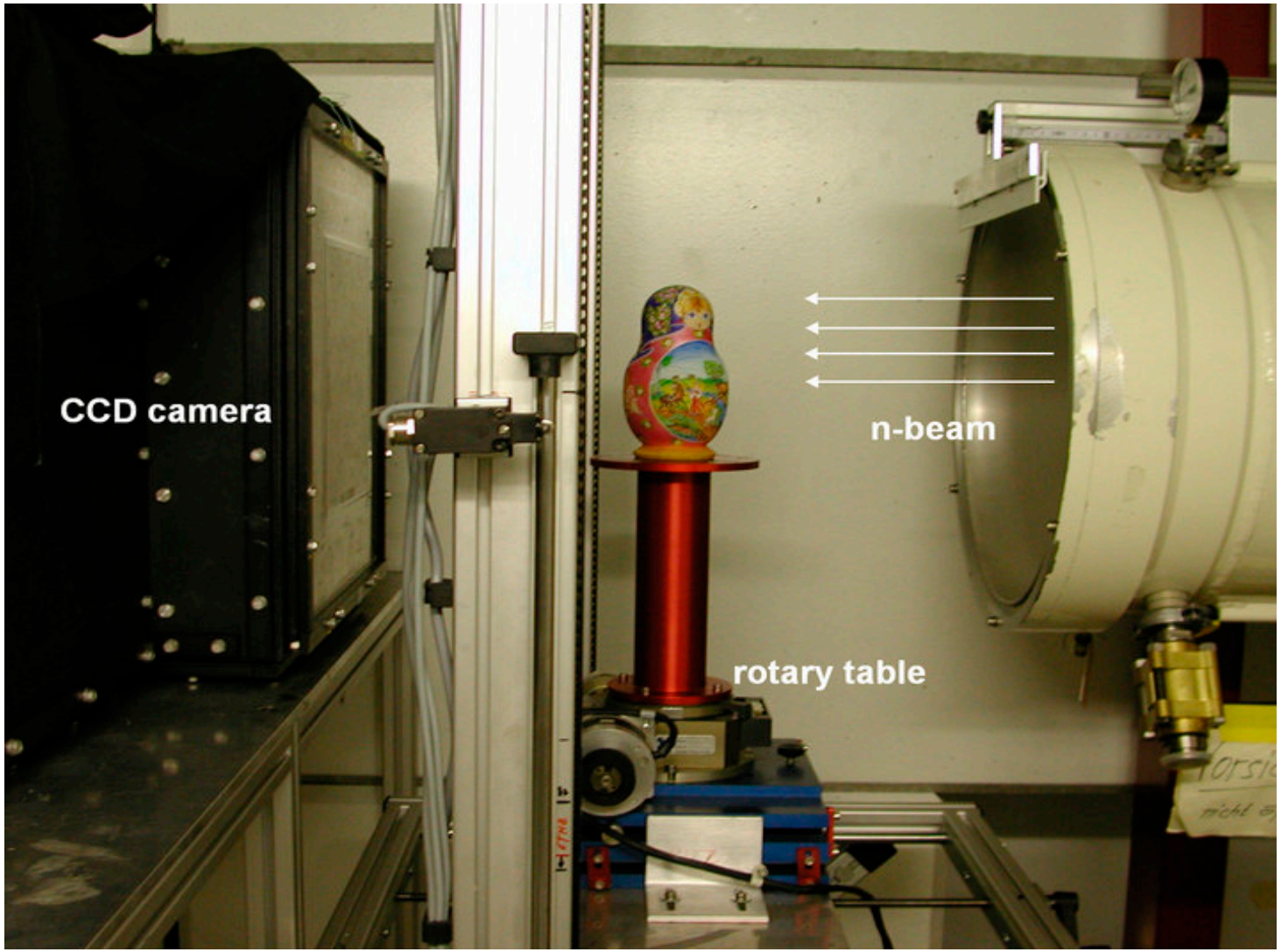


lab-based

X-ray microCT



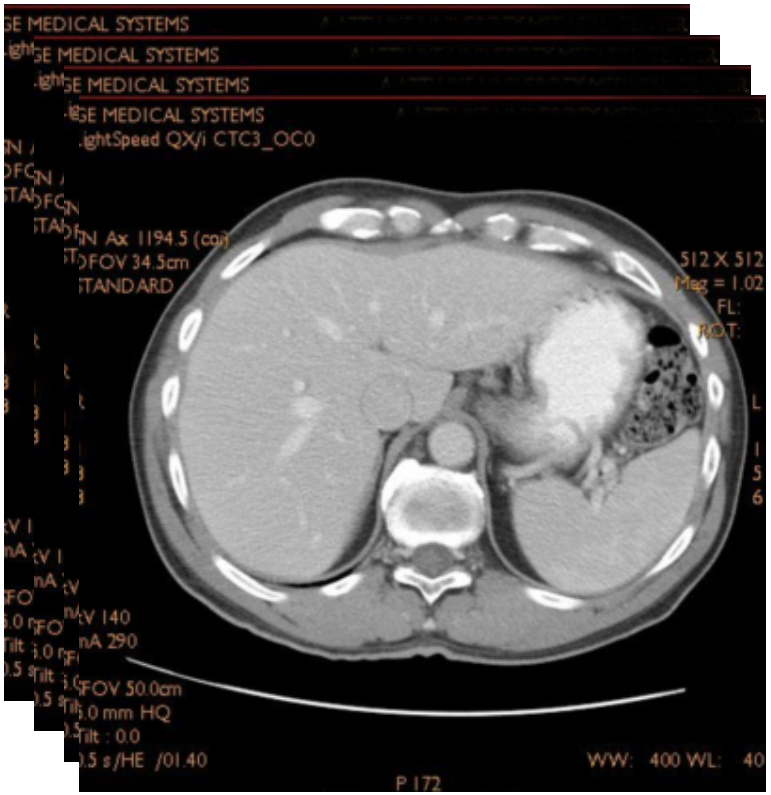
Neutron CT Setup



CT Reconstruction

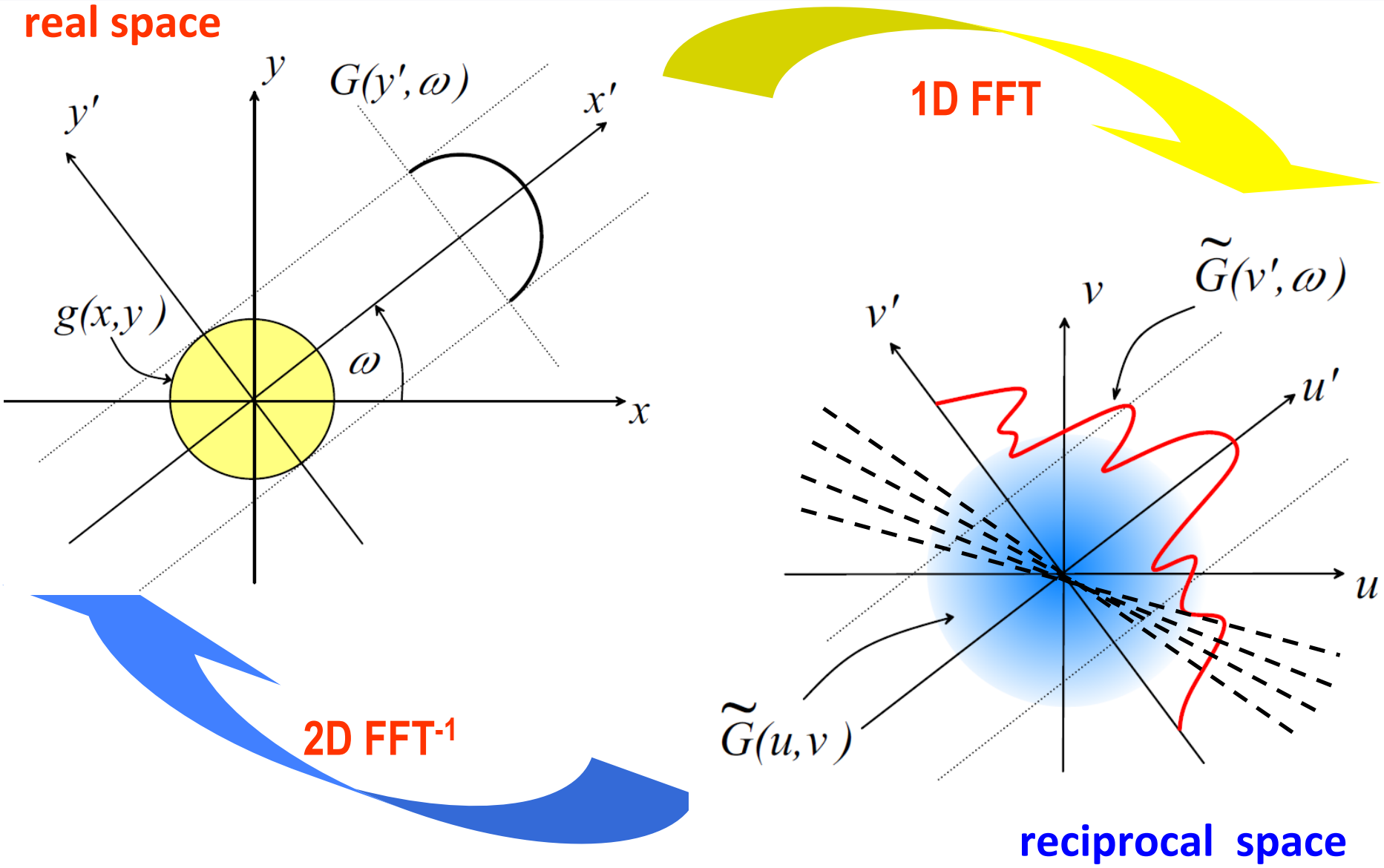
1000 projections with
1000 x 1000 pixels

