# Parametric Decay Instability (PDI) in Tokamaks

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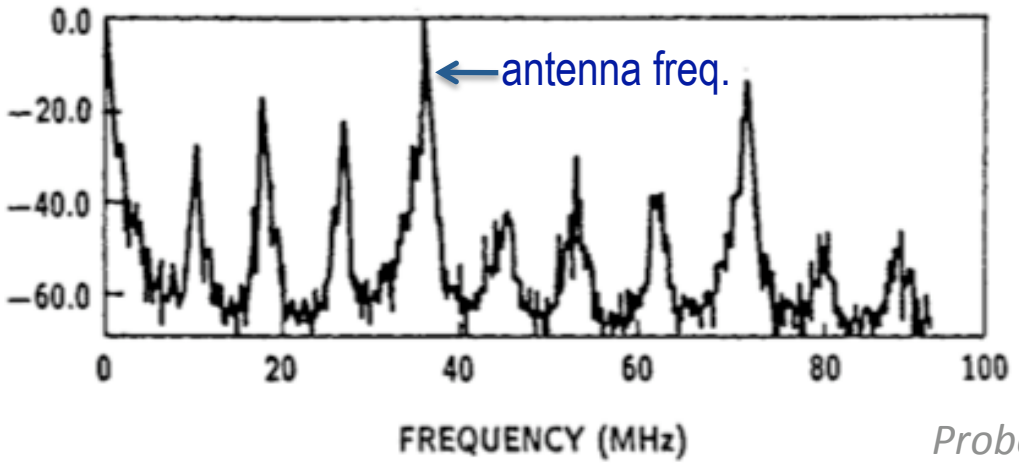
# What is PDI?

- In a heating experiment, linear waves can be excited by external sources.
- A high amplitude wave (pump wave @ $f_0$) can non-linearly excite other waves (daughter waves @ $f_1$, $f_2$).
  - Particles with oscillatory velocity $u(f_0)$.
  - Fluctuating density $n(f_2)$.
  - The above two quantities produce a current *nu*, which can drive a plasma wave at $f_1(=f_2-f_0)$.
  - The plasma wave ($f_1$) and the oscillatory velocity at pump frequency ($f_0$) produces a ponderomotive force that drives the low frequency perturbation.

# PDI and Edge Ion Heating in IBW and ICRF Experiments

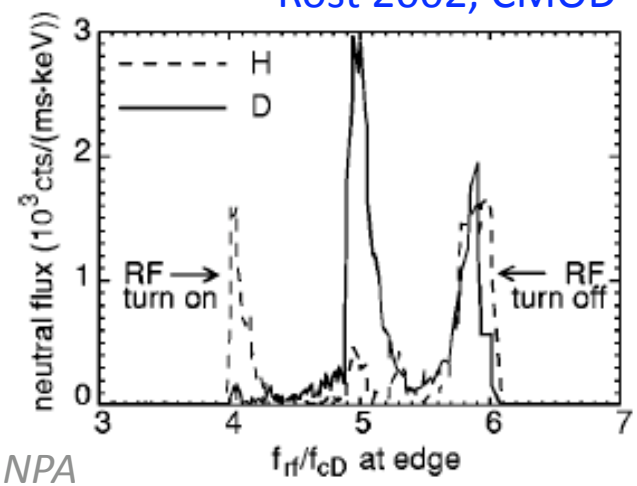
a

Pinsker 1993, DIII-D




Rost 2002, CMOD

b

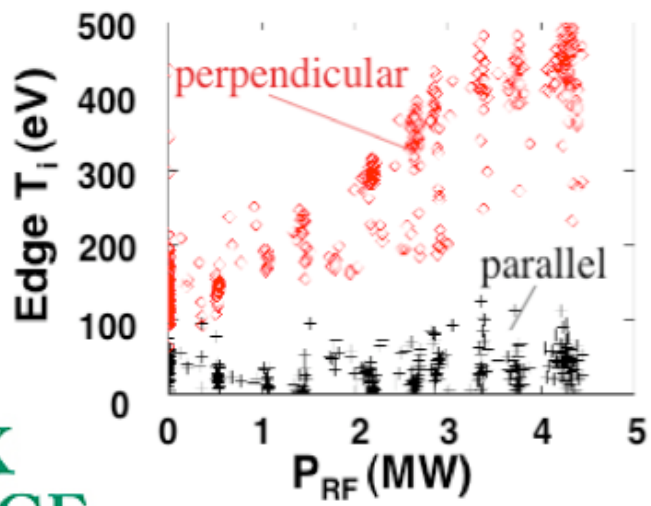



*Probe NPA*

*ERD Reflectometer*

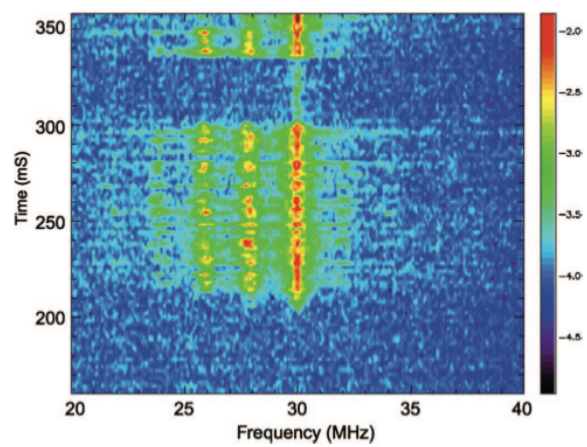
c

Biewer 2005, NSTX




Wilgen 2006, NSTX

d



# Motivation

- PDI has been widely observed in tokamak ICRF and IBW heating experiments (NSTX, Alcator C-MOD, DIII-D, TST-2, HT-7, etc.).
- PDI in the scrape-off-layer (SOL) can lead to significant power loss, degrading the rf heating efficiency in the bulk plasma (eg. direct IBW).
- On NSTX, edge PDI loss accounts for 30% of total rf power (a lower bound estimate).
- We need a more quantitative understanding of PDI.

# Outline

- Kinetic theory and model equations
- Numerical methods
- AORSA 1D simulations and PDI dispersion calculations
  - Alcator C-MOD
  - NSTX
- Summary
- Future work

# Oscillating Frame, Kinetic Theory

- Maxwellian plasma in a oscillating frame,

$$\frac{\partial \mathbf{u}}{\partial t} = \Omega \mathbf{u} \times \mathbf{b} + \frac{Ze}{m}\mathbf{E}(t) \leftarrow \boxed{\text{dipole pump}} \quad , \qquad \frac{\partial \xi}{\partial t} = \mathbf{u}$$

- Vlasov equation in the oscillating frame:

$$\frac{\partial f_k}{\partial t} + i k_{\|} w_{\|} f_k - \Omega \frac{\partial f_k}{\partial \varphi} = -\mathbf{a}_k \cdot \nabla_w F \, e^{i\mathbf{k}\cdot\xi - i\delta_k} \quad , \qquad e^{i\mathbf{k}\cdot\xi} = \sum_n J_n(\mu) e^{-in\omega_0 t - in\beta}$$

