Progress in Equilibrium and Stability Modeling With M3D-C1

by

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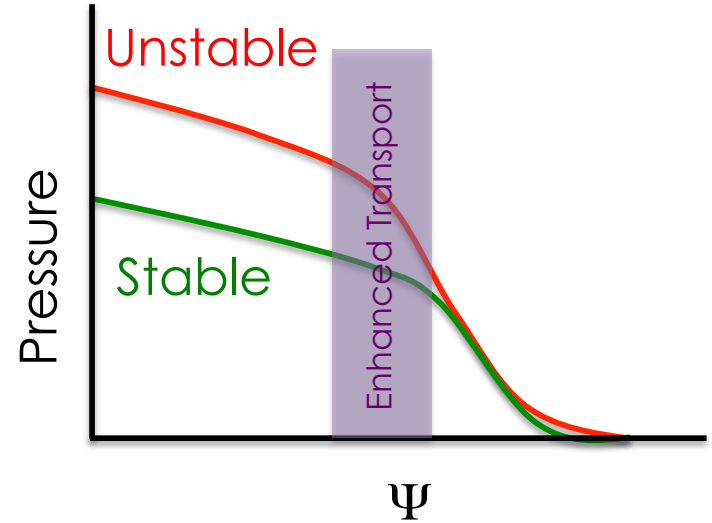
Goals for M3D-C1 Focus on Validation and Predictive Modeling

- **We lack validated, predictive models for salient effects of 3D perturbations**
  - Enhanced particle transport ("pump-out")
  - Torque and locking
  - ELM suppression



- **Predictive models for these effects require understanding both**
  - 3D equilibria (kinking / tearing response; $n \neq 0$)
  - Transport in 3D fields (changes to $n = 0$ profiles)

- **Lots of recent progress:**
  - Validation of 3D magnetic response models
  - Application of transport models in 3D tokamak equilibria
  - Development of capabilities for disruption modeling

Outline

- **Overview of M3D-C1**
- **Validation of 3D Equilibrium Calculations**
  - 2014 Joint Research Target on 3D Response Model Validation
- **Calculating transport in 3D equilibria**
- **Disruption modeling with M3D-C1**
  - Resistive wall model in M3D-C1
  - Halo, Hiro, and inductive wall currents during VDEs

Overview of M3D-C1

Full, Compressible, Two-Fluid Model is Implemented

$$\frac{\partial n}{\partial t} + \nabla \cdot \left(n_i \mathbf{v} \right) = 0$$

$$n_i m_i \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi_i$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \Gamma p \nabla \cdot \mathbf{v} = -\frac{1}{n_e e} \mathbf{J} \cdot \left(\Gamma p_e \frac{\nabla n_e}{n_e} - \nabla p_e \right) - \left(\Gamma - 1 \right) \nabla \cdot \mathbf{q}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{n_e e} \left(\mathbf{J} \times \mathbf{B} - \nabla p_e \right)$$

$$\Pi_i = -\mu \left[\nabla \mathbf{v} + \left(\nabla \mathbf{v} \right)^T \right] + \Pi_i^{gv} + \Pi_i^{\parallel}$$

$$\mathbf{q} = -\kappa \nabla T_i - \kappa_{\parallel} \mathbf{bb} \cdot \nabla T_e$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\Gamma = 5/3$$

$$n_e = Z_i n_i$$

- **Ion viscosity model optionally includes Braginskii gyroviscosity, parallel viscosity (poloidal flow damping)**
- **Open field line region of "plasma" region is treated as low-temperature, low-density plasma**

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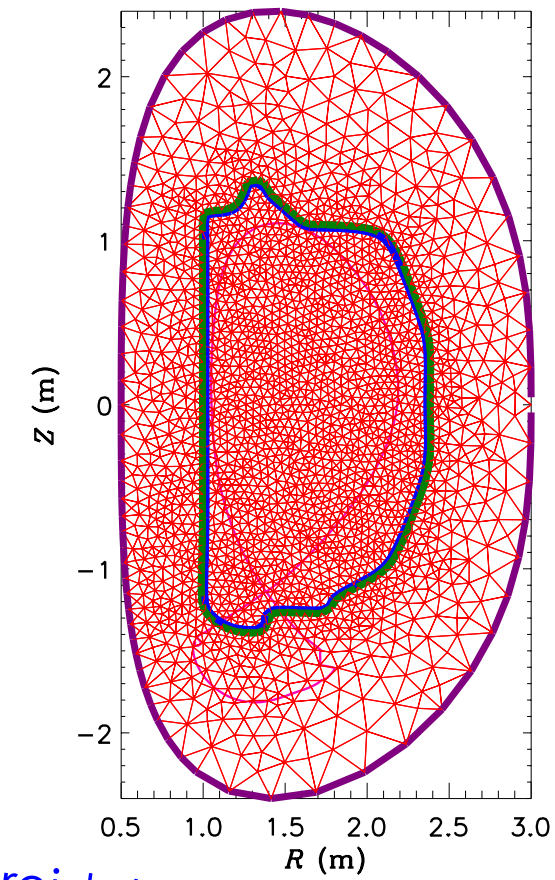
M3D-C1 Solves Linear and Nonlinear Two-Fluid MHD