CT reconstruction with
1000 virtual slices



=> 10^9 equations mit 10^9 unknown paramters

CT Reconstruction: Fourier Slice Theorem

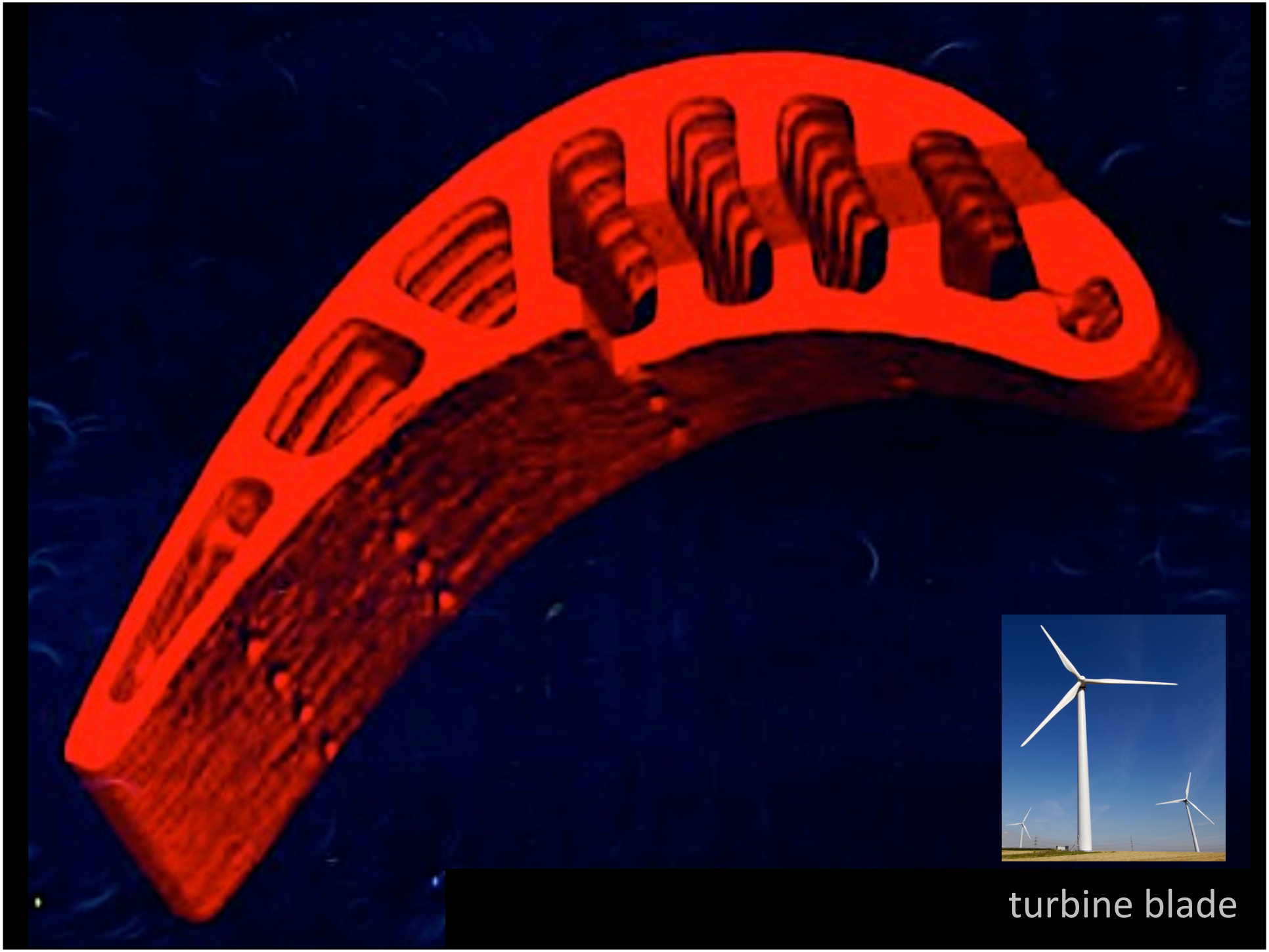


CT Reconstruction on GPUs

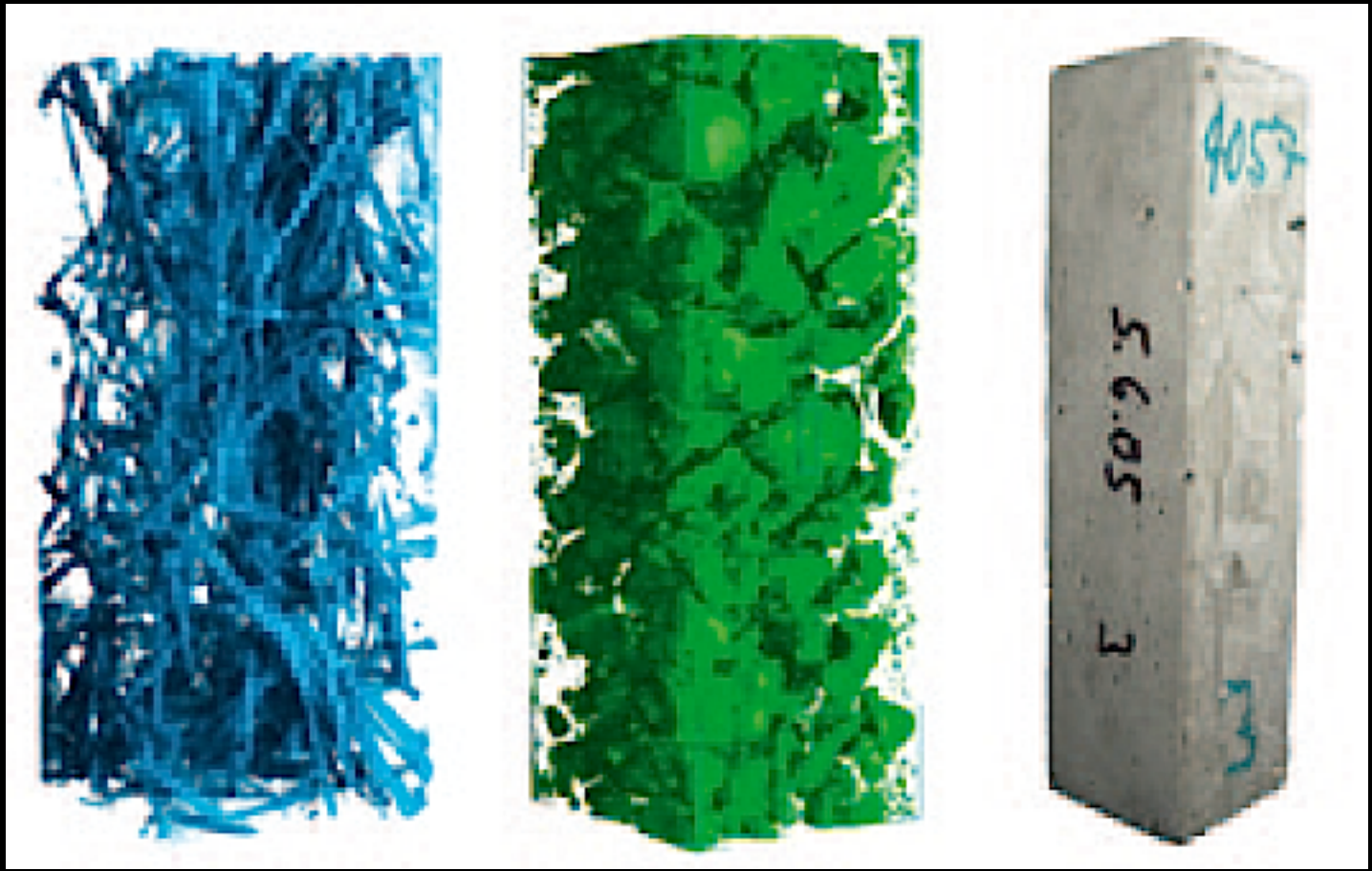
Graphical Processing Units

speed-up factor > 100

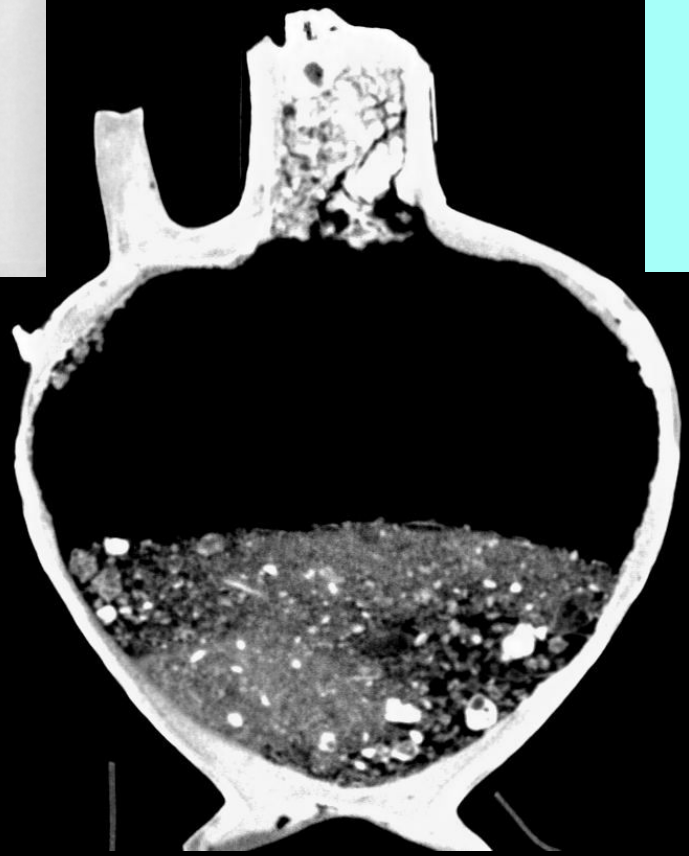
compared to CPUs

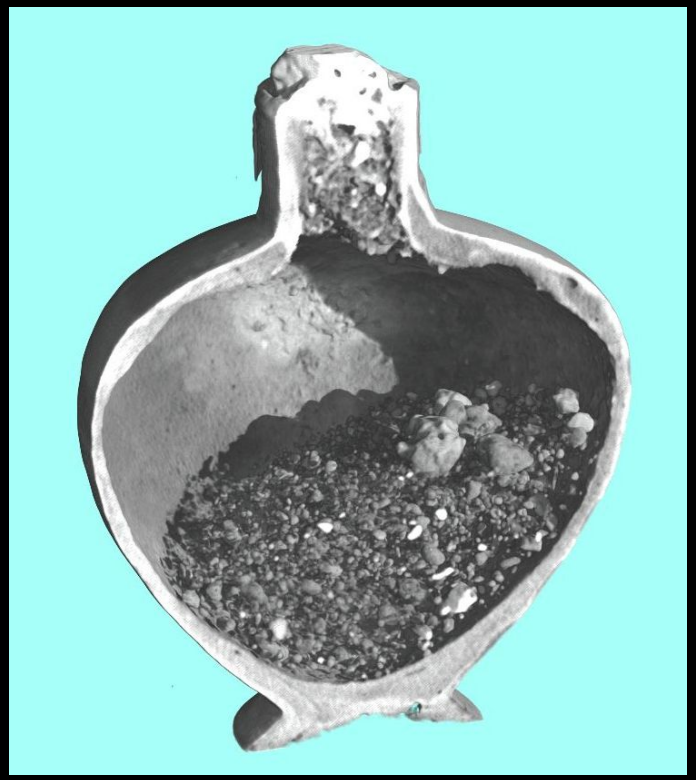


turbine blade



steel-reinforced concrete





sealed roman vase

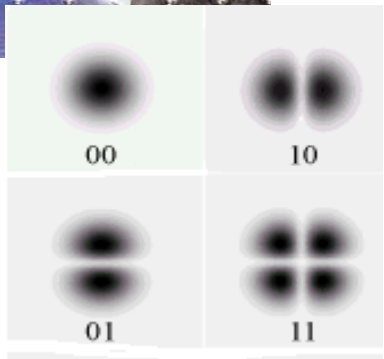
Frascati, INSIS, July 2012

Neutron Imaging – Part 2

Basics, Tomography, Coherence

Franz Pfeiffer
Department of Physics
Technical University Munich

Coherent vs. Incoherent Sources

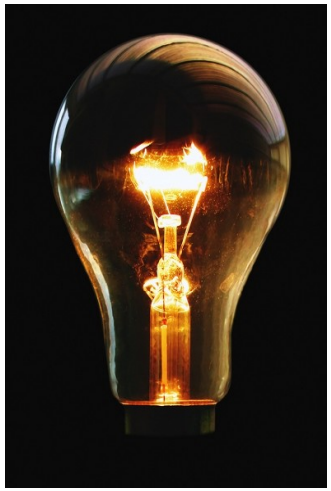


1. Lasers (single mode) – stimulated emission

- highly correlated *stimulated emission* process into one defined laser mode, e.g. TEM_{00} in laser cavity
- wave-front with high correlation in space and time

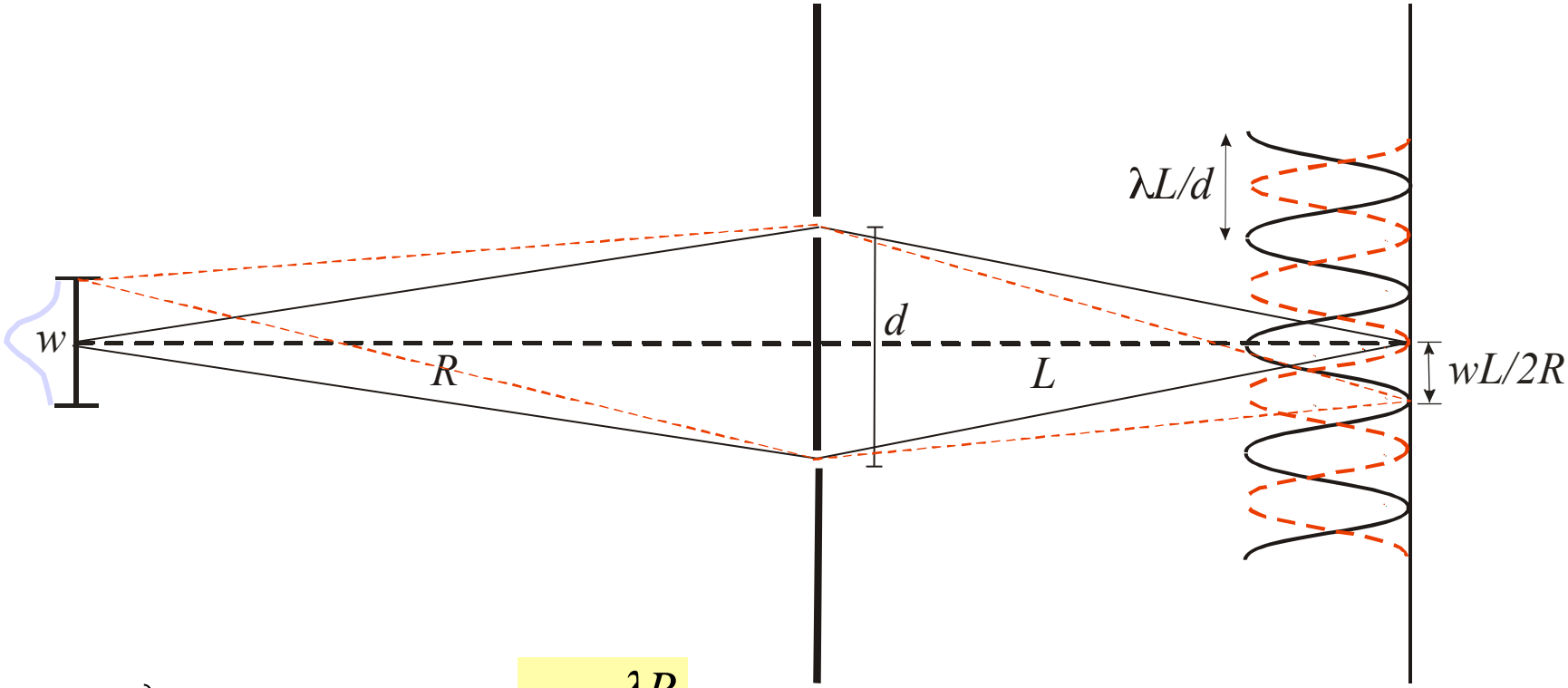
2. Chaotic light sources – spontaneous emission

- thermal sources, e.g. light bulbs
- x-ray tubes, x-ray synchrotron sources
- neutron spallation and reactor sources



<u>But:</u> If the size of the source is small compared to the distance of the observer, we can extract a partially coherent beam (van Cittert-Zernicke theorem) !

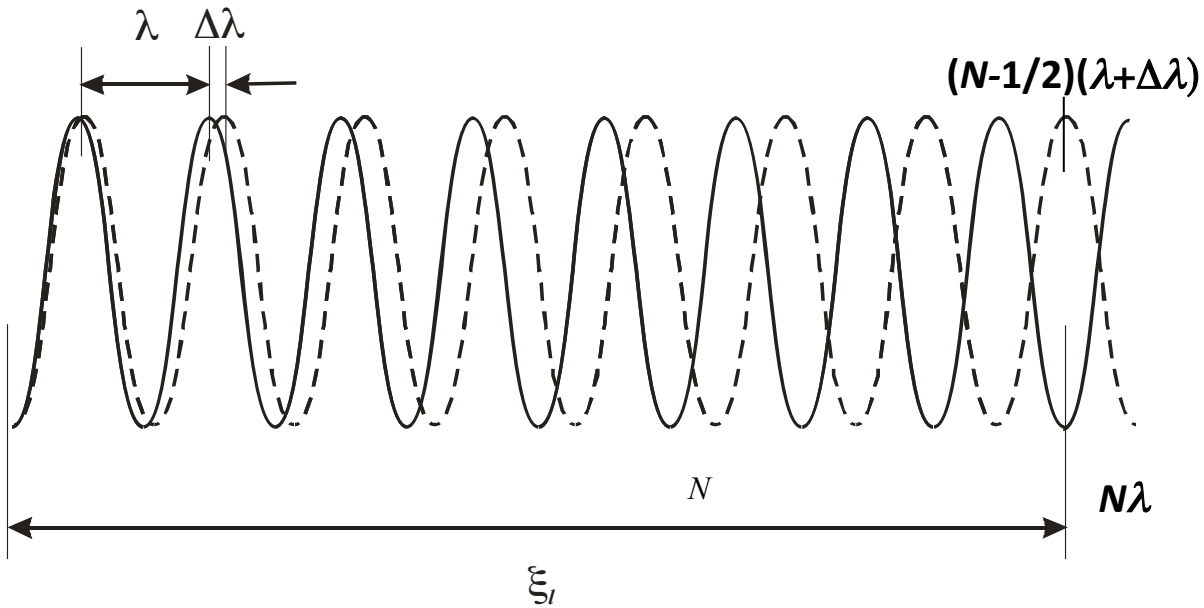
Transverse/ Spatial Coherence Length



In anti-phase if $d = \dfrac{\lambda R}{w}$

Transverse coherence length at distance R: $\xi_t \equiv \dfrac{\lambda R}{w}$

Longitudinal / Temporal Coherence Length



$$\left.\begin{aligned} N\lambda &= (N-1/2)(\lambda+\Delta\lambda) \\ \xi_l &\equiv N\lambda \end{aligned}\right\} \quad \xi_l \approx \frac{1}{2}\cdot\frac{\lambda^2}{\Delta\lambda}.$$