- Solutions

$$p = m + n$$

$$f_k = \sum_p f_k^{(p)} e^{-i(\omega + p\omega_0)t} \quad , \text{ with } \quad \mathbf{a}_k = \sum_m \mathbf{a}_k^{(m)} e^{-i(\omega + m\omega_0)t}$$

- Electrostatic theory: charge density + Poisson's equation $\Longrightarrow$ Nonlinear Coupling

# PDI Model Equations

- Pump wave is solved independently, and assumed constant in the SOL

$$-\frac{c^2}{\omega_0^{\,2}}\left(\nabla\times\nabla\times\mathbf{E}_0\right)+\mathbf{K_0}\cdot\mathbf{E}_0=-\frac{i}{\varepsilon_0\omega_0}\mathbf{J}_{antenna}$$

- Daughter waves are solved by

Nonlinear coupling terms

$$-\frac{c^2}{\omega_1^{\,2}}\left(\nabla\times\nabla\times\mathbf{E}_1\right)+\mathbf{K_1}\cdot\mathbf{E}_1=\boxed{\mathbf{C}_1(\mathbf{E}_0,\mathbf{E}_1,\mathbf{E}_2)}-\frac{i}{\varepsilon_0\omega_1}\mathbf{J}_{noise,1}$$

$$-\frac{c^2}{\omega_2^{\,2}}\left(\nabla\times\nabla\times\mathbf{E}_2\right)+\mathbf{K_2}\cdot\mathbf{E}_2=\boxed{\mathbf{C}_2(\mathbf{E}_0,\mathbf{E}_1,\mathbf{E}_2)}-\frac{i}{\varepsilon_0\omega_2}\mathbf{J}_{noise,2}$$

- Slection rules:

$$\omega_0=\omega_1+\omega_2$$

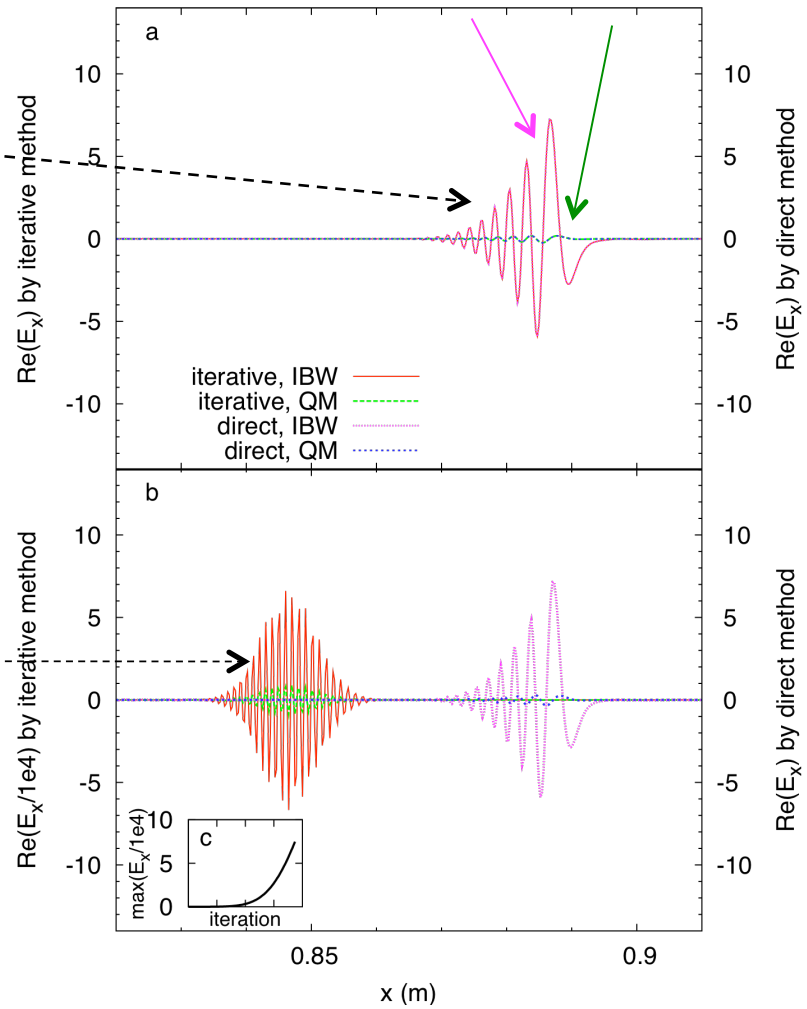
$$\mathbf{k}_0=\mathbf{k}_1+\mathbf{k}_2$$

# Successive Approximations Diverge for Large Coupling

Fast wave ➔ IBW+Quasi-Mode

Iterative method agrees with direct method

Weak coupling

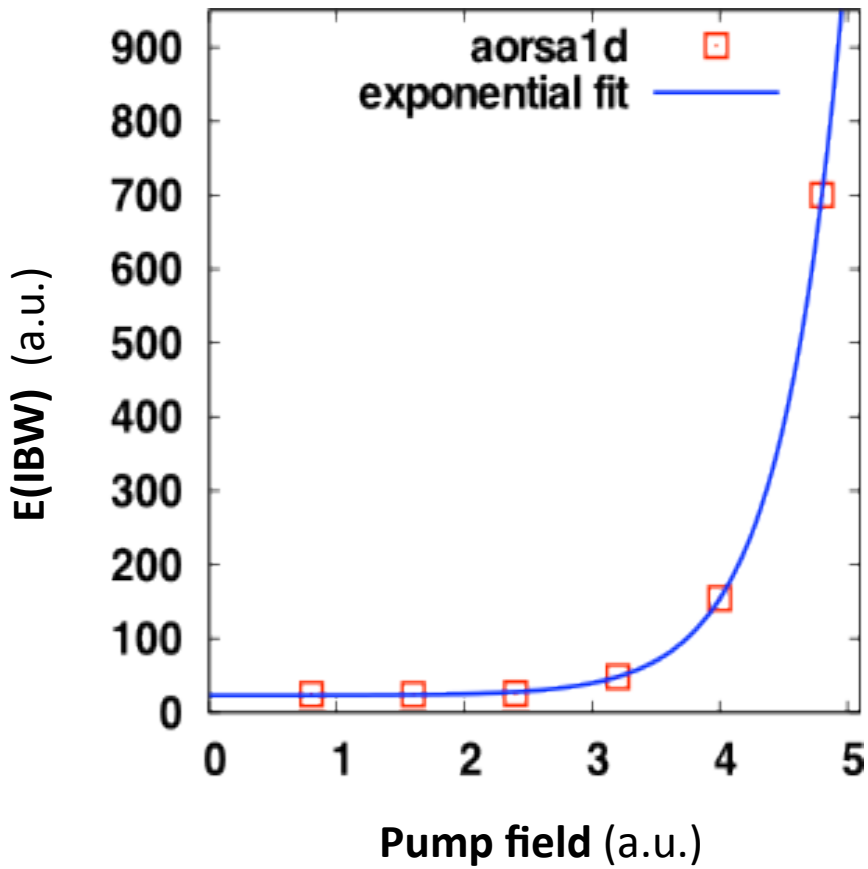



Iterative method diverges

Stronger coupling

# PDI Simulations for Alcator C-MOD

Exponential growth of the daughter field




| | |
|---|---|
| $f_2$ | 53.4 MHz |
| $f_3$ | 26.6 MHz |
| $T_e$ | 15 eV |
| $T_i$ | 3 eV |
| $n_e$ | $1 \times 10^{19}$ $m^{-3}$ |
| $B_0$ | 5.3 T |
| $r_t$ | 0.9 m |
| $k_y$ | 2000 $m^{-1}$ |
| $n_\phi$ | 40 |