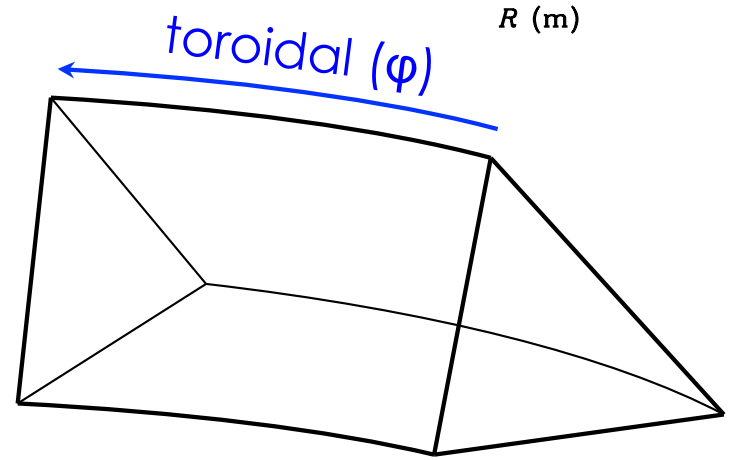
- **Linear calculations:**
  - 2-Fluid equations are linearized about axisymmetric equilibrium
  - Solution has $\exp[-in\varphi]$ dependence (n's are independent) → "2D"
  - <u>Stability:</u>
    - time-dependent
    - initial value with random initial conditions
    - homogeneous system (amplitude is arbitrary)
  - <u>Perturbed equilibrium:</u>
    - time-independent (usually)
    - boundary value problem, with bc's set by external applied field
    - inhomogeneous system (amplitude set by boundary conditions)

- **Nonlinear calculations:**
  - Time-dependent, initial value
  - Can run in "delta-f" mode or "full-f" mode (full-f includes sources)

M3D-C1 Uses High-Order Elements on an Unstructured Mesh

- **The poloidal plane is discretized using triangular, C^1, degree-5 polynomial elements**
- **Linear calculations: a single toroidal Fourier mode is considered**
- **Nonlinear calculations: toroidal direction is discretized using cubic Hermite elements**
  - Preserves local coupling (block-tridiagonal)
  - Preserves C^1 property in all directions
  - Allows non-uniform toroidal resolution
- **(R, φ, Z) coordinates**





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3D Response Validation

2014 Joint Research Target: Improve Predictive Understanding of the Role That 3D Fields Play in Tokamak Plasmas

Annual target:
Conduct experiments and analysis to investigate and quantify plasma response to non-axisymmetric (3D) magnetic fields in tokamaks.
Effects of 3D fields can be both beneficial and detrimental, and research will aim to validate theoretical models in order to predict plasma performance with varying levels and types of externally imposed 3D fields. Dependence of response to multiple plasma parameters will be explored in order to gain confidence in predictive capability of the models.

- **JRT representatives:**
  - DIII-D: E. Strait (chair), N. Ferraro
  - C-Mod: E. Marmar, J. Rice
  - NSTX: J.-K. Park, J. Canik

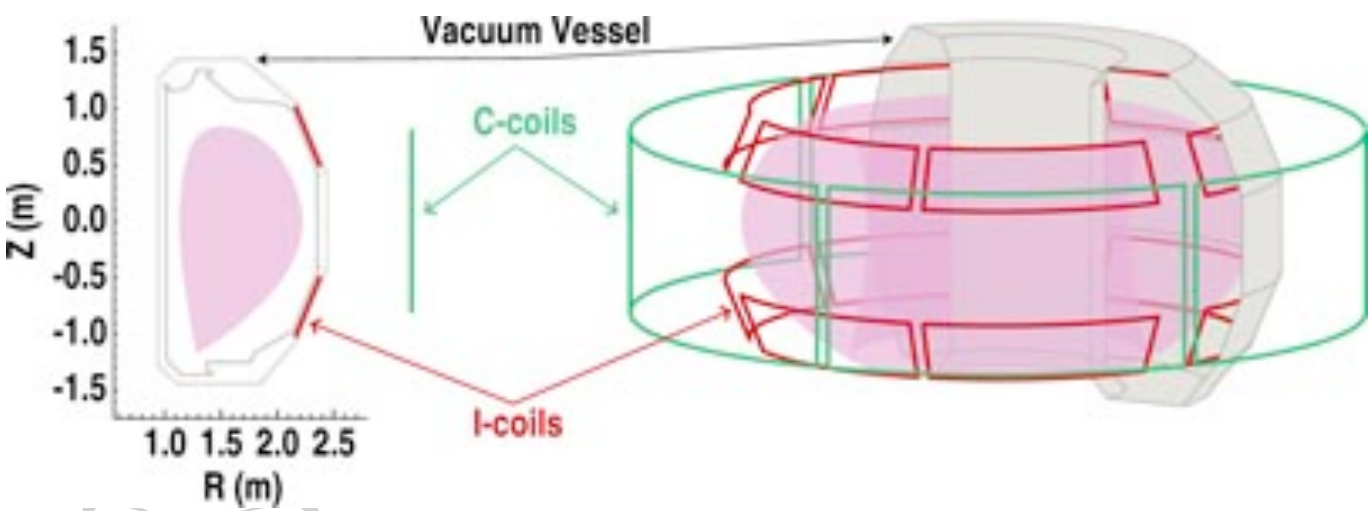
DIII-D Conducted Four Experiments for 2014 JRT