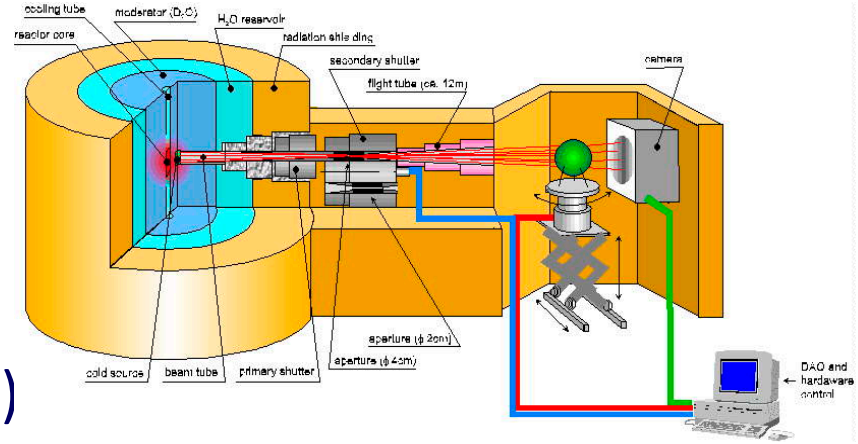
Typical Neutron Coherence Lengths

Transverse coherence length:

$$\xi_t = \lambda R/w$$

ξ_t **= 1 µm**

ξ_t **= 10 µm** (for 2 mm aperture)



Longitudinal coherence length:

$$\xi_l = \lambda^2/2\Delta\lambda$$

ξ_l **= 1 nm** (50 % BW)

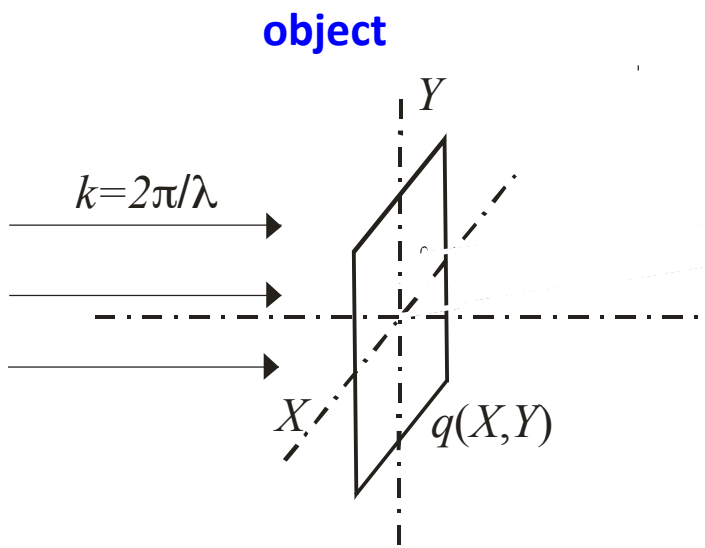
ξ_l **= 5 µm** (0.01 % BW)

distance: $R \sim 20$ m

source size: $w \sim 20$ mm

wave length: $\lambda \sim 1$ nm

Coherent Image Formation: Complex Object Transmission



object's refractive index:

$$n(x,y,z) = 1 - \delta(x,y,z) + i\beta(x,y,z)$$

$$\delta = Nb_c\lambda^2 / 2\pi$$

N : **atom density,**
b_c: **coherent scattering lenght**

$$\beta = \mu\lambda / 4\pi$$

μ: **absorption coefficient**

object's phase shift:

$$\phi(X,Y) = \frac{2\pi}{\lambda}\int^{object}\delta(X,Y,Z)dZ$$

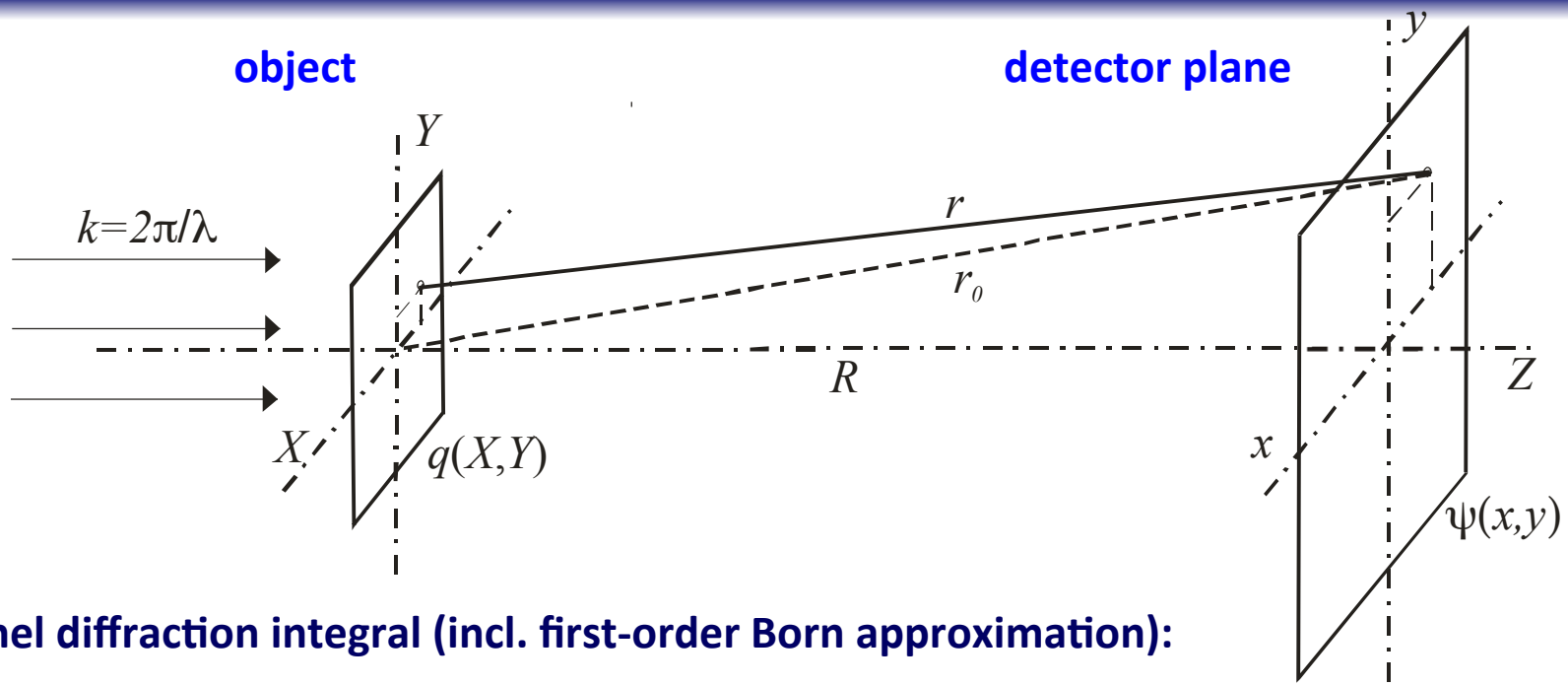
object's transmission:

$$T(X,Y) = \exp\left[-\int^{object}(4\pi\beta(X,Y,Z)/\lambda)dZ\right]$$

object's complex exit wave:

$$q(X,Y) = \sqrt{T(X,Y)}\exp[i\phi(X,Y)]$$

Coherent Image Formation: Neutron Wave Propagation



Fresnel diffraction integral (incl. first-order Born approximation):

$$\psi(x,y) = -\frac{i}{\lambda}\iint q(X,Y)\cdot\frac{\exp(ikr)}{r}dXdY$$

$$\approx -\frac{i\exp(ikr_0)}{\lambda r_0}\iint \overbrace{q(X,Y)\exp\left(-ik\frac{xX+yY}{r_0}\right)}^{\text{Fourier transform of q(X,Y)}}\underbrace{\exp\left(ik\frac{X^2+Y^2}{2r_0}\right)}_{\text{Fresnel term, accounts for spherical curvature of wave front.}}dXdY$$

'paraxial' or small-angle approximation

Coherent Image Formation: Neutron Wave Propagation

Neutron wave propagation recipe:

1) Take your complex object transmission function: $\boldsymbol{\psi_i(x,y)}$
2) Fourier Transform:

$$\mathbf{FT[\psi_i(x,y)] = \tilde{\psi}_i(u,v)}$$

3) Multiply with Fresnel Propagator:

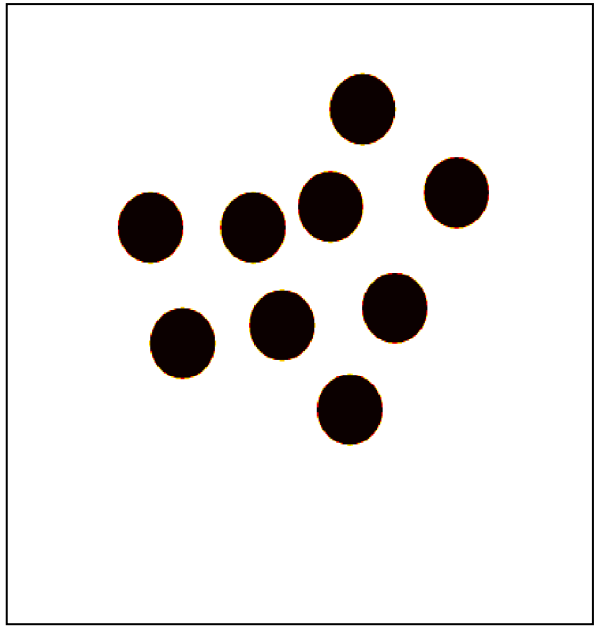
$$\mathbf{\tilde{\psi}_f(u,v) = \tilde{\psi}_i(u,v) * \tilde{P}(u,v)}$$

4) Fourier Transform back:

$$\mathbf{\psi_f(u,v) = FT^{-1}[\tilde{\psi}_f(x,y)]}$$

Numerical Examples

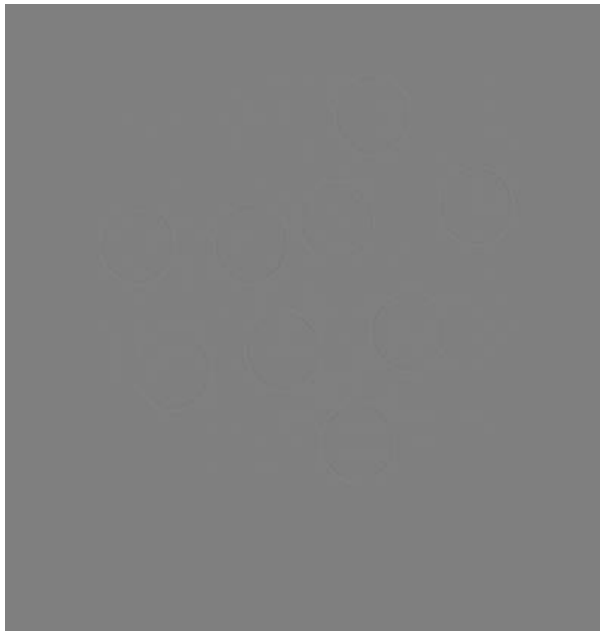
object phase



coherent imaging experiment

(propagation)

detector amplitude



$$q(X,Y) = \sqrt{T(X,Y)}\exp[i\phi(X,Y)]$$
$$\approx \exp\left[i\phi(X,Y)\right]$$

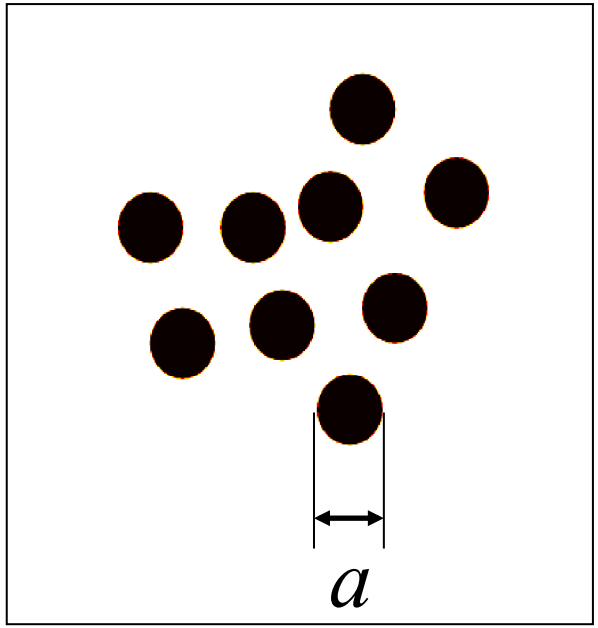
pure phase object !

contact regime

$$r = 0$$

Numerical Examples

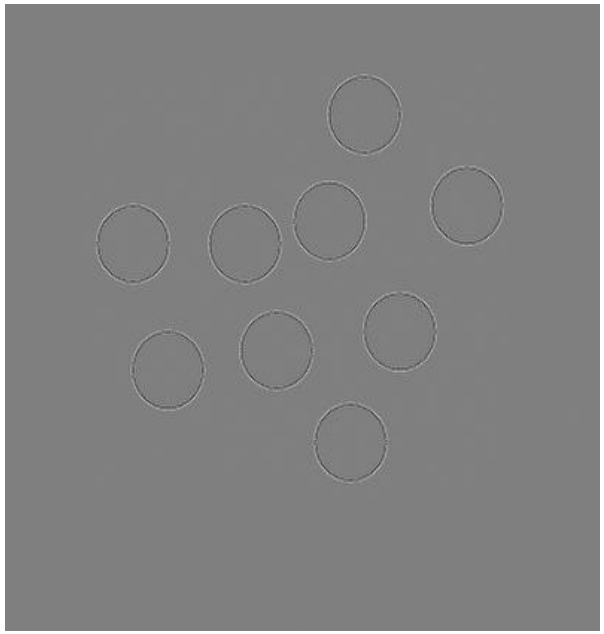
object phase



coherent imaging experiment

(propagation)