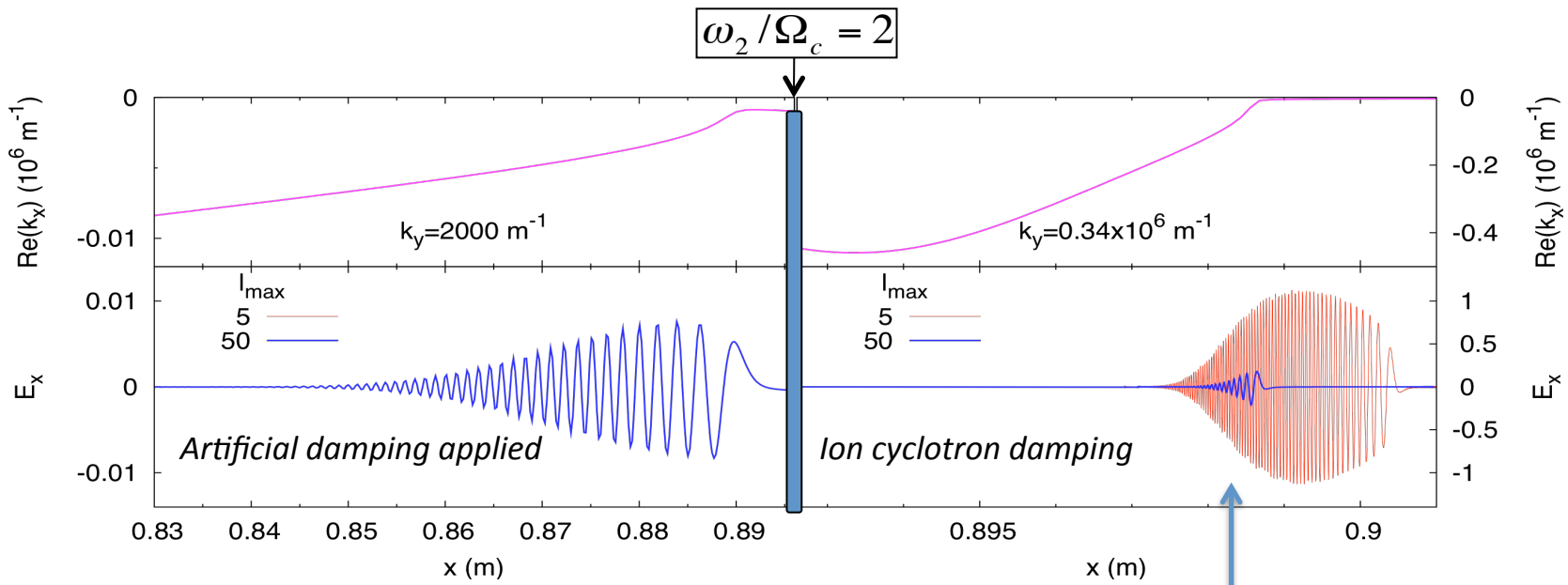
# Which waves to simulate?

- PDI daughter waves satisfy the selection rules.

$$\omega_0 = \omega_1 + \omega_2$$
$$\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$$

- One of the daughter waves is a linear plasma wave.

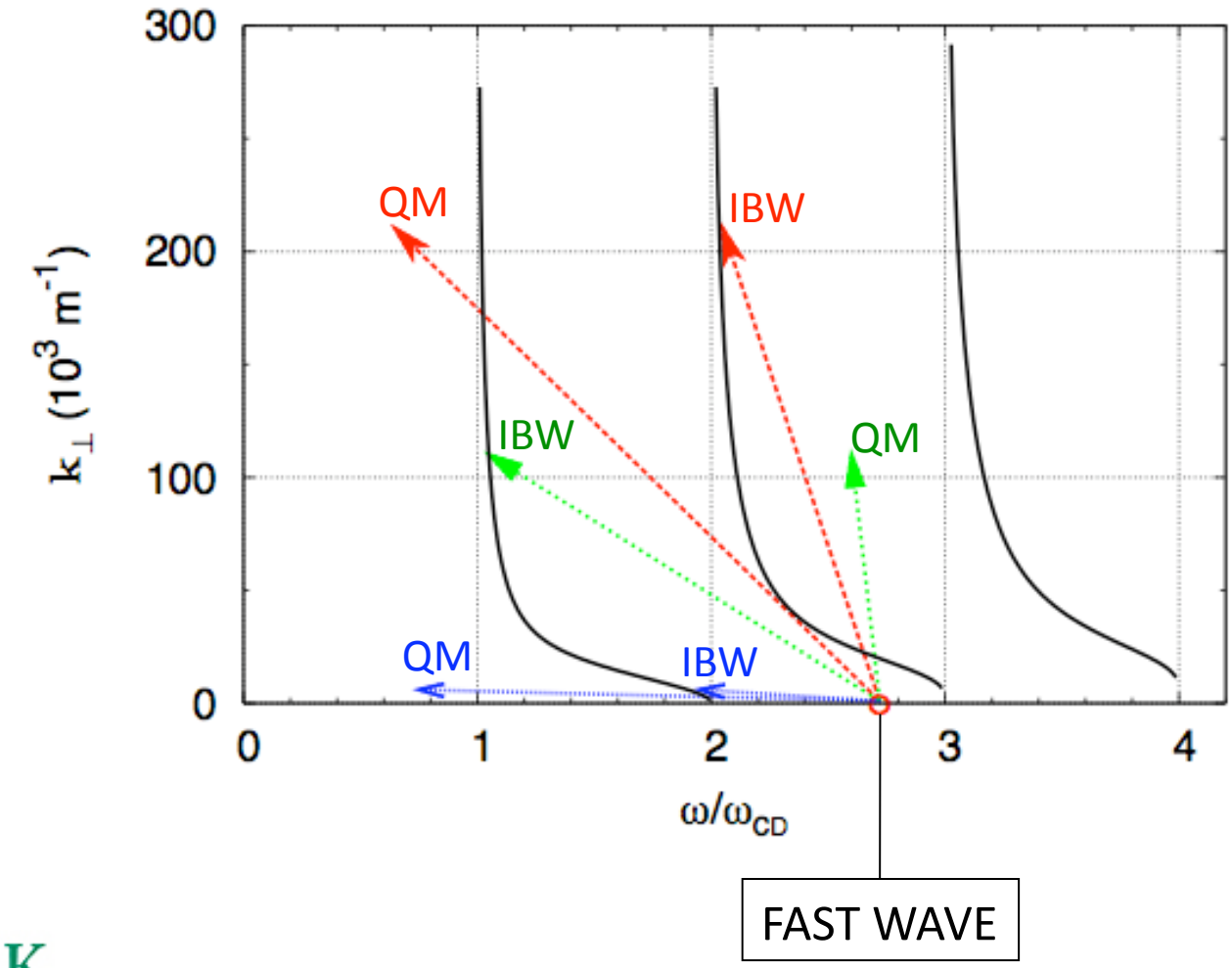
# Two Branches of IBW's




- $k_\perp(left) << k_\perp(right)$
- Damping(left)<<damping(right)
- Slow convergence for large $k_\perp$