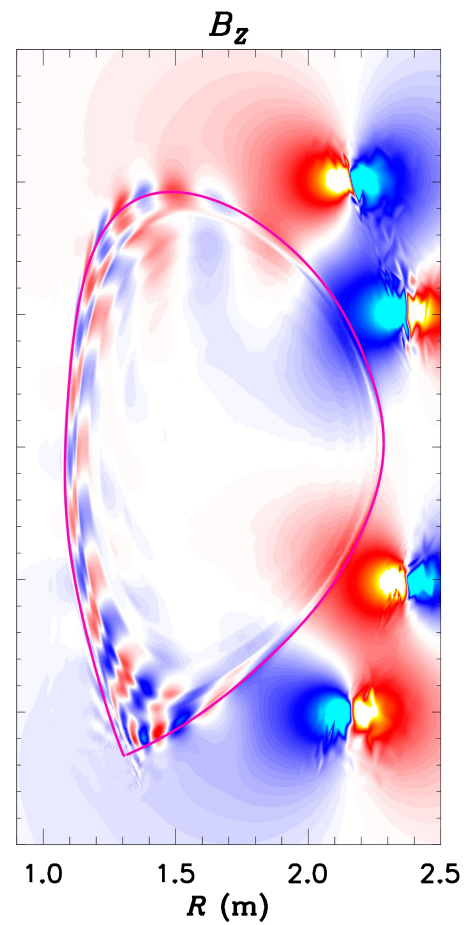
1. **Plasma for 3D code validation**

2. **Plasma Response in RMP ELM-suppression regime**

3. **Error field correction with *n* > 2**

4. **Plasma response to TBM-like perturbation**





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Experiment #1: Plasma Response Data for Model Validation

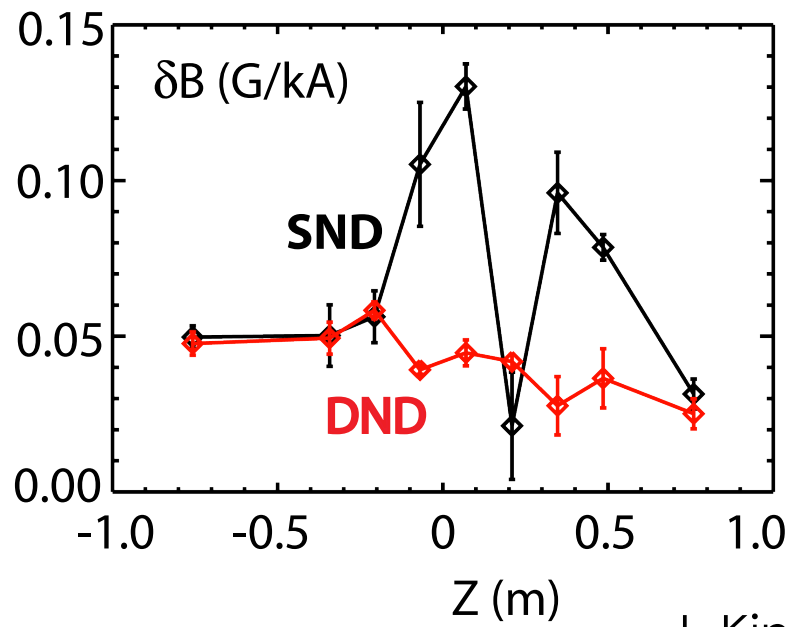
- **Goal: experimental validation of 3D equilibrium and stability codes (VMEC, IPEC, MARS, M3D-C1, and others)**

- **Data on *n* = 3 response obtained in symmetric double-null discharges (DND)**
  - Symmetry allows benchmarking of stellarator codes
  - Plasma parameters (q_{95}, β) scanned to test response models

- **Clear 3D response data provides a strong test of numerical models**

- **<u>Unexpected result</u>: High field side response is significantly smaller than in single-null discharges (SND)**