detector amplitude

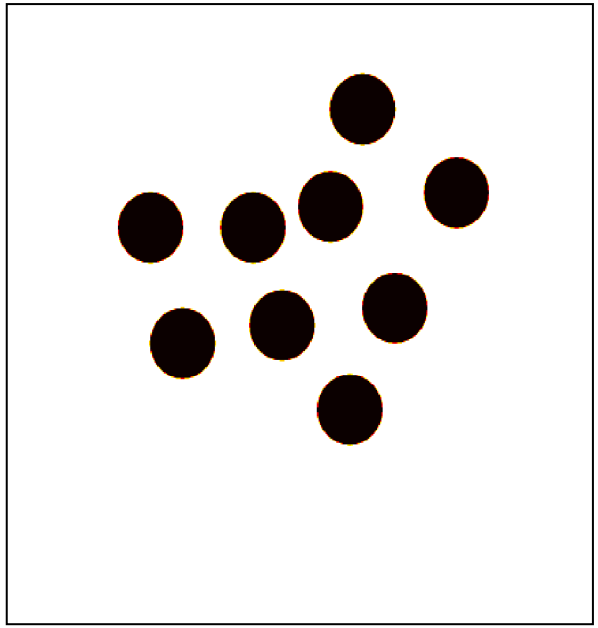


near contact regime

$$r << a^2 / \lambda$$

Numerical Examples

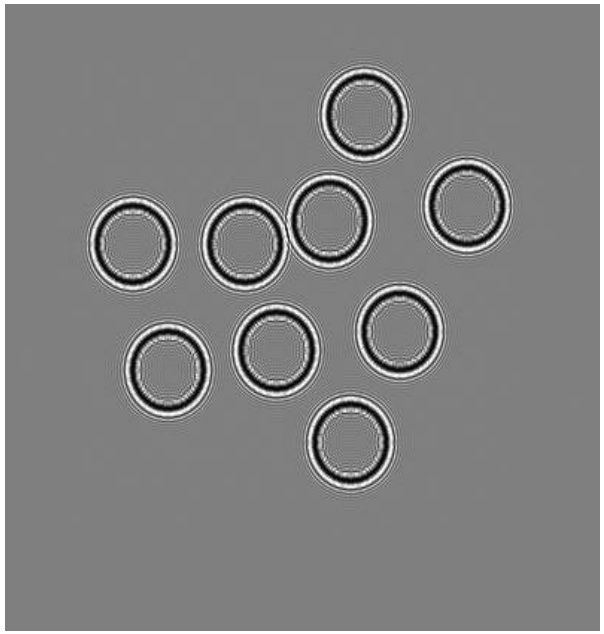
object phase



coherent imaging experiment

(propagation)

detector amplitude

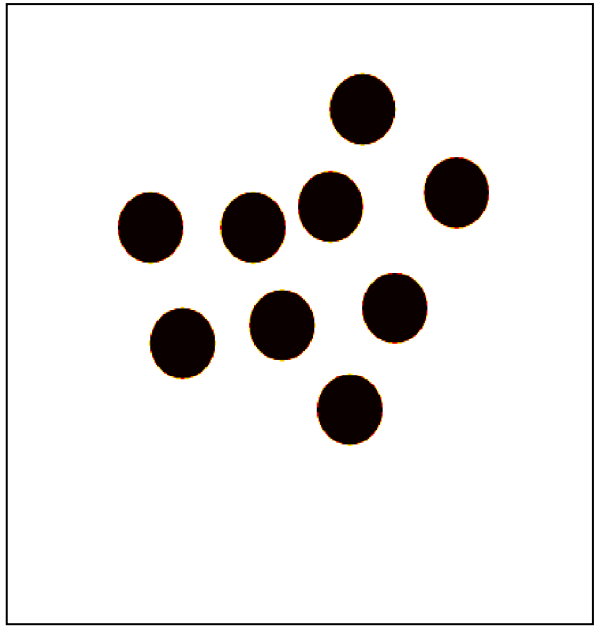


near Fresnel regime

$$r < a^2 / \lambda$$

Numerical Examples

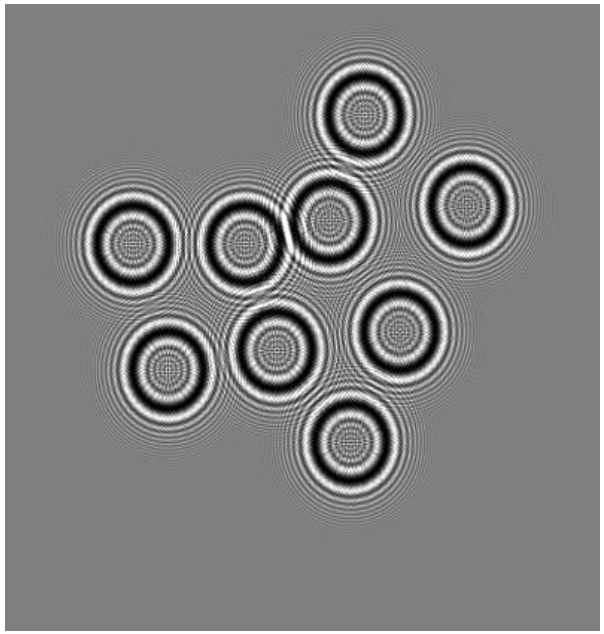
object phase



coherent imaging experiment

(propagation)

detector amplitude

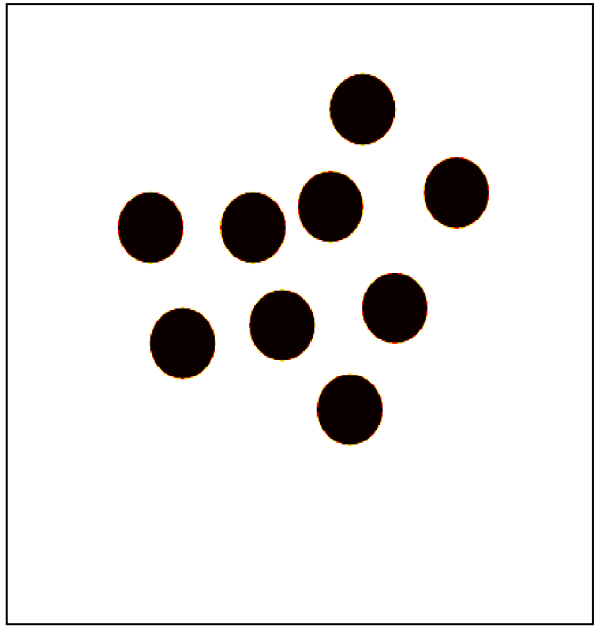


Fresnel regime

$$r \approx a^2 / \lambda$$

Numerical Examples

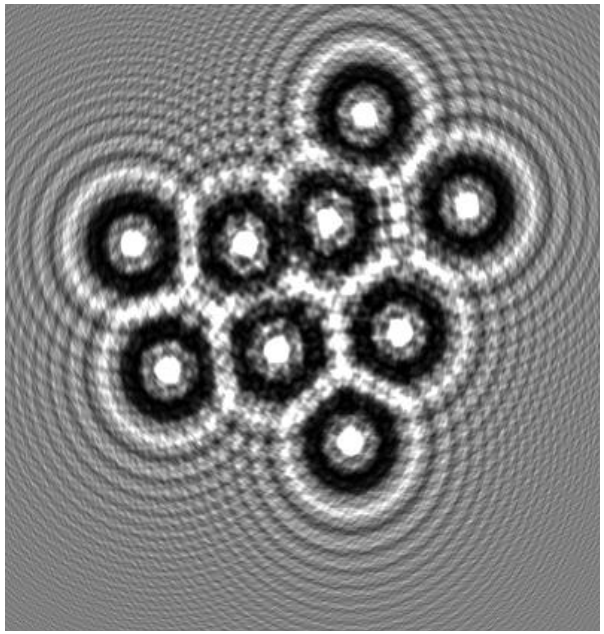
object phase



coherent imaging experiment

(propagation)

detector amplitude

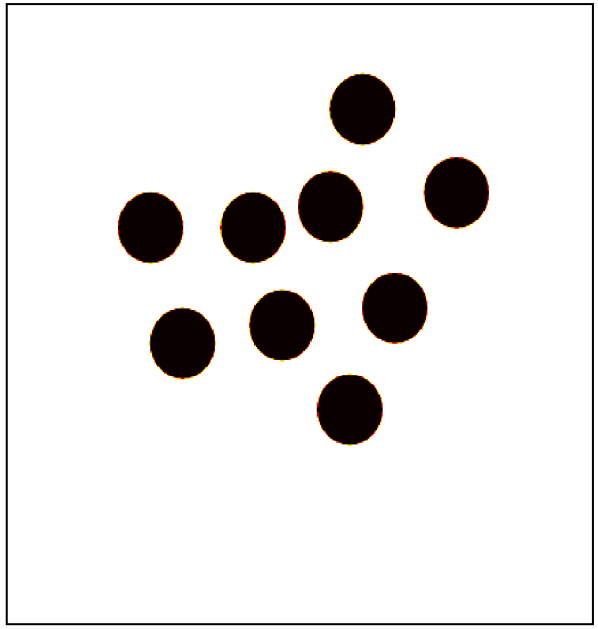


holographic regime

$$r \geq a^2 / \lambda$$

Numerical Examples

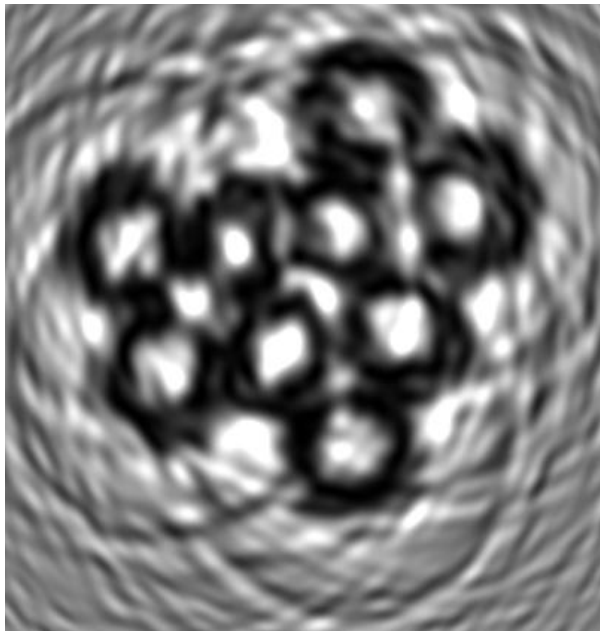
object phase



coherent imaging experiment

(propagation)