# IBW Dispersion Relation & Decay Channels

*CMOD Minority heating regime*



# PDI Dispersion Relation

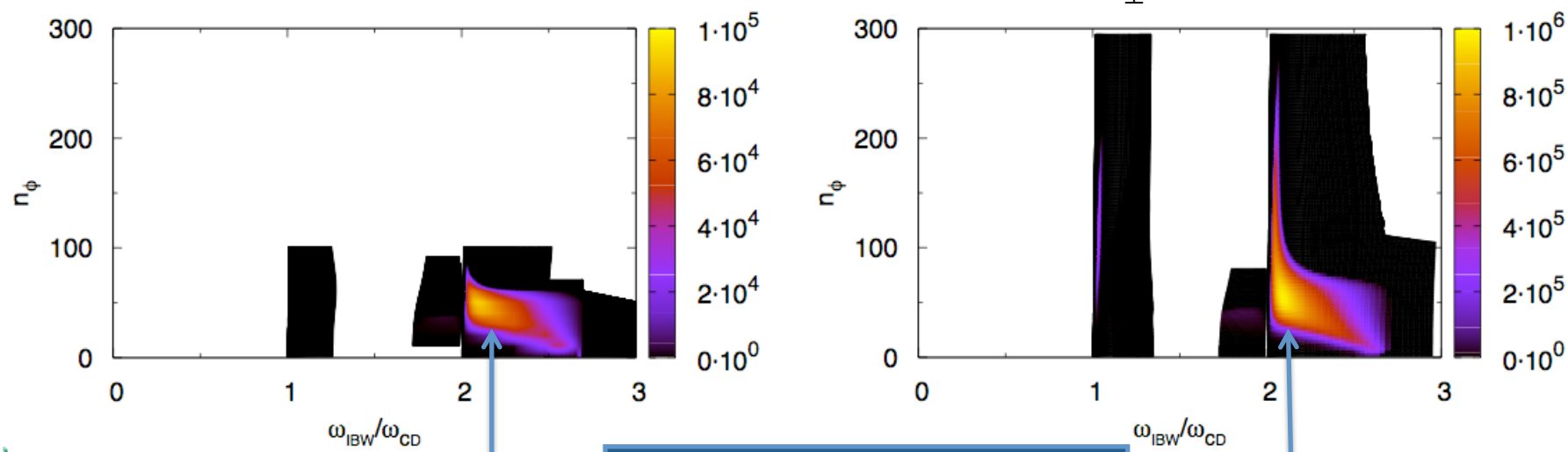
*CMOD Minority heating regime*

$$\varepsilon(\omega_1,k_1)\varepsilon(\omega_2,k_2) = -\frac{\mu_{D2}\mu_{D3}}{4}(\chi_e(\omega_1,k_1)-\chi_e(\omega_2,k_2))(\chi_i(\omega_1,k_1)-\chi_i(\omega_2,k_2)),$$

$$\mu_D = \mu_e - \mu_i \ ,$$

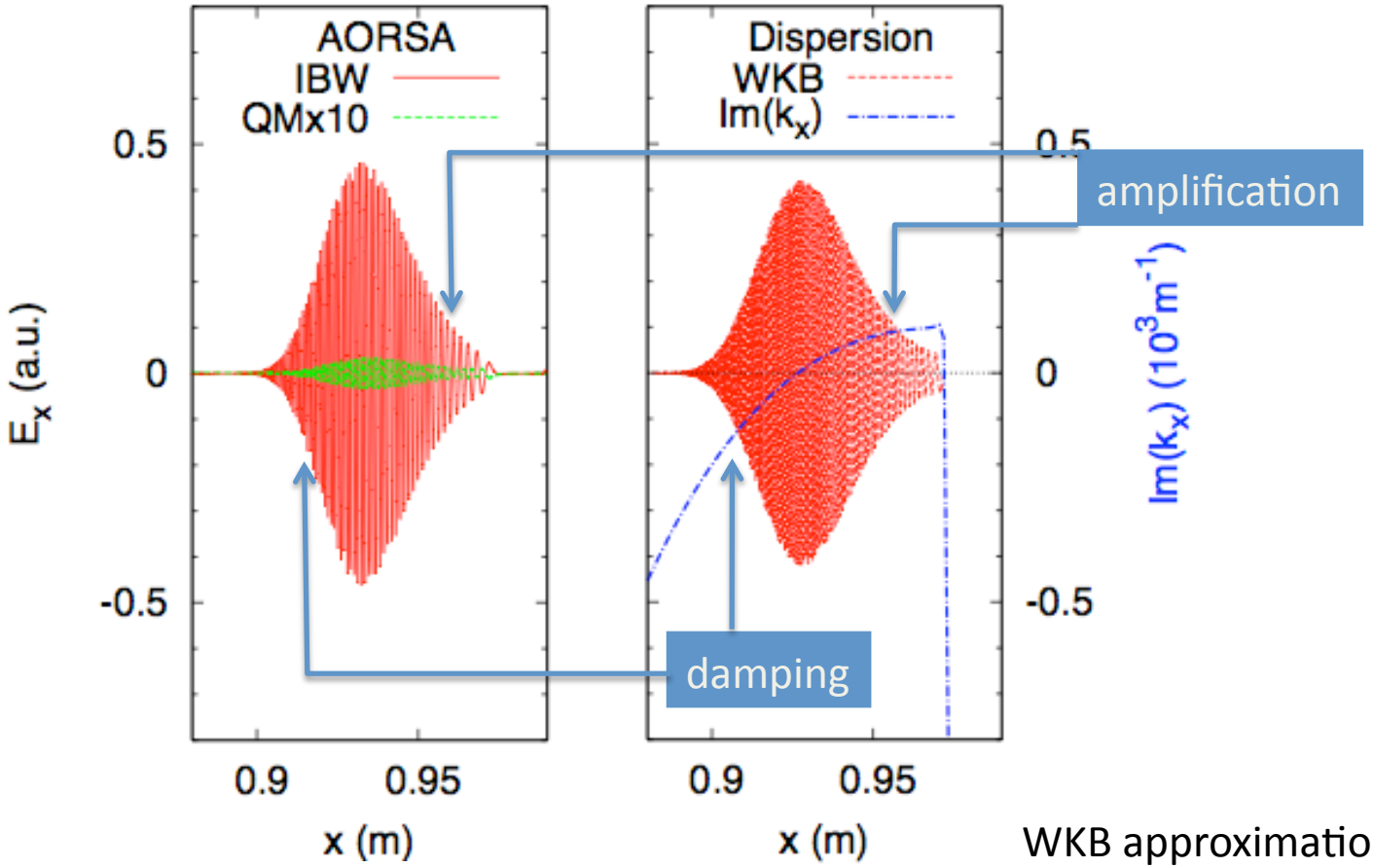
$$\mu = \frac{q}{m}\left[\left(\frac{\mathbf{E}_\perp\cdot\mathbf{k}_\perp}{\omega_0^2-\omega_c^2}+\frac{E_\parallel k_\parallel}{\omega_0^2}\right)^2+\frac{|\mathbf{E}_\perp\times\mathbf{k}_\perp|^2\omega_c^2}{(\omega_0^2-\omega_c^2)^2\omega_0^2}\right]^{1/2},$$