High field side response, n=3



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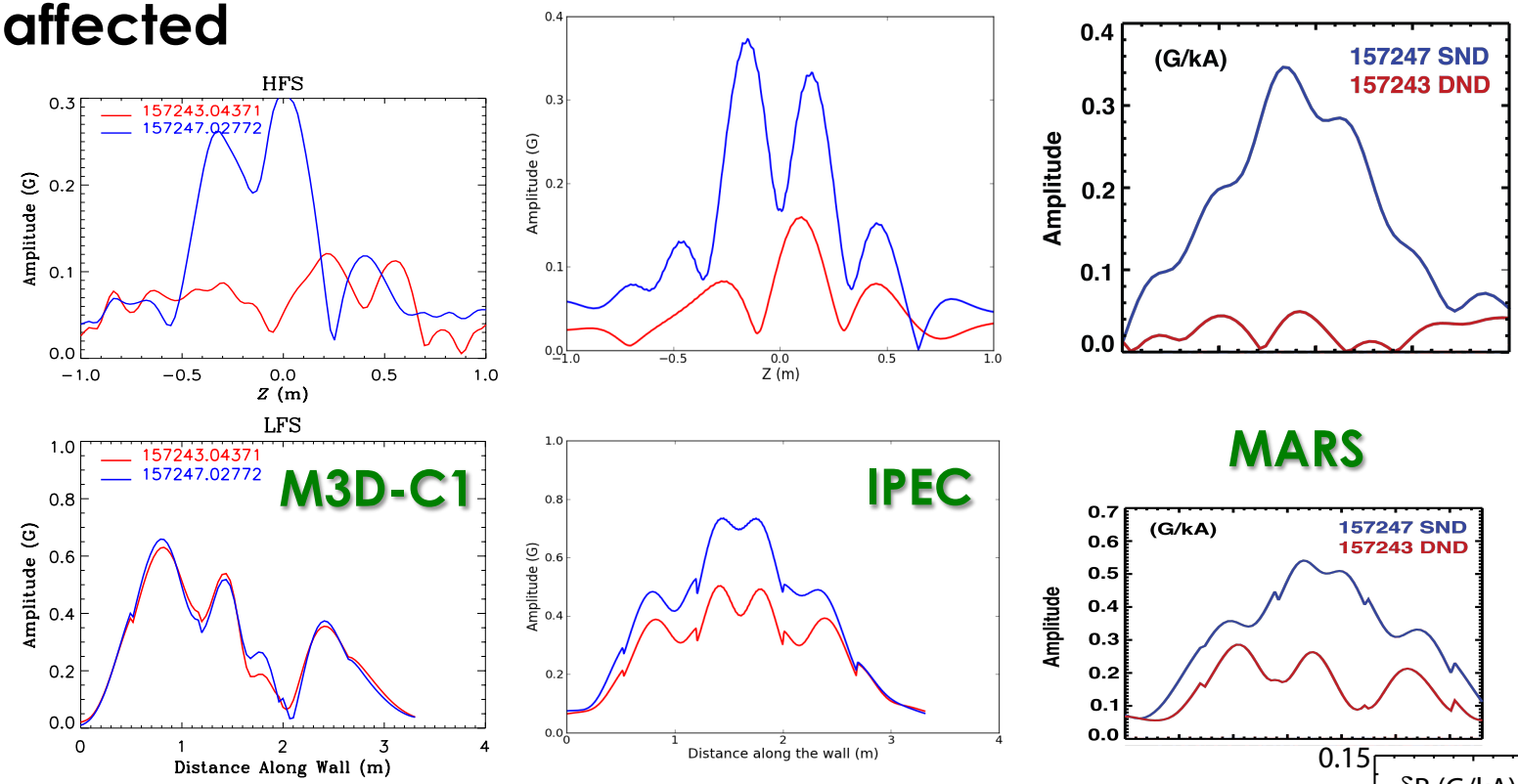
Linear, Ideal Modeling Reproduces Dependence of Response on Geometry; Consistent with P-B Mode

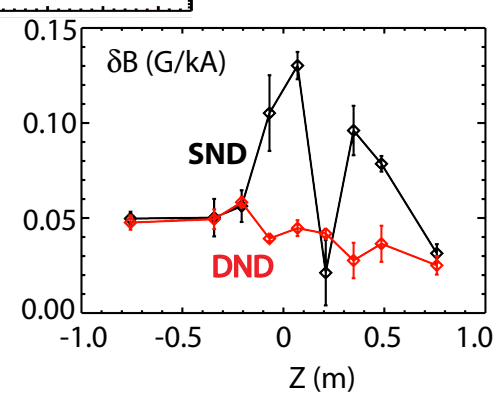
- In double-null geometry, HFS response is significantly reduced relative to single-null; LFS response less affected

High Field Side

Low Field Side



- Quantitative validation is awaiting experimental analysis



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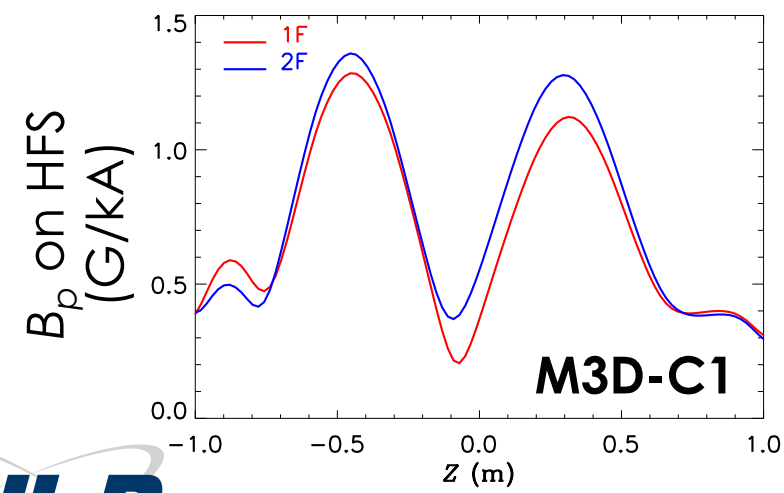
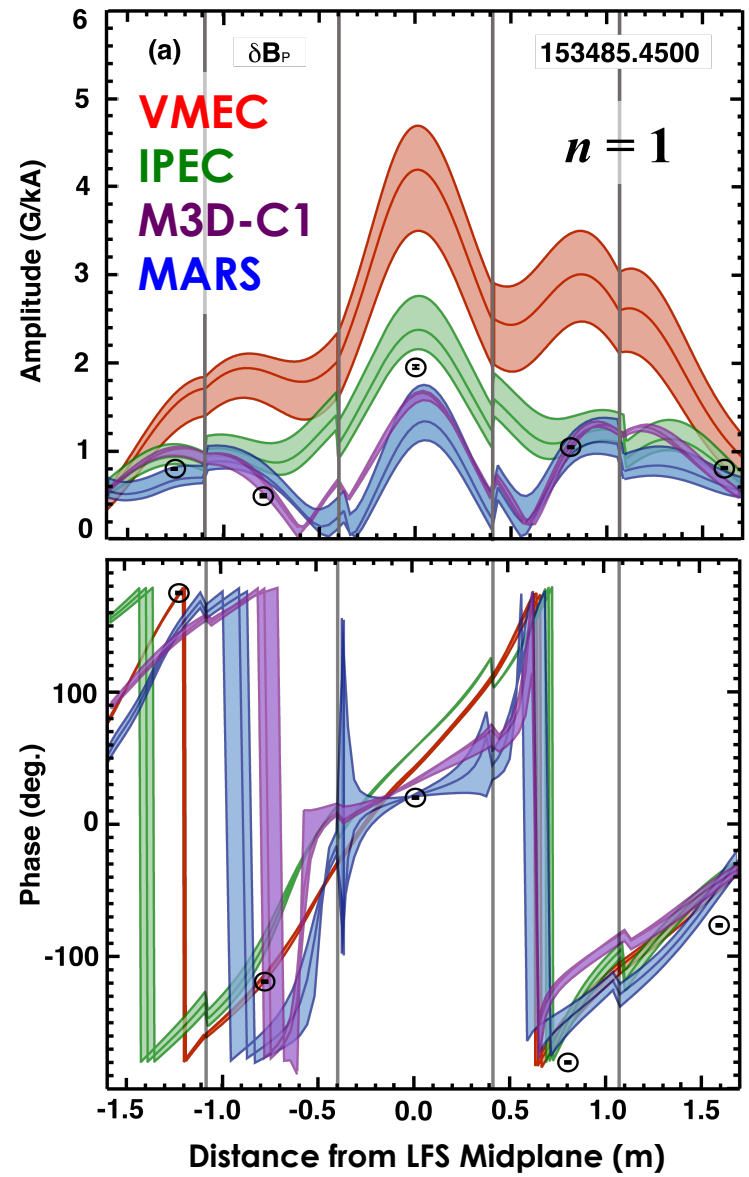
Linear Calculations Show Encouraging Agreement With Experimental Data