detector amplitude

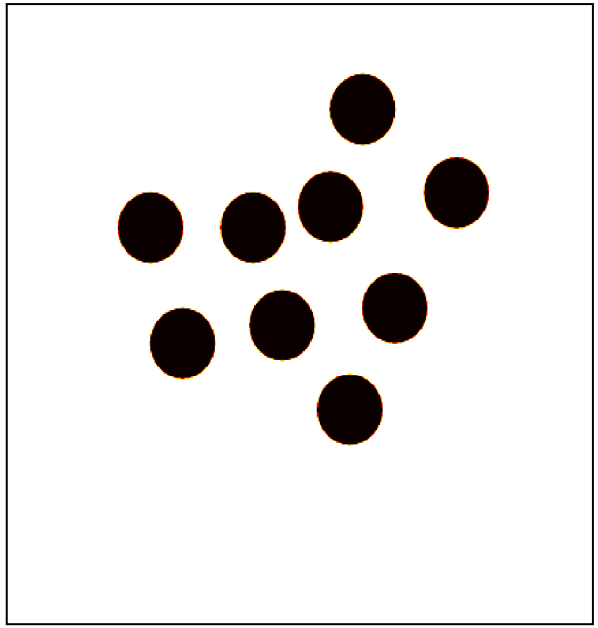


holographic regime

$$r > a^2 / \lambda$$

Numerical Examples

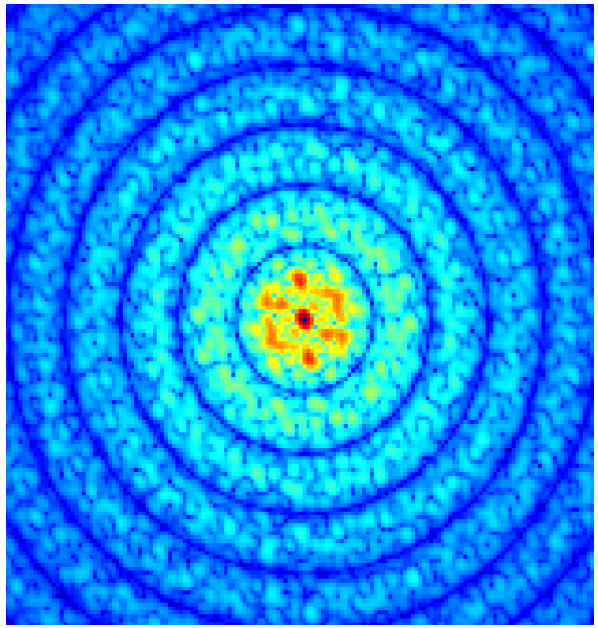
object phase



coherent imaging experiment

Fourier transform

detector amplitude

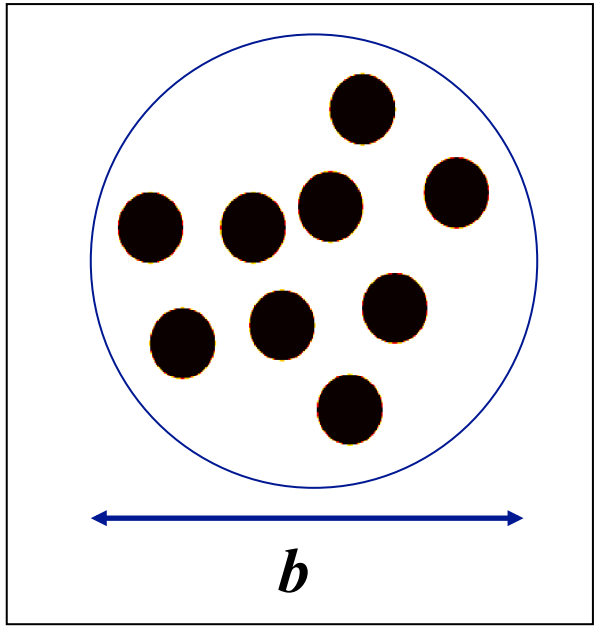


Fraunhofer far-field

$$r >> a^2 / \lambda$$

Speckles

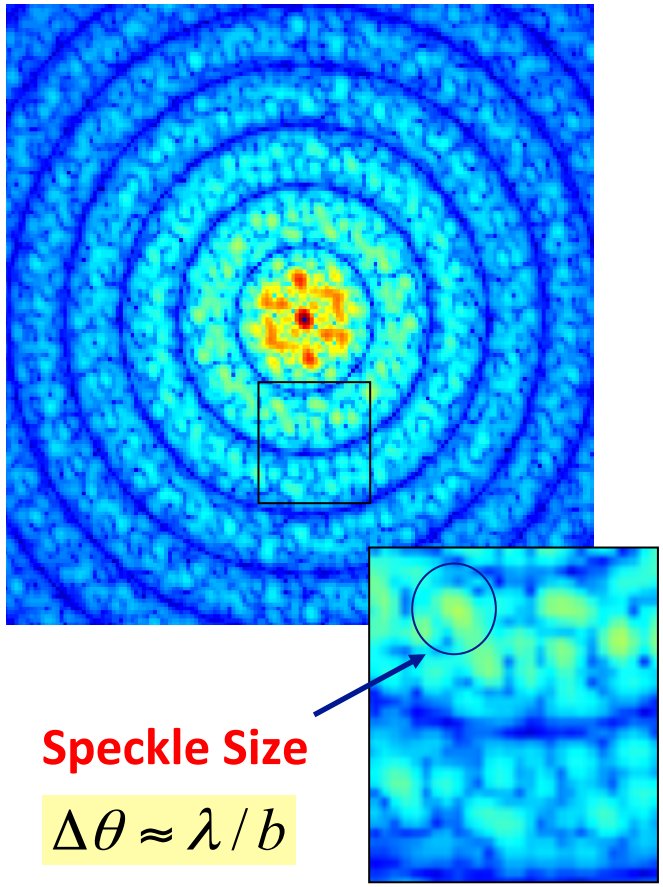
object phase



coherent imaging experiment

Fourier transform

detector amplitude



Fraunhofer far-field

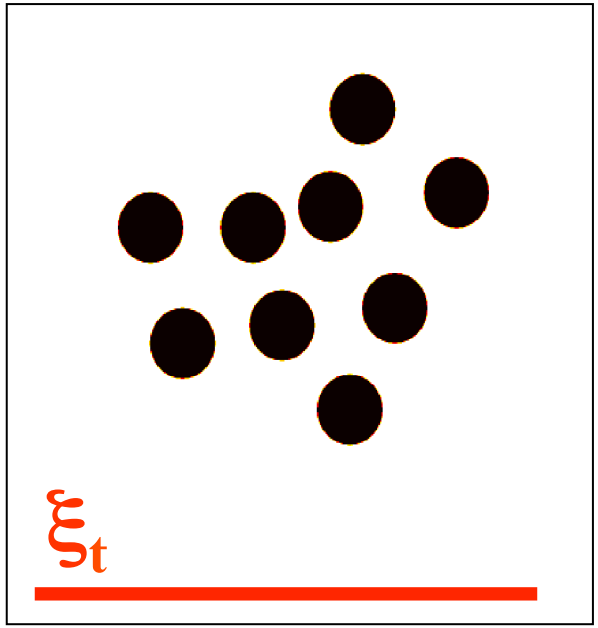
$$r >> a^2 / \lambda$$

Speckle Size

$$\Delta\theta \approx \lambda / b$$

Incoherent vs. coherent scattering

object phase

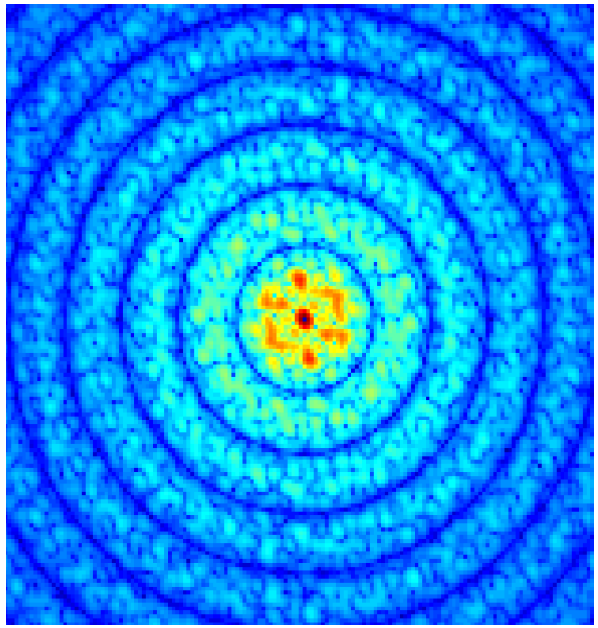


'coherent' case

coherent imaging Experiment

Fourier transform

detector amplitude

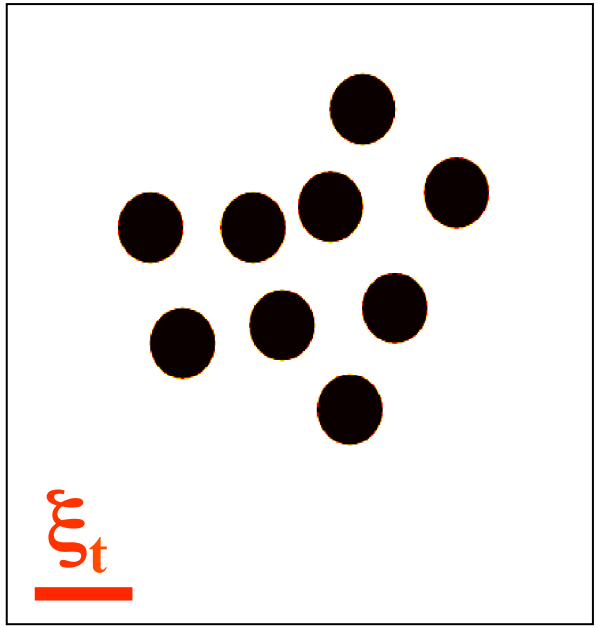


Fraunhofer far-field

$$r >> a^2 / \lambda$$

Incoherent vs. coherent scattering

object phase

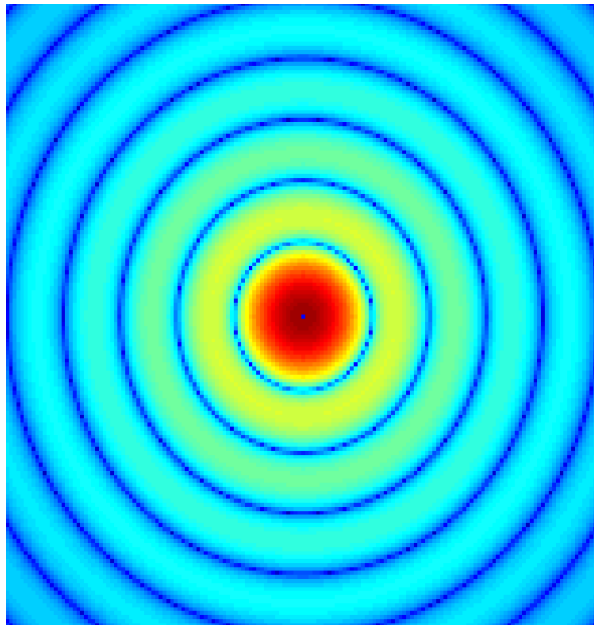


'incoherent' case

coherent imaging Experiment

Fourier transform

detector amplitude

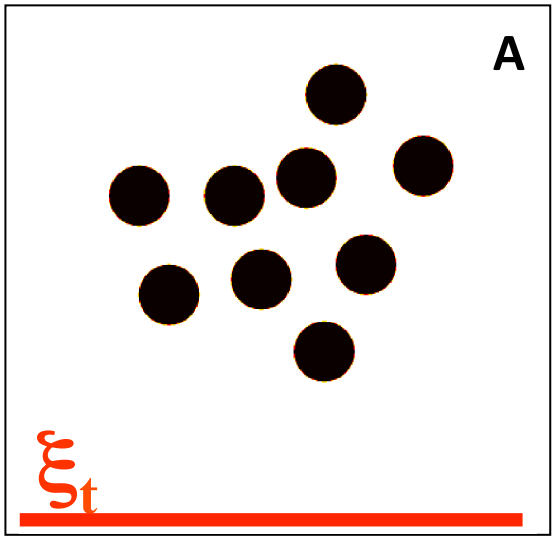


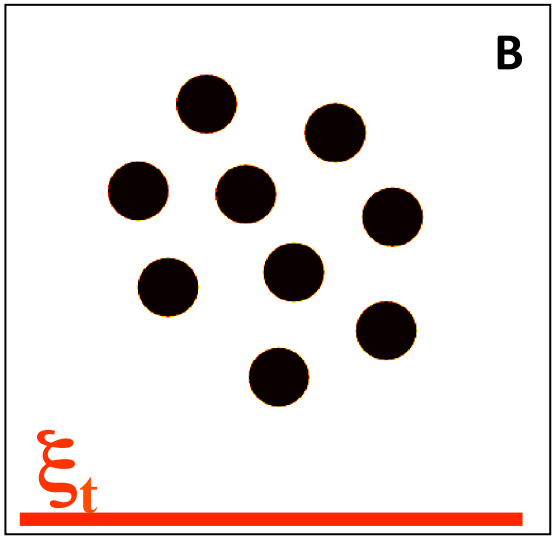
Fraunhofer far-field

$$r >> a^2 / \lambda$$

Speckle structure

object phase





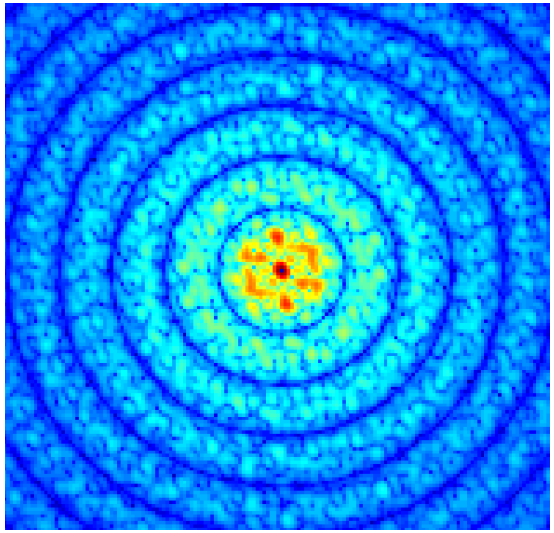
coherent imaging experiment

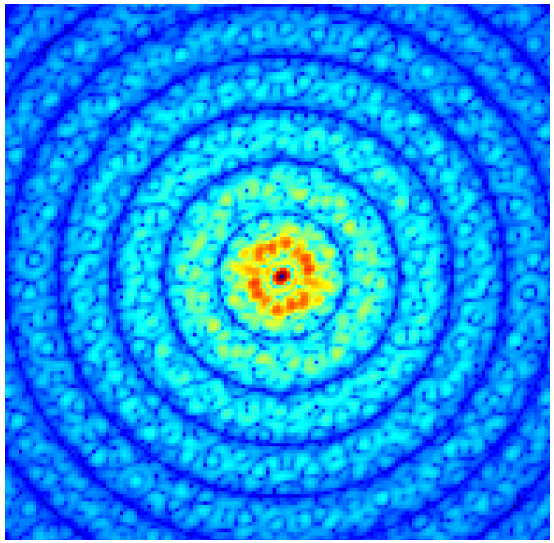
Fourier transform

Fraunhofer far-field

$$r >> a^2 / \lambda$$

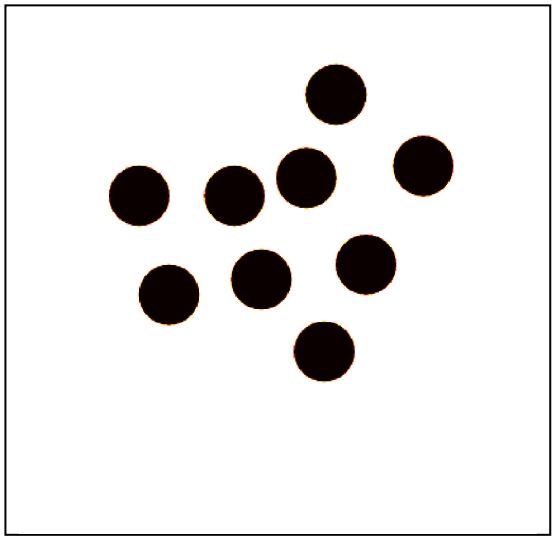
detector amplitude





Far-field & near-field phase retrieval

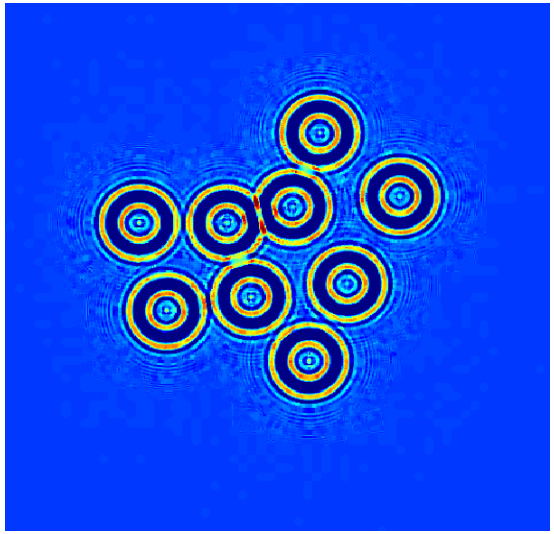
object phase | **detector amplitude**

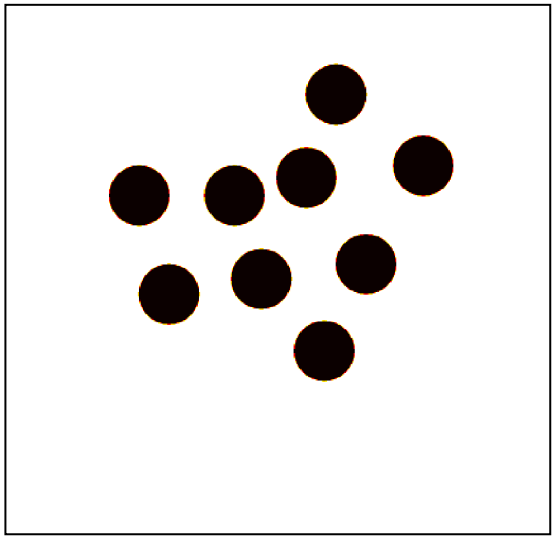


?