$\mathbf{E}_\perp = 9kV/m$ Growth rate $\mathbf{E}_\perp = 45kV/m$




MOST UNSTABLE MODE

# Diamond-shape-wave Excitation



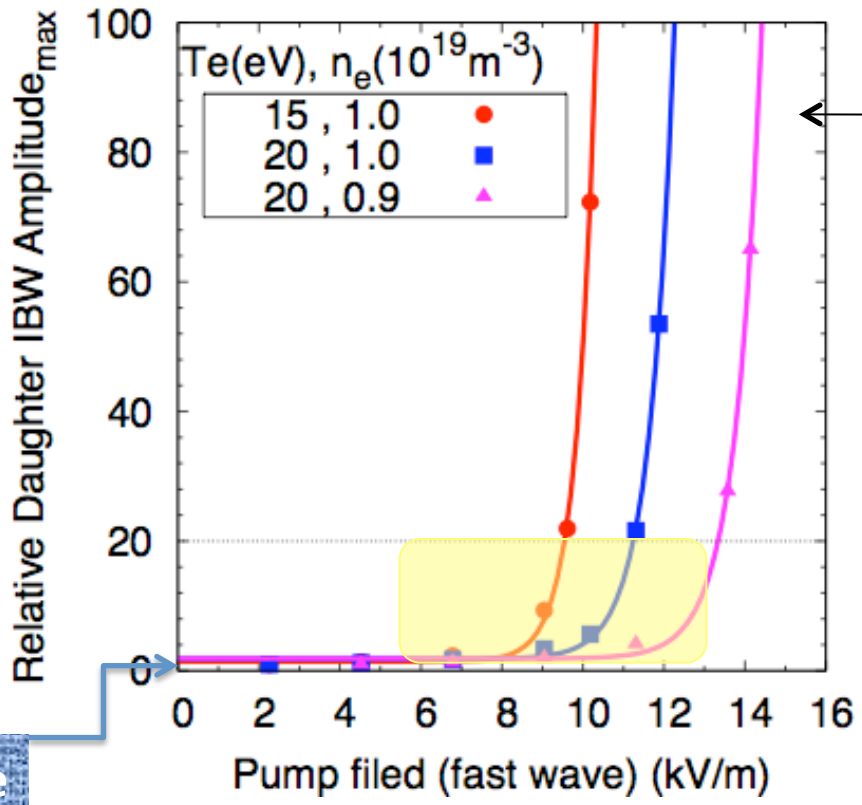

WKB approximation

$$g(x) = \frac{a\sin(\mathrm{Re}(k_x)x)}{\sqrt{\mathrm{Re}(k_x)}} e^{\int \mathrm{Im}(k_x)dx}$$

# PDI Threshold Consistent with Experiment

*CMOD Minority heating regime*

Fast wave ➔ IBW+QM



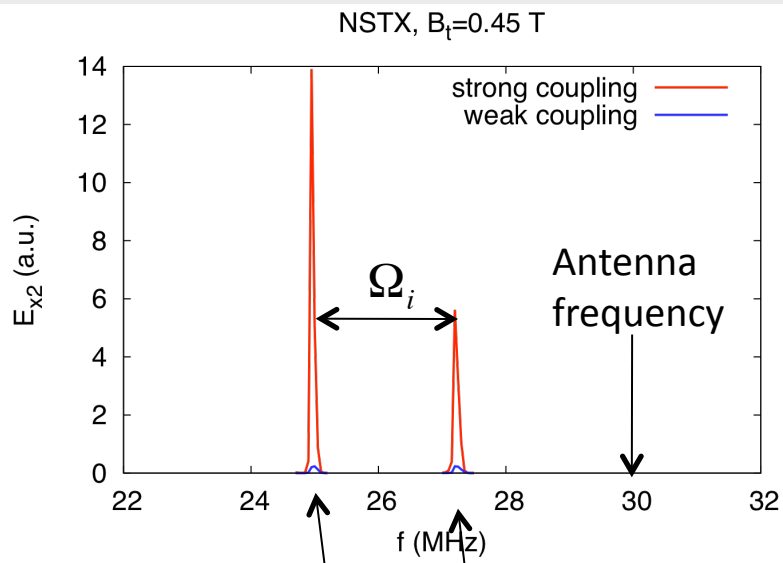

noise

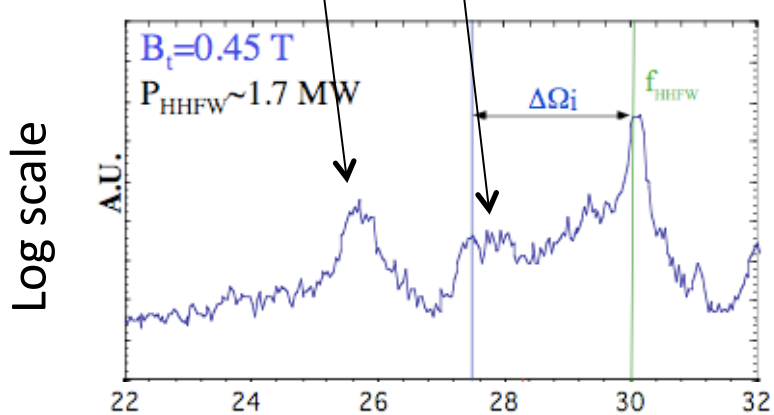
Exponential growth

✓ Linear 3D AORSA simulations predict typical fast wave edge amplitude = (5, 12) kV/m @0.5MW rf power.

✓ Scaling trends for the PDI threshold behavior with density, temperature are consistent with Ref. [Rost 2002].