- **Different codes show different sensitivities to bootstrap current**
  - M3D-C1 seems least sensitive, probably because there is no q_{edge}
- **VMEC (run by S. Lazerson) is outlier**
- ***Uncertainties due to reconstruction and numerical methods are larger than effects of non-ideal physics in this case ($\beta \sim \beta^{no\text{-}wall} / 2$)***

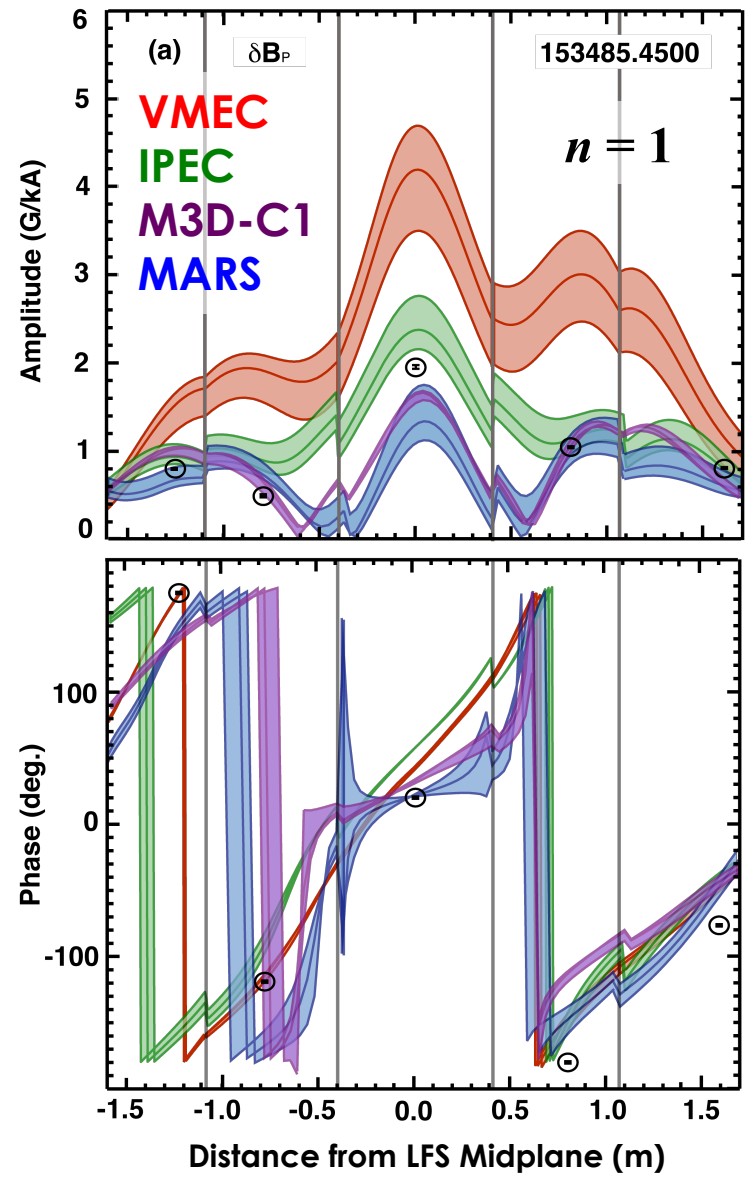




Courtesy J. King

Discrepancies Among Codes are Not Entirely Understood

- **MARS and IPEC are using the same model (linear ideal MHD)**
  - Potential differences in edge profiles / cutoff
- **VMEC imposes onerous constraints**
  - No stochasticity is allowed near rational surfaces or separatrix (this is not imposed by linear ideal-MHD!)
- **VMEC shows slow convergence in radial resolution in test case (S. Lazerson)**
  - Is this understood?