Fresnel regime

$r \approx a^2 / \lambda$



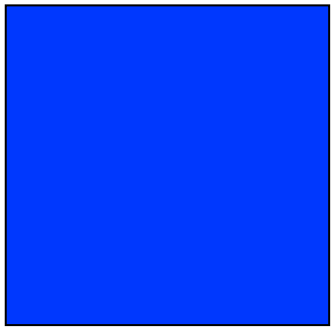


?

contact regime

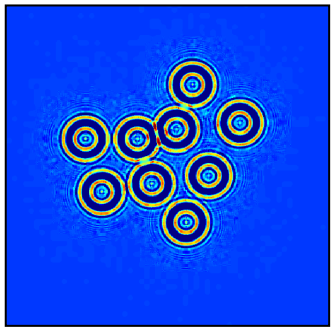
$r = 0$

Near-field, holographic, and far-field phase retrieval



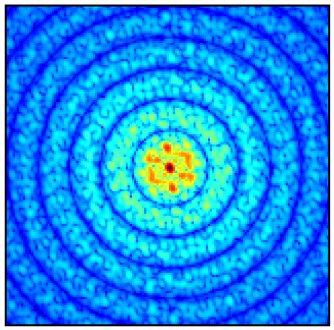
Near-field regime (large FOV, small distances):

- crystal interferometers
- crystal analyzers
- grating interferometers



Fresnel/ Holographic (medium FOV, medium distance):

- transport of intensity equation (back-propagation)
- in-line holography



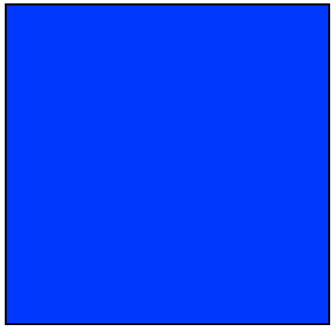
Fraunhofer far-field (small FOV, large distance):

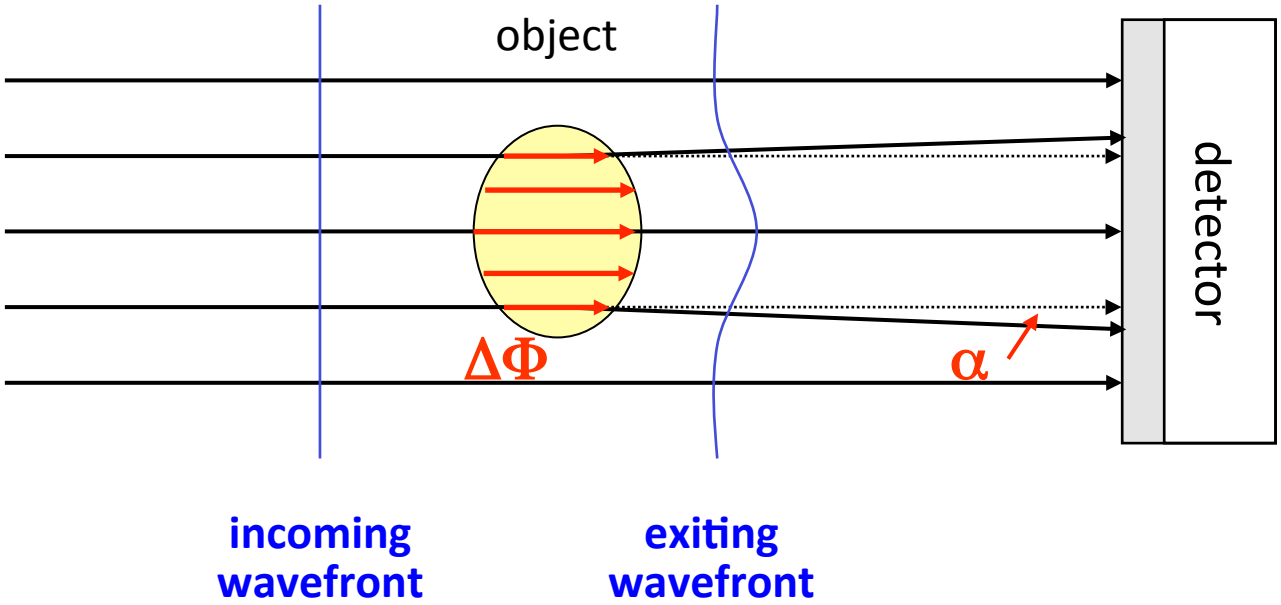
- iterative phase retrieval algorithms
- fourier transform holography
- …

required brilliance

Phase retrieval in the near-field regime

nearfield regime

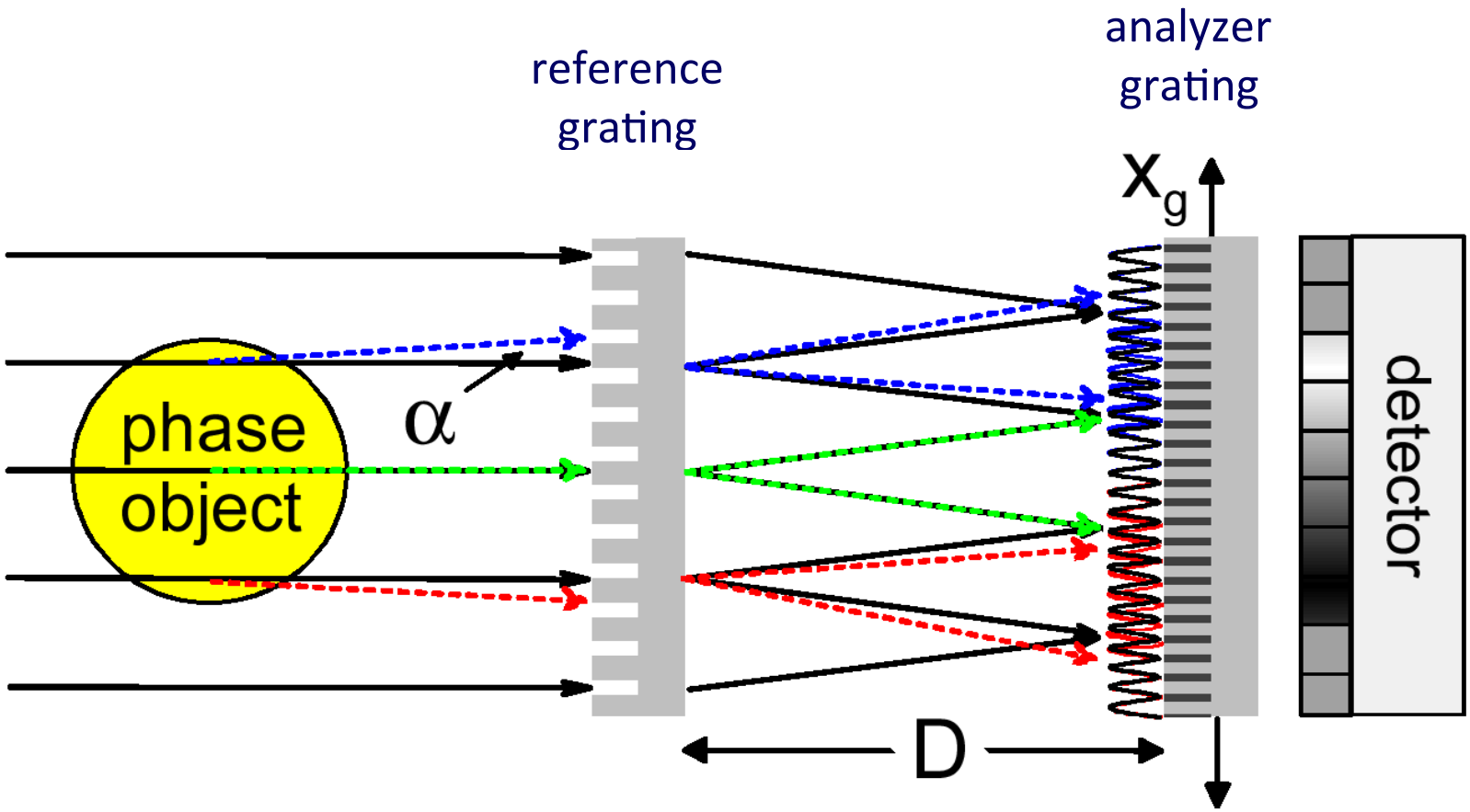




⇒ **Snell's law: the deflection angle contains information about the phase (gradient) of the object !**

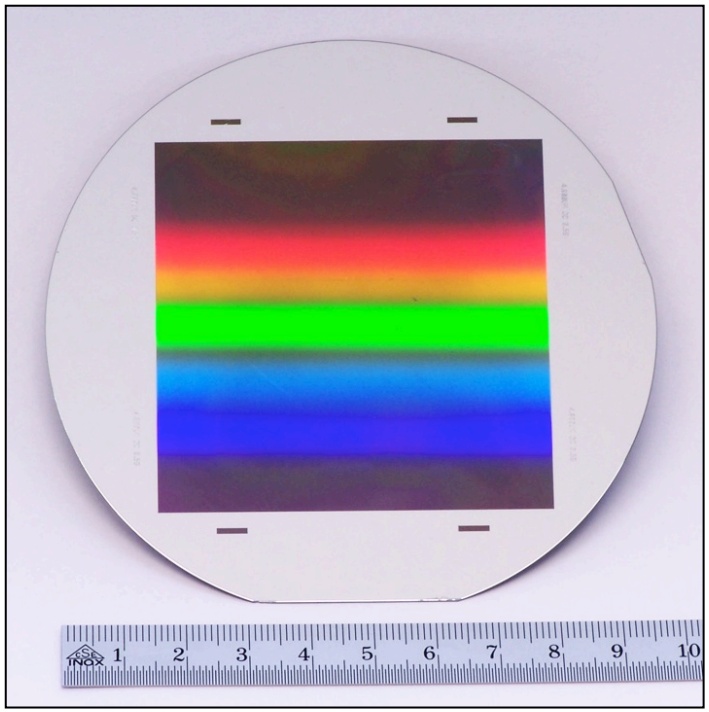
$$\alpha = \frac{\lambda}{2\pi} \nabla \Phi$$

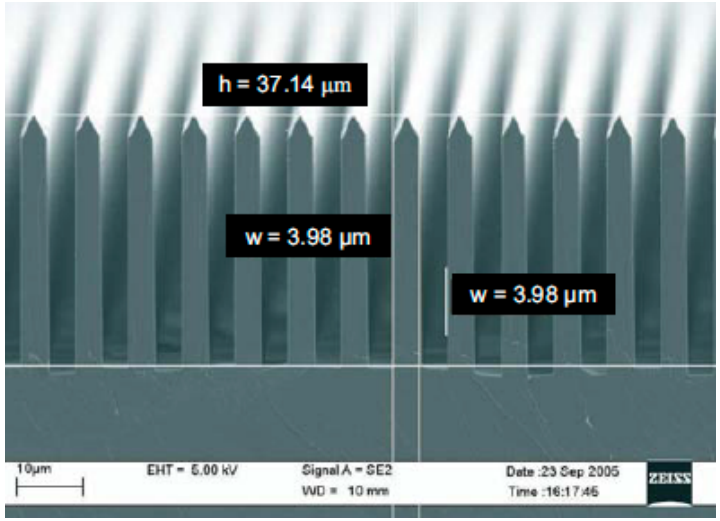
Phase retrieval using a grating interferometer