# PDI Simulations for NSTX

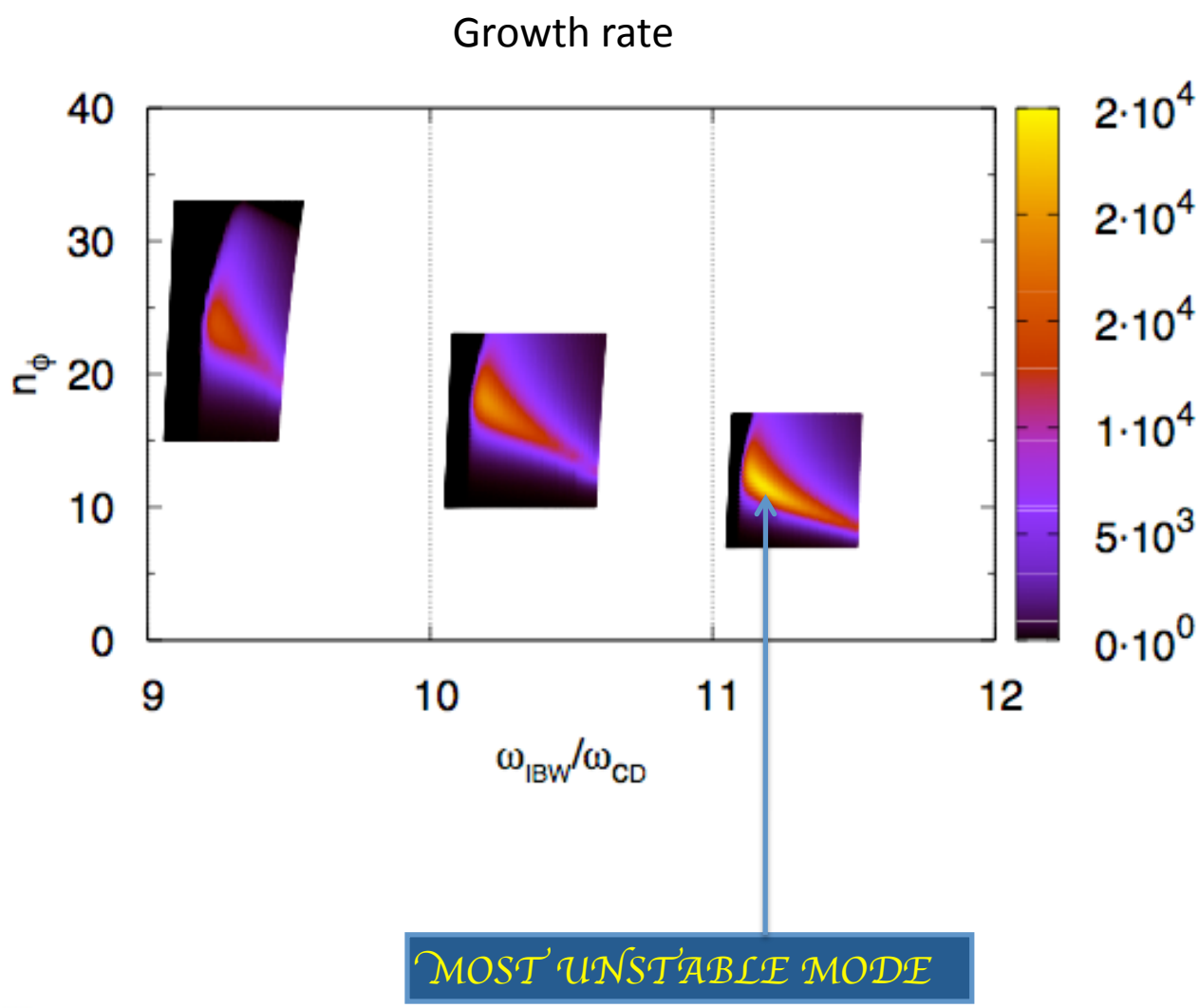


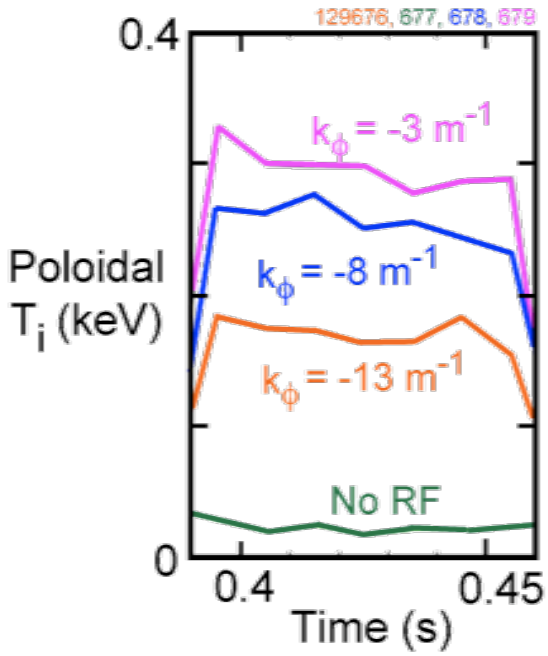

probe measurements [Diem,APS04]

- Daughter IBW amplitudes vs. frequency at a fixed point in the edge.
- Both weak and strong PDI are simulated.
- Peaks are separated by the cyclotron frequency at the probe ($\Omega_i$).
- Simulation parameters:
  - $T_e = T_i = 50$ eV,
  - $n_e = 2 \times 10^{18}$ m$^{-3}$,
  - $B_\phi = 0.45$ T
  - $k_y = 7000$ m$^{-1}$, $n_\phi = 15$

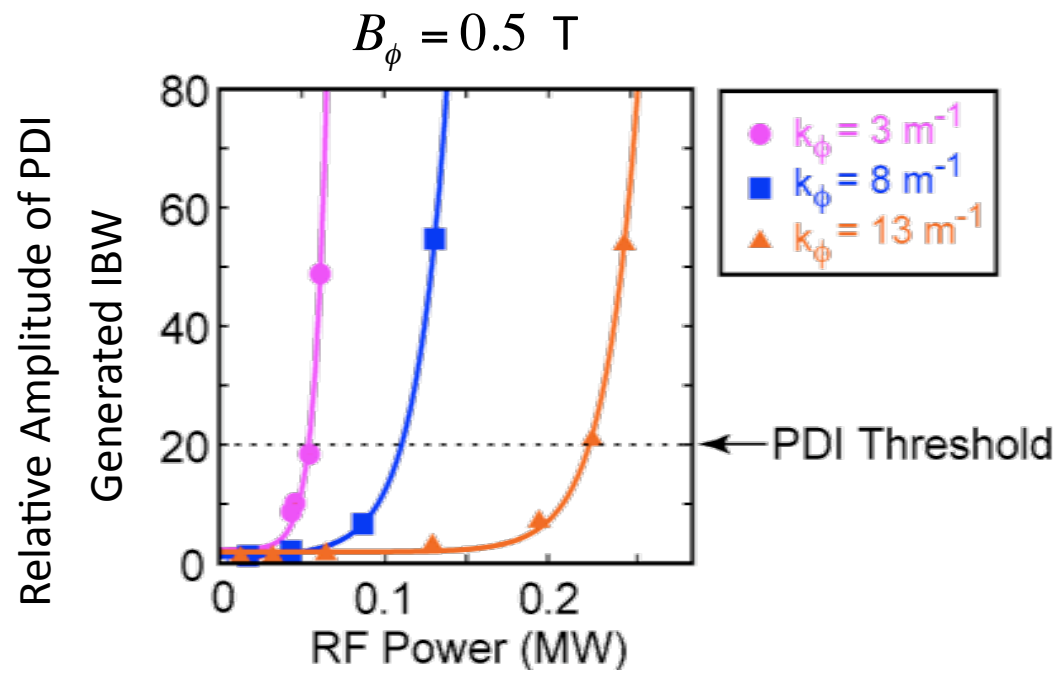
# PDI Dispersion Relation



# PDI $k_\phi$ Dependence Consistent with Edge Ion Heating Measurements




$P_{RF}$ = 1.2-1.3 MW




- ✓ Predicted power threshold is consistent with previous reflectometer measurements (100 to 300kW), (Wilgen 2005).