Courtesy J. King

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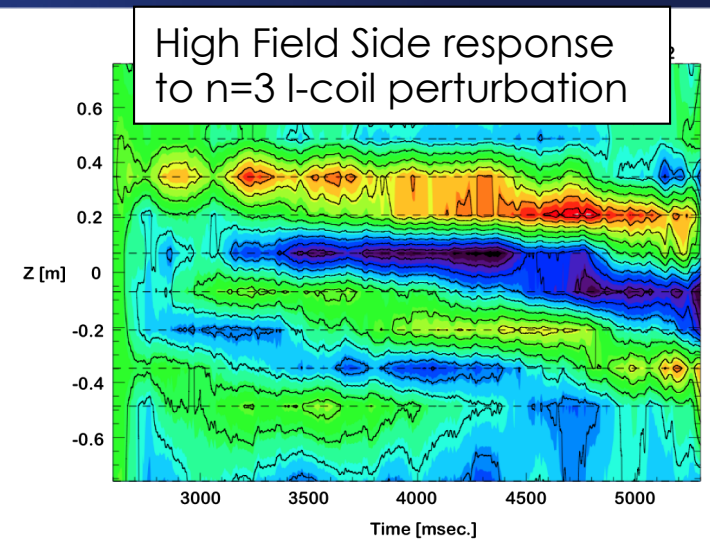
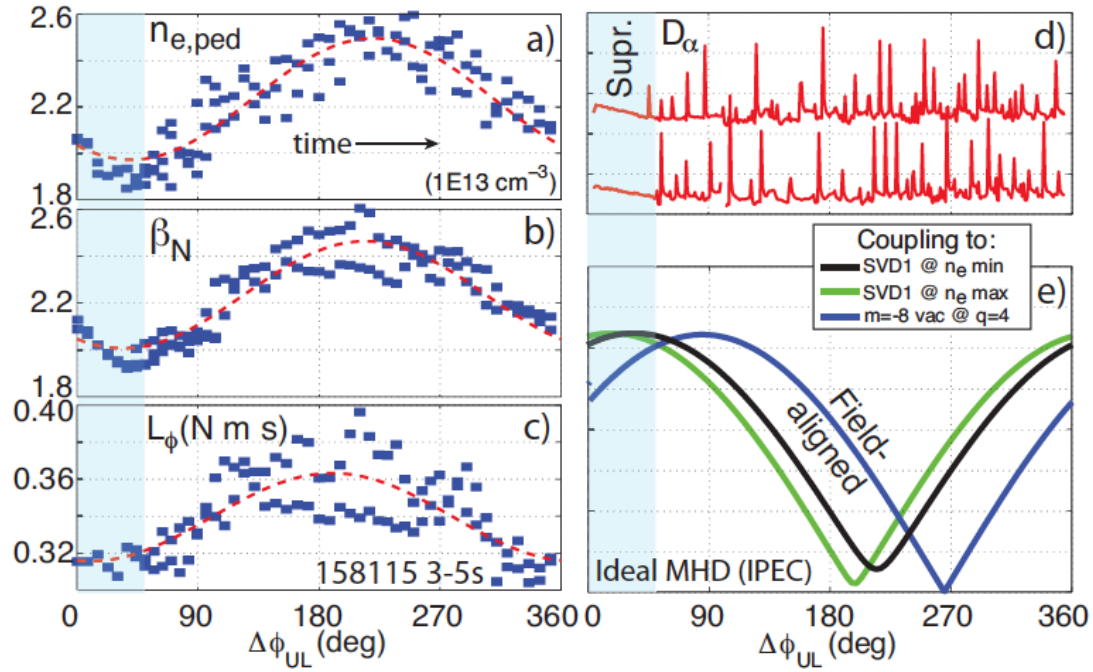
Experiment #2: Plasma Response in RMP ELM Suppression Regime

- **Goal is to observe magnetic / transport response during ELM suppression**
- **Experiments find consistent trend on HFS with q_{95}**
  - Codes reproduce this trend
- **Nonlinear transition to / from ELM suppression is not captured**
  - Appears to be correlated with quasilinear torque