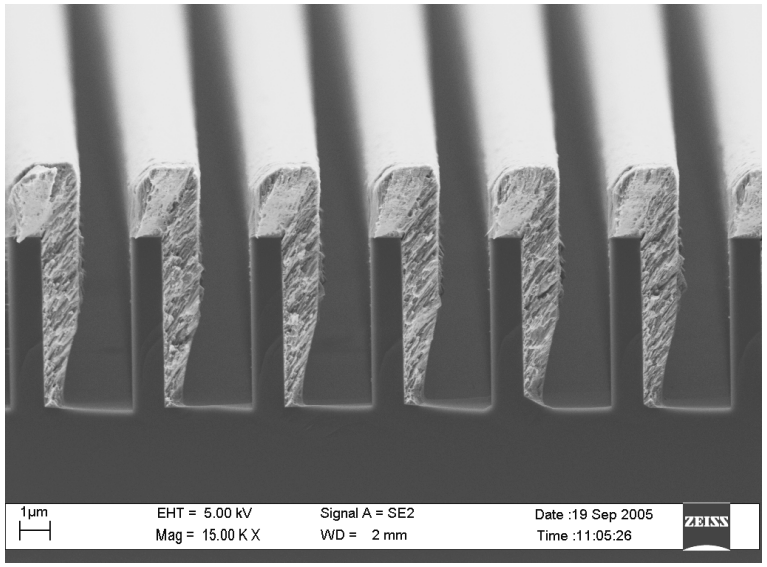
Pfeiffer et al | Phys Rev Lett | 2006

Phase retrieval using a grating interferometer





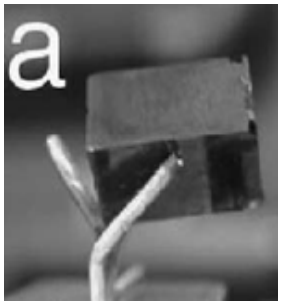
Si phase grating



Si/ Gd analyzer grating

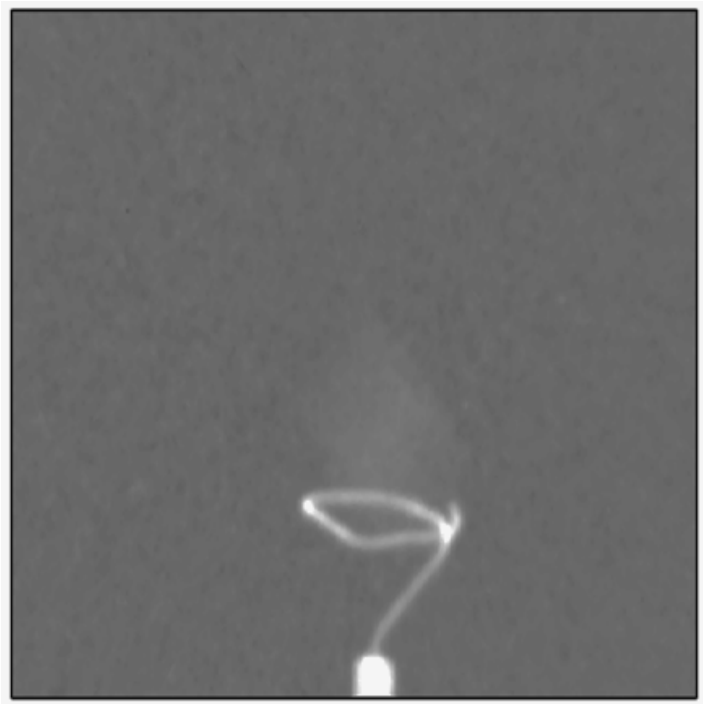
Gruenzweig et al | Rev Sci Inst | 2008

Phase retrieval using a grating interferometer

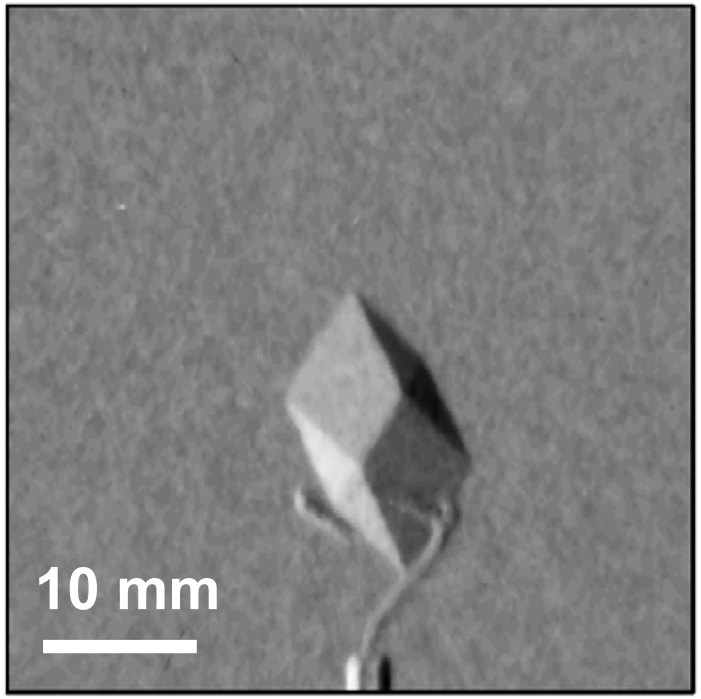


Si cube

amplitude of the complex object transmission function

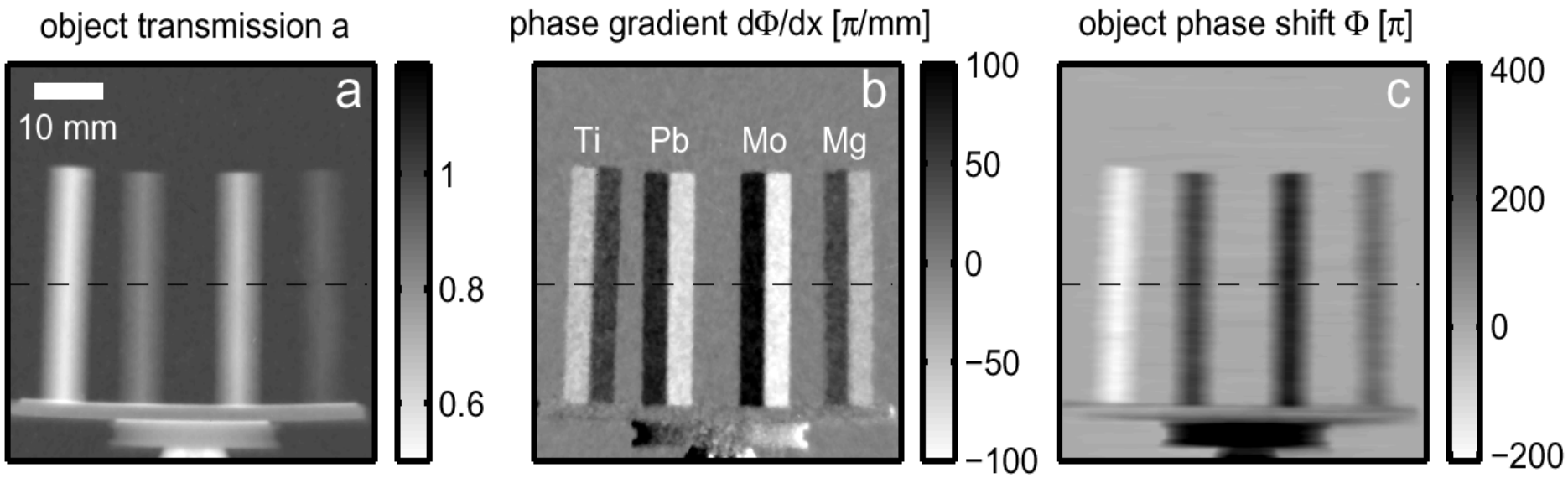


phase gradient of the complex object transmission function



experiments at ICON/SINQ (2005), unpublished

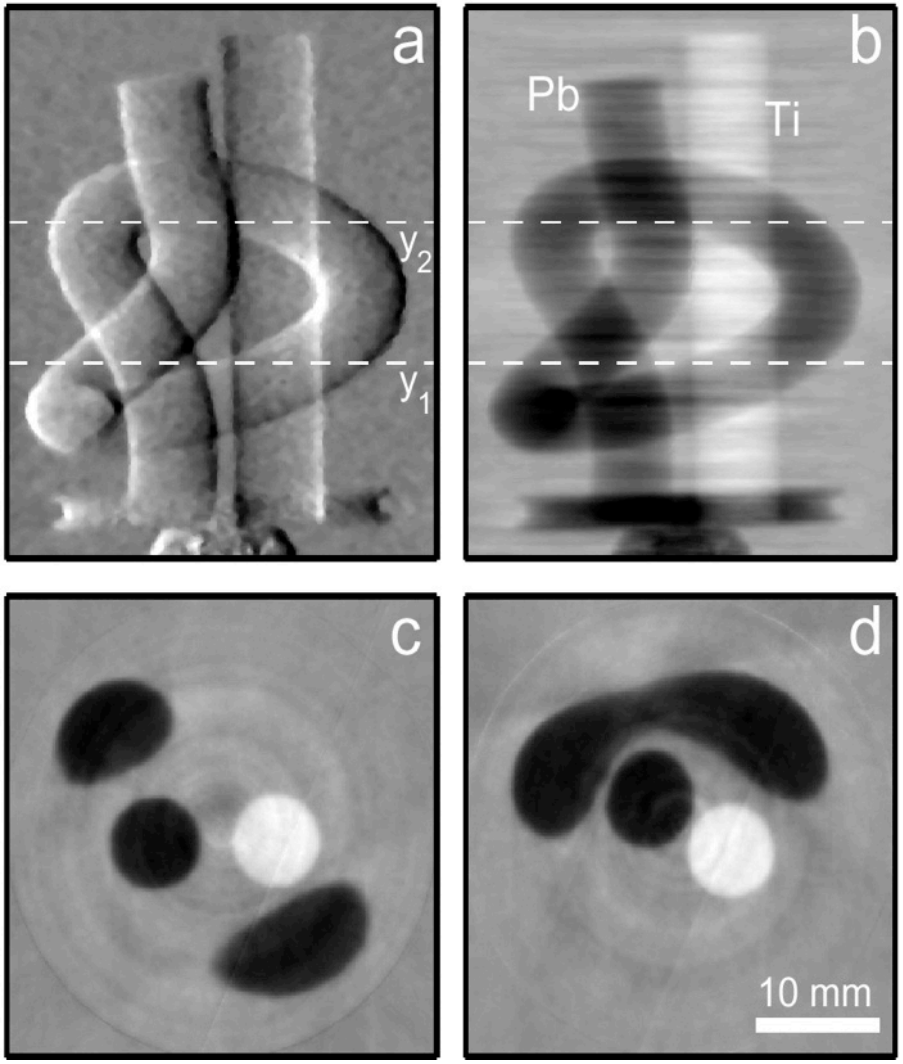
Phase retrieval using a grating interferometer