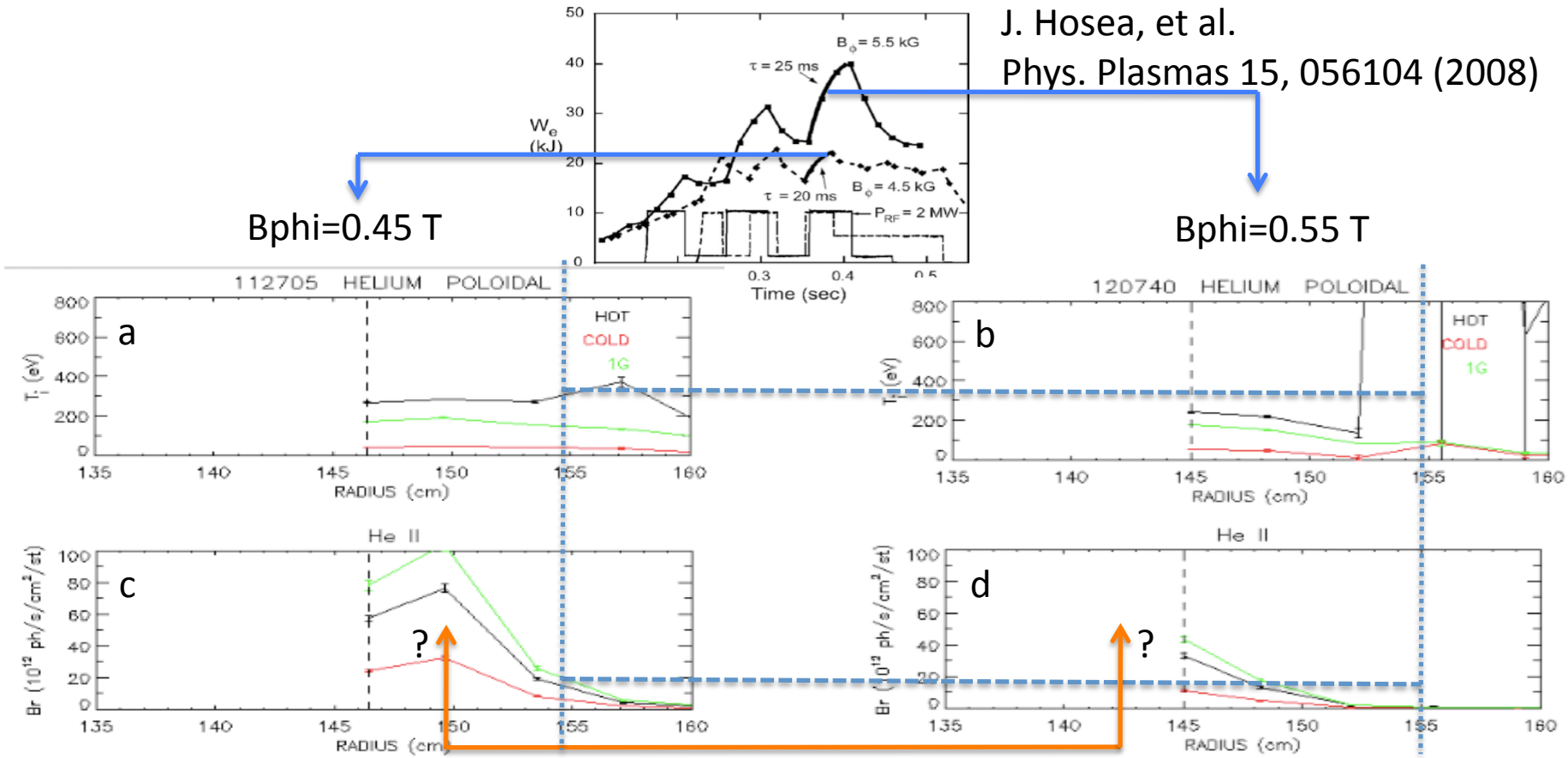
# Summary

- Fast wave->IBW+QM is successfully excited using the extended AORSA code.
- The most unstable modes are found to be the waves excited just above the cyclotron harmonic.
- PDI simulations are consistent with the threshold reported in minority heating experiments on Alcator C-MOD.
- PDI simulations show the threshold decreases with $k_\phi$ , consistent with the edge ion heating measurements.
- Predicted PDI threshold in NSTX is about 100 KW, consistent with previous reflectometer measurements.