R. Nazikian, C. Paz-Soldan

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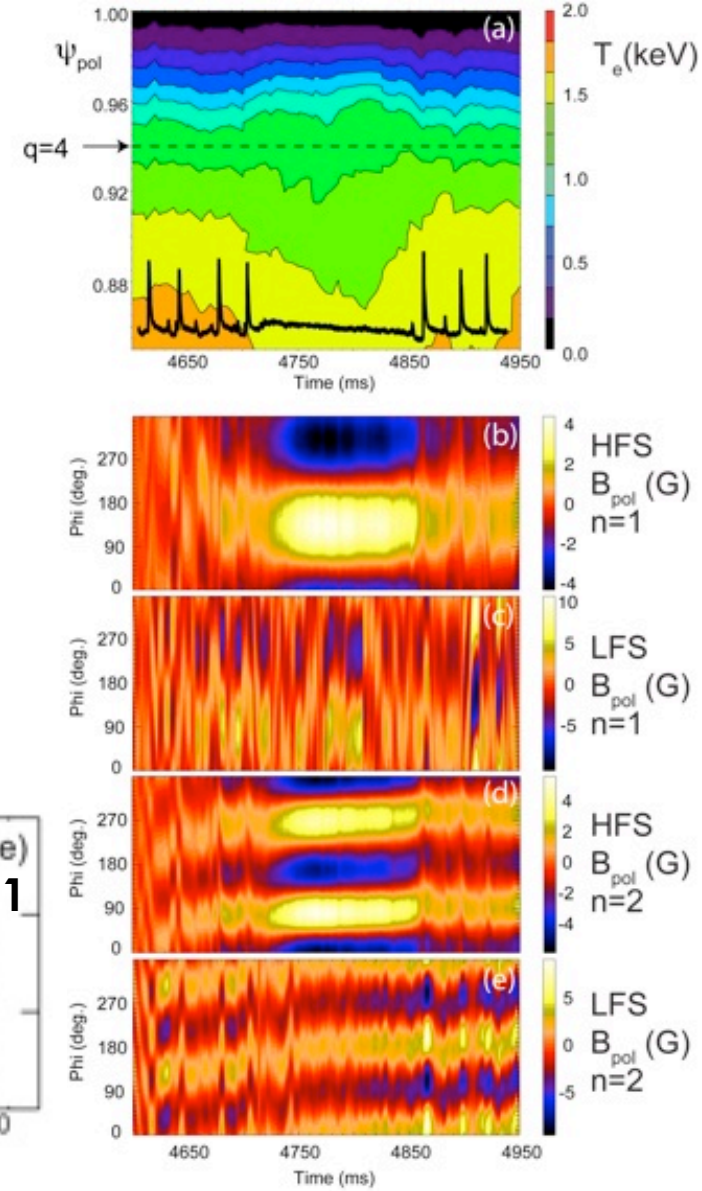
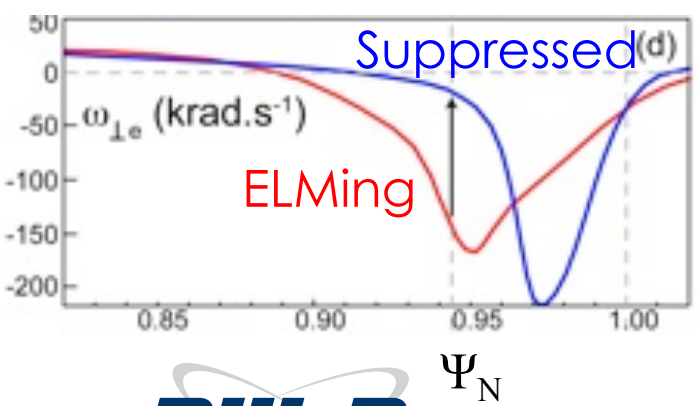
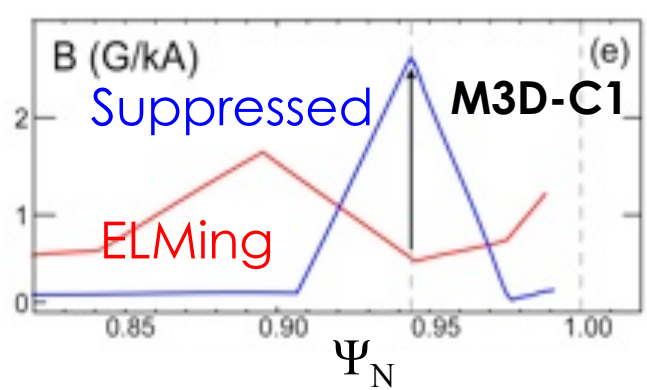
Clear Magnetic Signatures Are Found Upon Entering / Leaving ELM Suppression

- **Plasma enters and exits ELM suppression as *n* = 2 poloidal spectrum is changed**
- **Flattening of T_e seen at pedestal top**
- **Strong *n* = 1 and *n* = 2 signal seen on HFS**
- **Looks a lot like island penetration…**
  - M3D-C1 finds strong resonant field (tearing) drive at top of pedestal when suppressed







R. Nazikian

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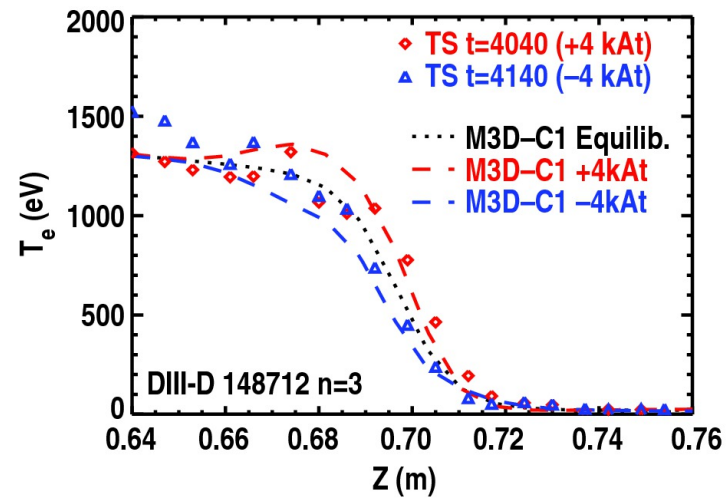
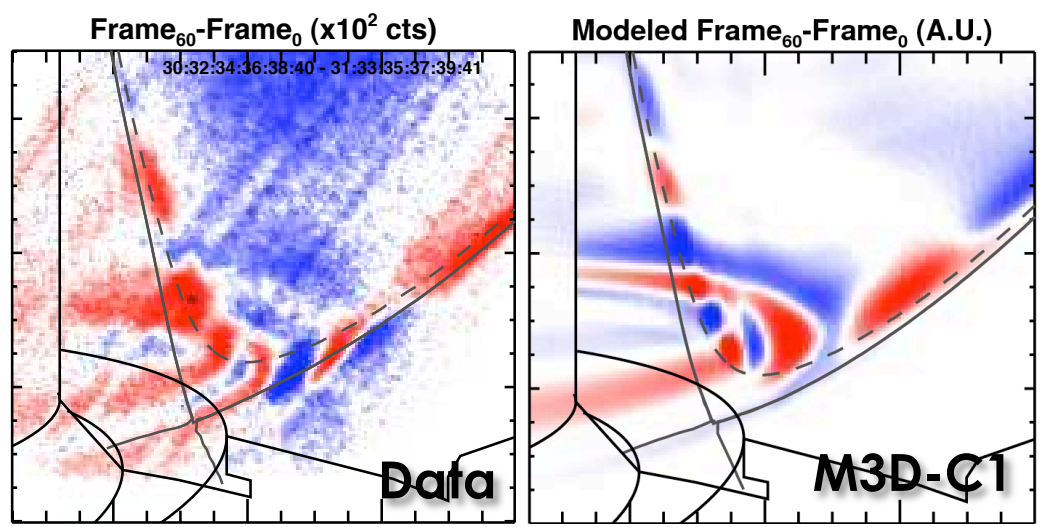
M3D-C1 Calculations of Helical Displacements are Consistent with Thomson and SXR data