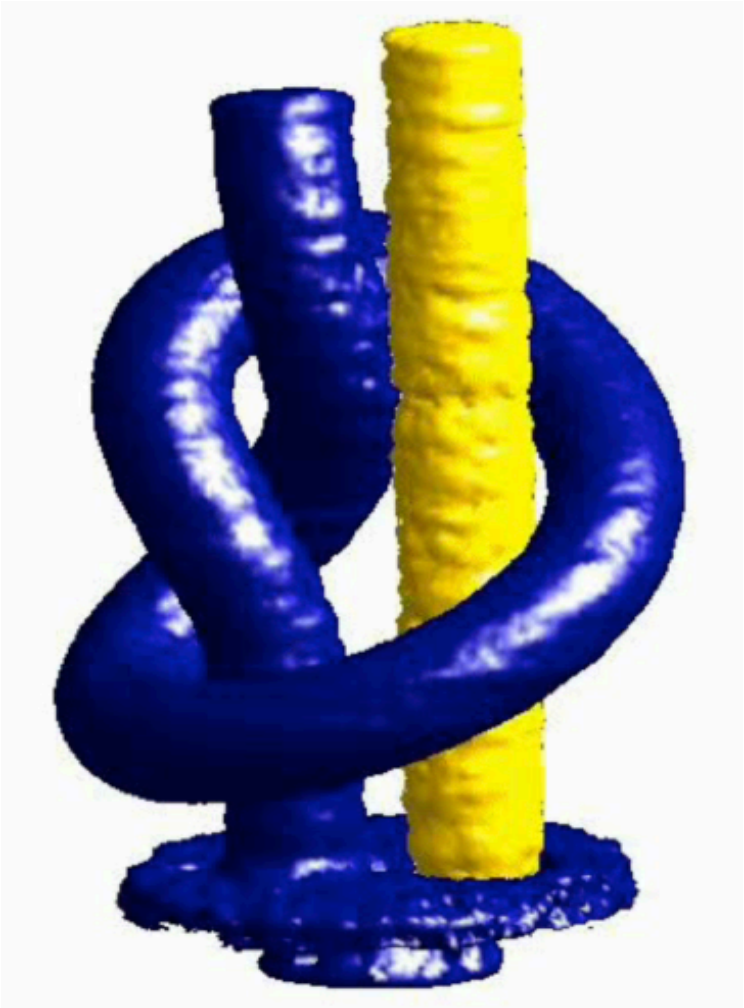


quantitative ✓

Pfeiffer et al | Phys Rev Lett | 2006

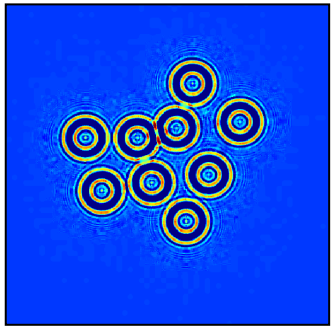
Quantitative neutron phase-contrast CT

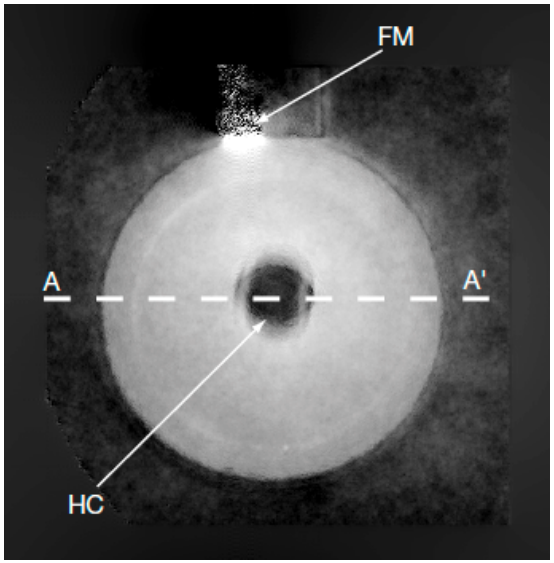


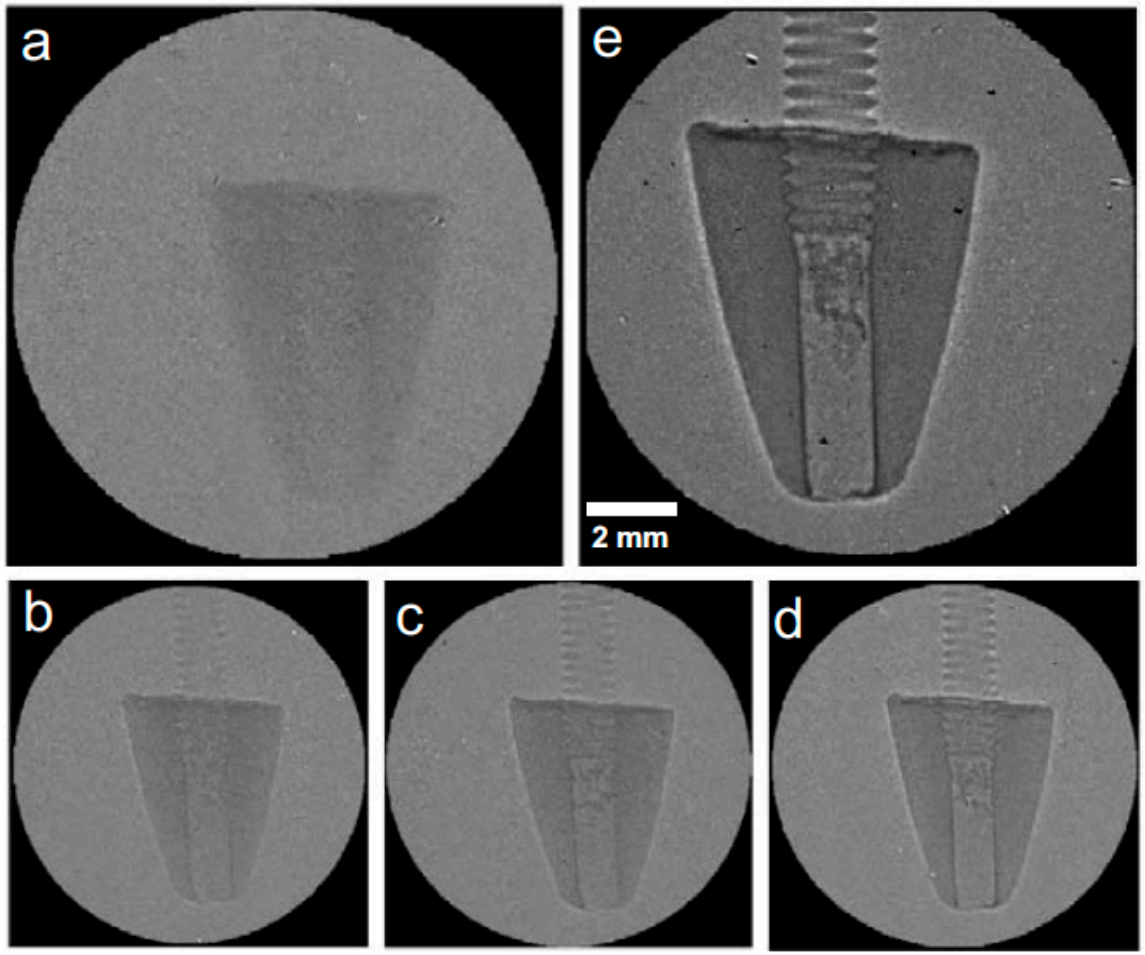


Near-field phase retrieval – Back-Propagation

Fresnel regime



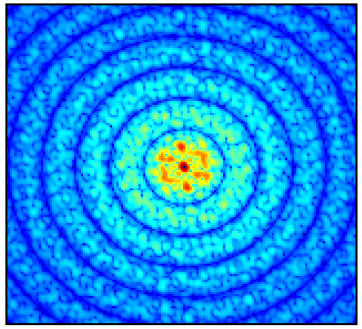




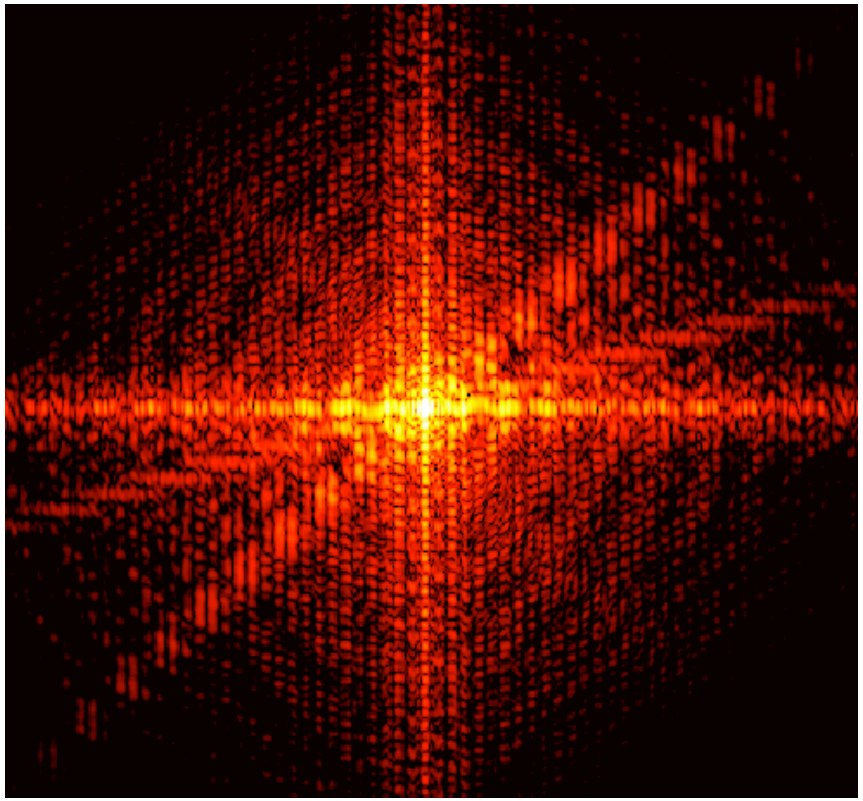
Allmann et al | Nature | 2000

Phase Retrieval from Coherent Far-field Diffraction Data

Far-field regime

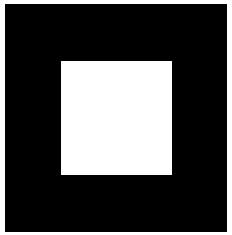


FFT

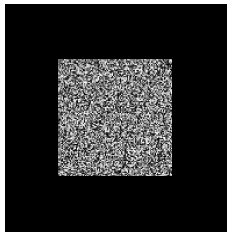


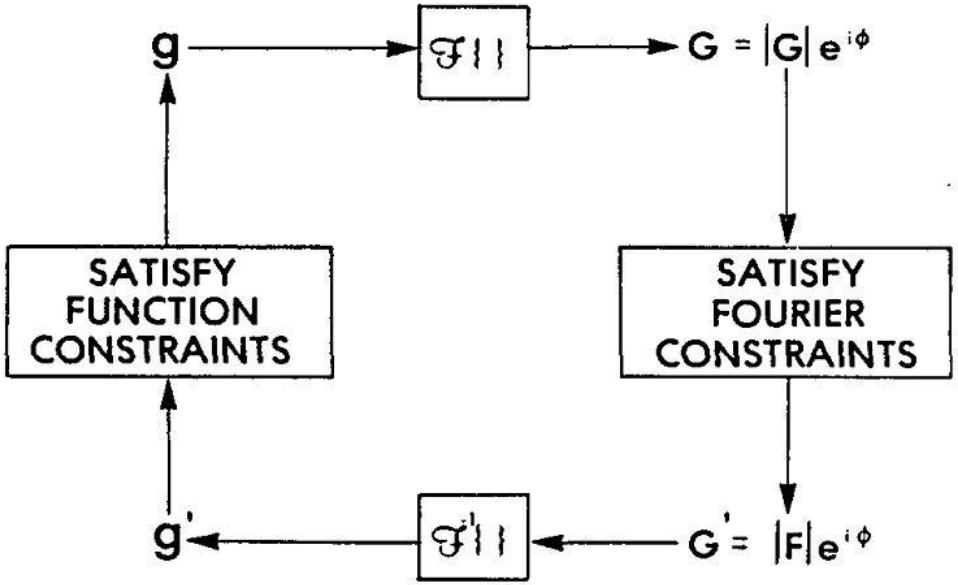
Far-field phase retrieval – iterative algorithms

amp

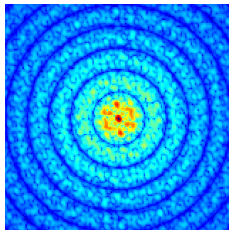


phase





amp



phase

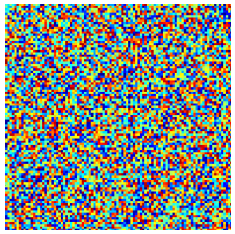


Fig. 1. Block diagram of the error-reduction (Gerchberg-Saxton) algorithm.

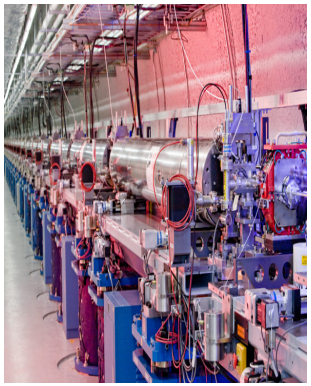
real space

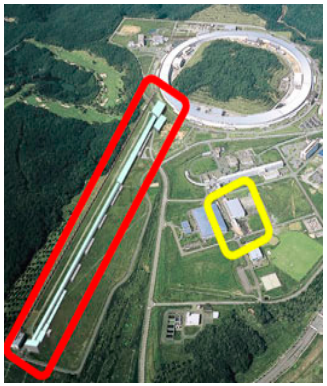
Fourier space

Fienup | Appl Opt | 1982

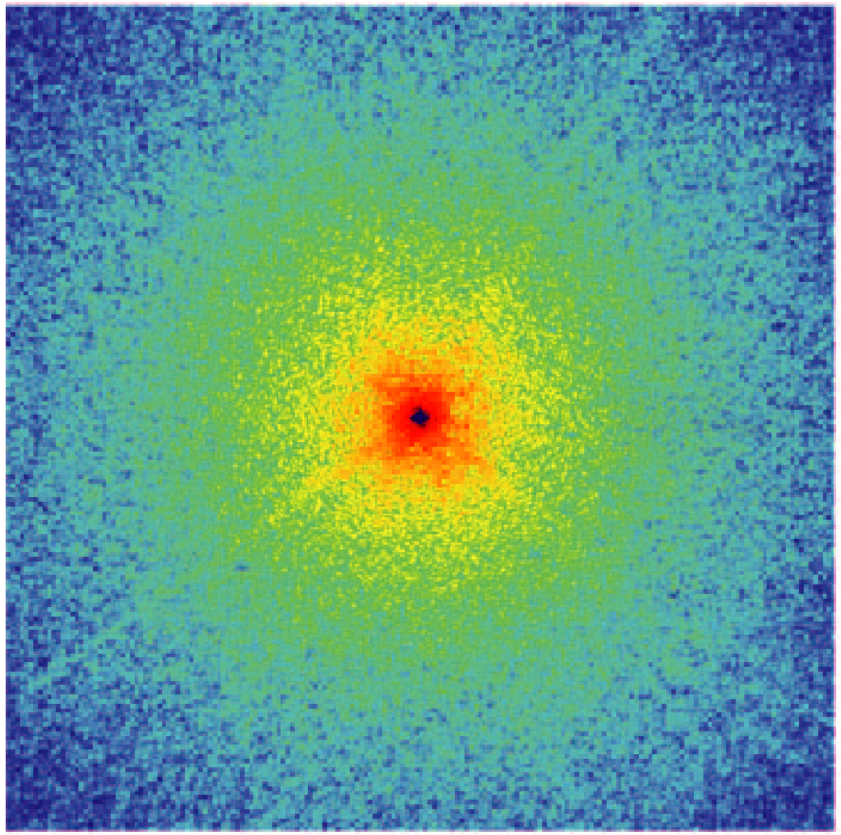
Coherent Diffractive X-Ray Microscopy

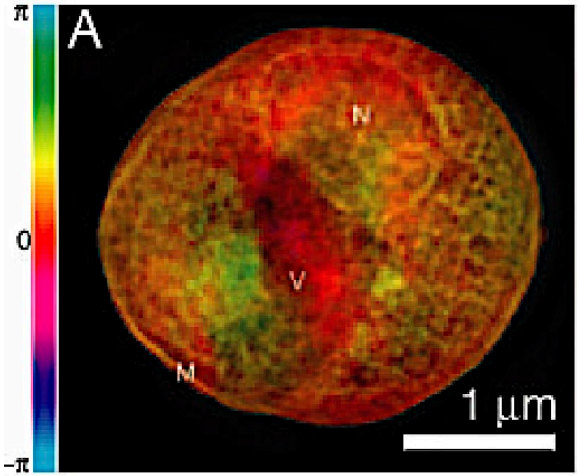
high-brilliance x-ray source





coherent x-ray diffraction pattern



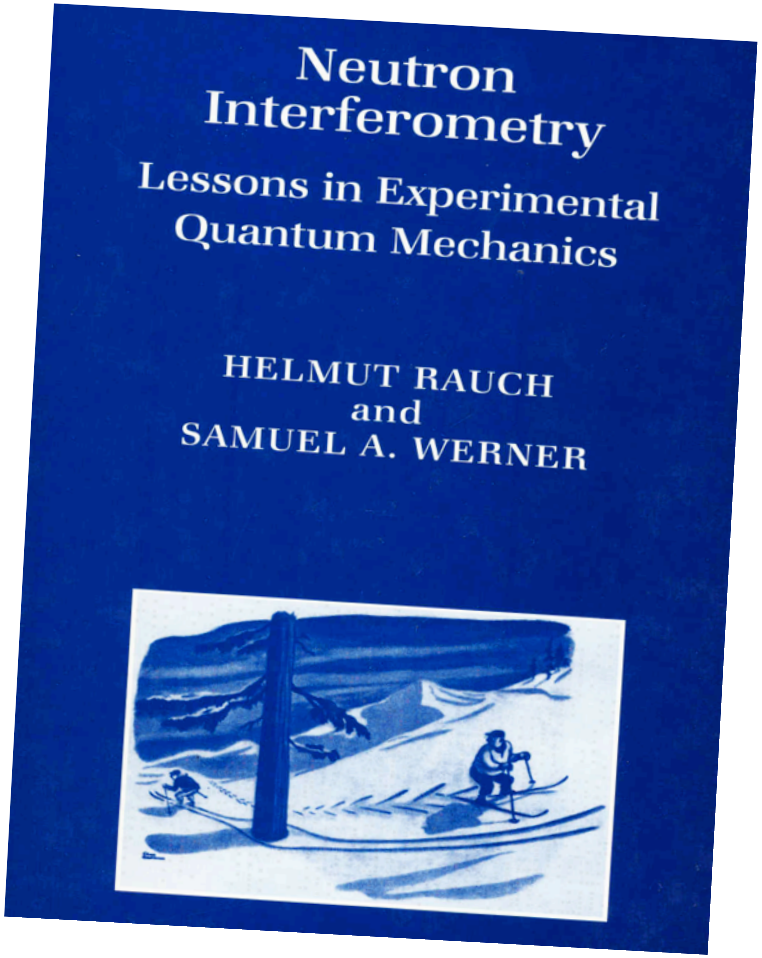


reconstructed phase

Shapiro et al | PNAS | 2006

The Future ?

Interaction	Potential	Phase shift
Nuclear	$\frac{2\pi\hbar^2}{m} b_c \delta(\boldsymbol{r})$	$-N b_c \lambda D$
Magnetic	$-\boldsymbol{\mu} \cdot \boldsymbol{B}(\boldsymbol{r})$	$\pm \frac{\mu B m \lambda D}{2\pi\hbar^2}$
Gravitation	$m\boldsymbol{g} \cdot \boldsymbol{r}$	$\frac{m^2 g \lambda A \sin\alpha}{2\pi h^2}$
Coriolis	$-\hbar\boldsymbol{\omega}(\boldsymbol{r} \times \boldsymbol{k})$	$\frac{2m}{\hbar} \boldsymbol{\omega}_e \cdot \boldsymbol{A}$
Aharonov–Casher (Schwinger)	$-\boldsymbol{\mu} \cdot (\boldsymbol{v} \times \boldsymbol{E})/c$	$\pm \frac{2\mu}{\hbar c} \boldsymbol{E} \cdot \boldsymbol{D}$
Scalar Aharonov–Bohm	$-\boldsymbol{\mu} \cdot \boldsymbol{B}(t)$	$\pm \frac{\mu B T}{\hbar}$
Magnetic Josephson	$-\boldsymbol{\mu} \cdot \boldsymbol{B}(t)$	$\pm \omega \cdot t$
Fizeau	—	$-N b_c \lambda D \left(\frac{w_x}{v_x - w_x} \right)$
Geometry (Berry)	—	$\Omega/2$



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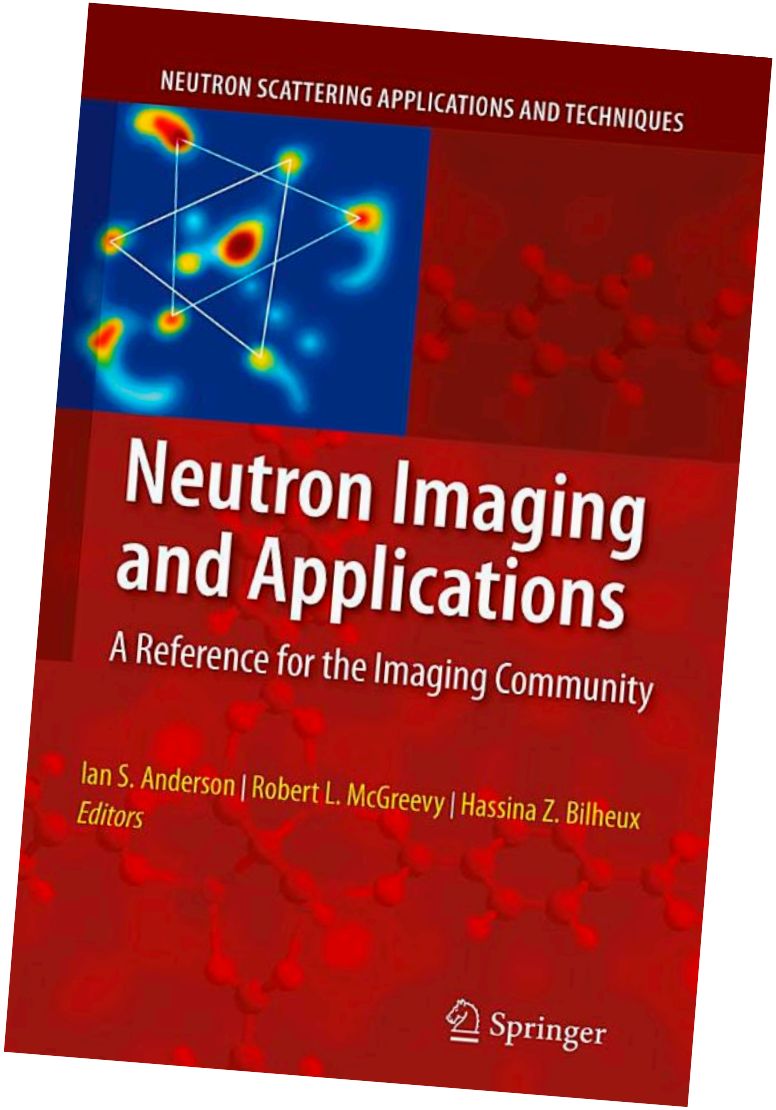
- …

Frascati, INSIS, July 2012

Thank you for your attention !



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