# Future Work

## **Core heating** vs. **edge ion heating in NSTX**

J. Hosea, et al.
Phys. Plasmas 15, 056104 (2008)



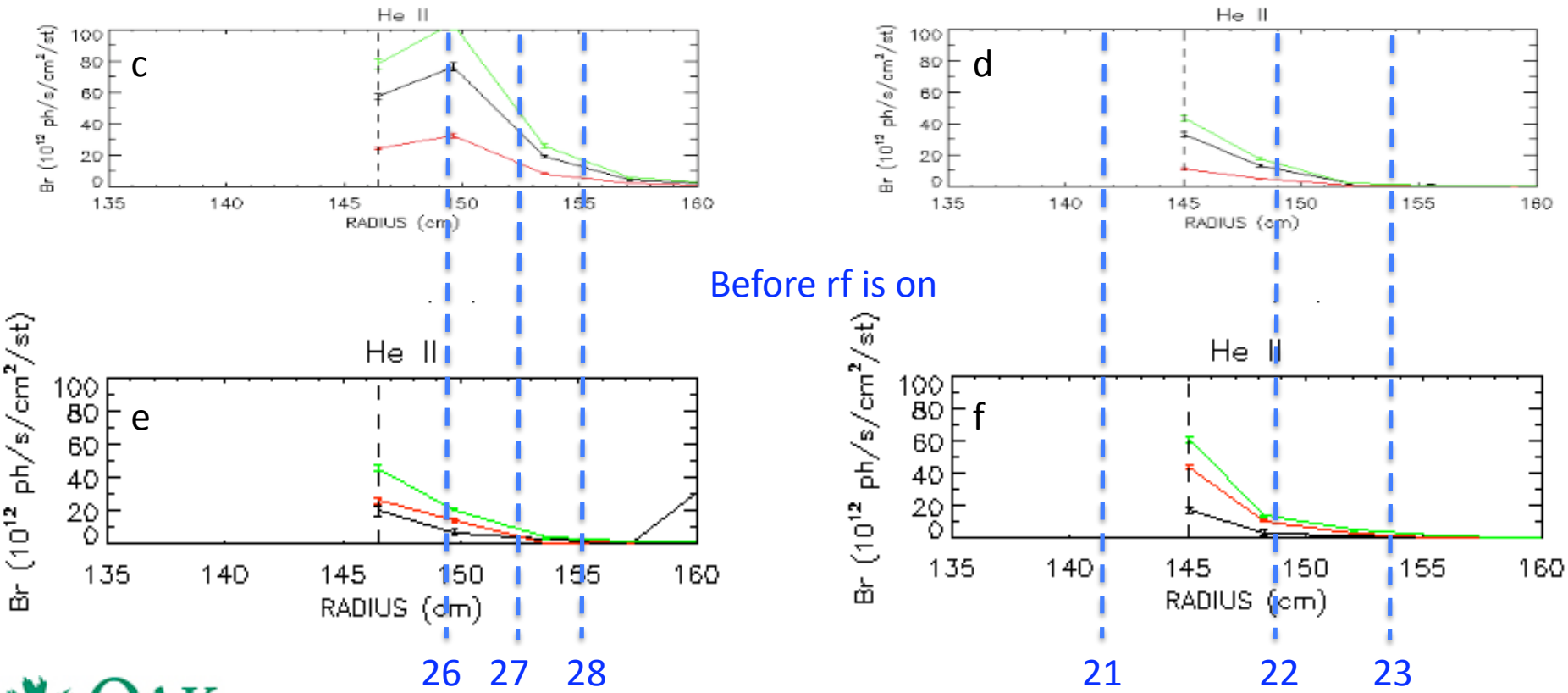
# Future Work

## **Core heating** vs. **edge ion heating in NSTX**

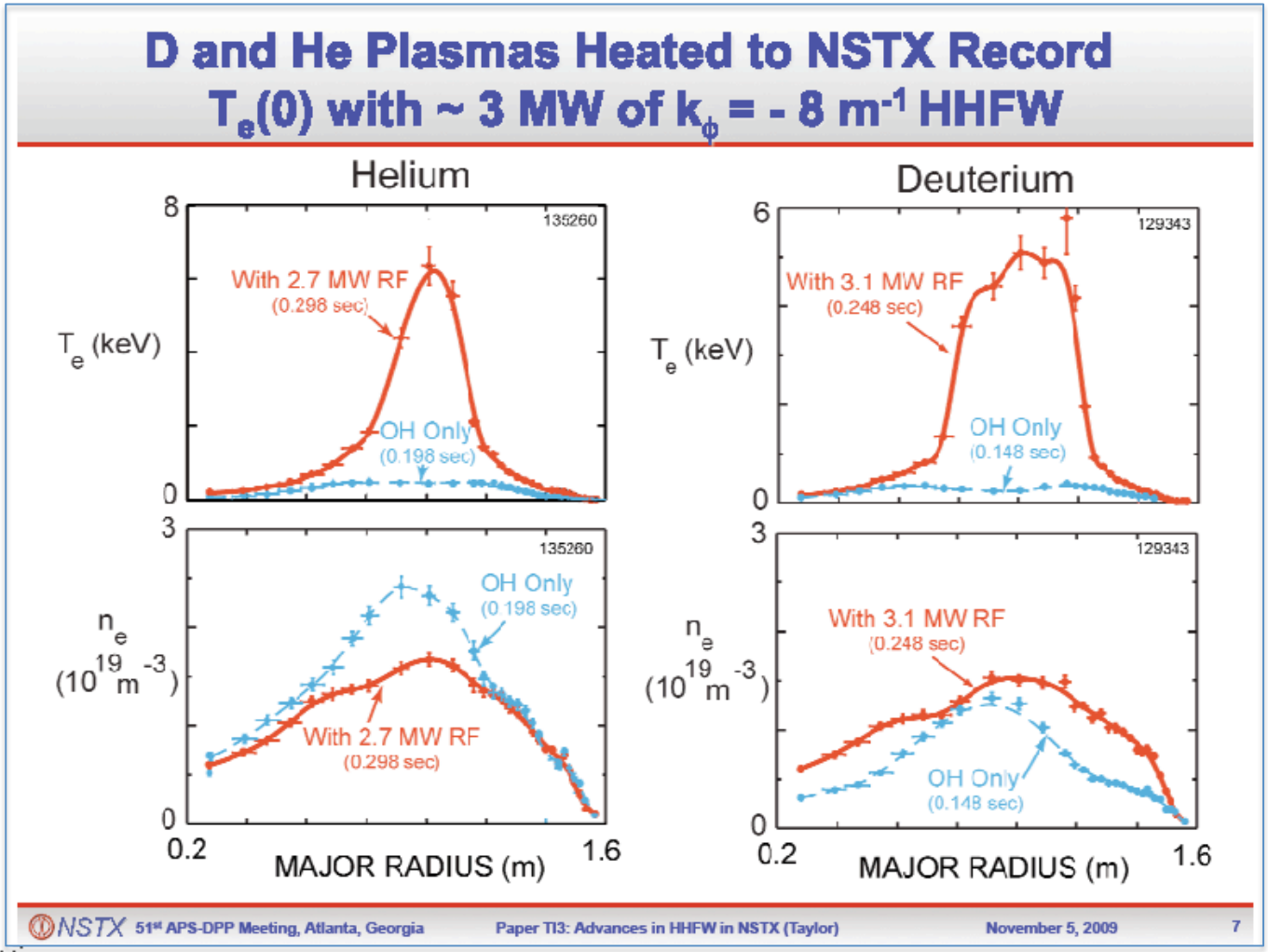
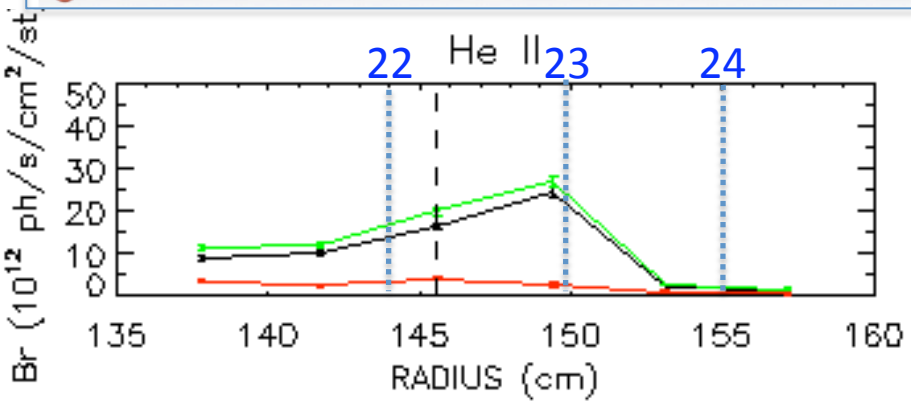
Bphi=0.45 T

After rf is on

Bphi=0.55 T




Before rf is on

# Another Example

G. Taylor, APS-DPP 2009