- **Non-axisymmetric response is observed by flipping or rotating phase of applied fields**
- **Thomson shows profile shift when phase is changed**
  - Consistent with 3D modeling, but could be due to phase-dependent $n = 0$ profile changes
- **SXR imaging shows poloidal structure**
  - Clearly shows field-aligned helical perturbation
  - Radial structure is very different from vacuum modeling





Shafer, Unterberg

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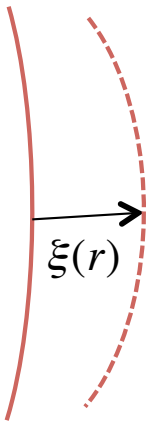
Linearity Can Break Down in Pedestal and Near Rational Surfaces

- **"Displacement" may be defined by movement of isotherms**



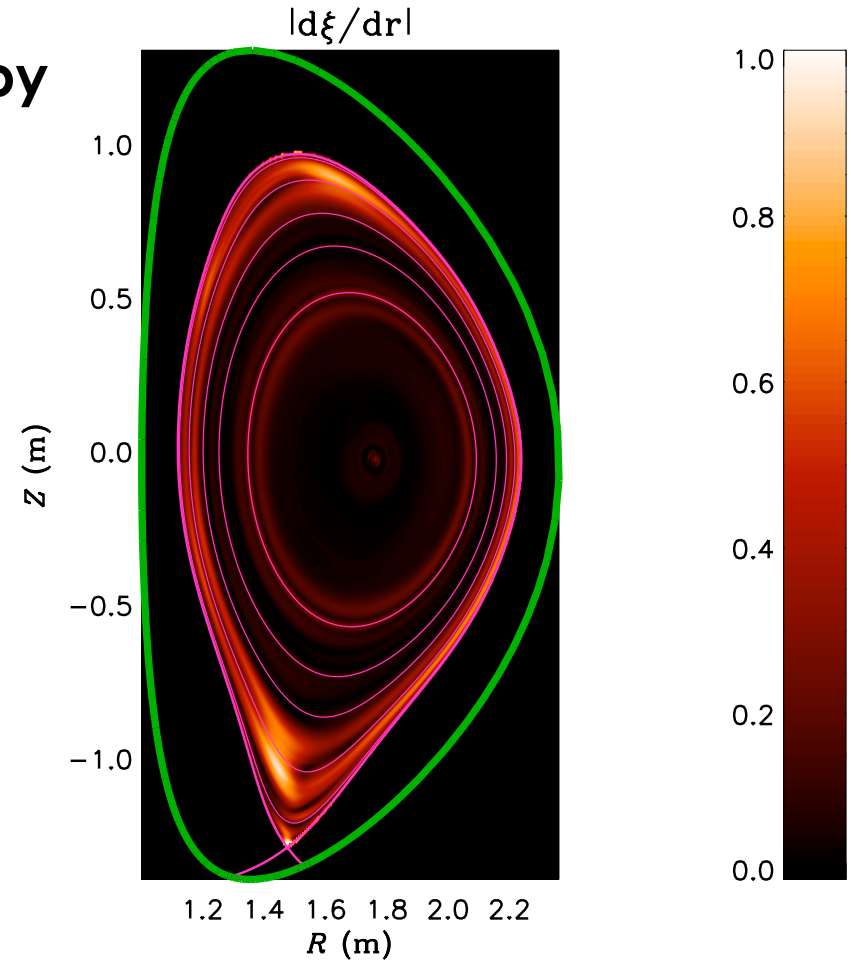
$$T_0(r+\xi)+\delta T(r+\xi)=T_0(r)$$

$$\left[T_0(r)+\frac{dT_0}{dr}\xi\right]+\delta T(r)=T_0(r)$$

$$\xi=-\frac{\delta T}{dT_0/dr}$$

- **Overlap of adjacent surfaces is possible, especially near mode-rational surfaces, edge, & x-point**

Overlap criterion: $\left|\frac{d\xi}{dr}\right|>1$



DIII-D 117327 $n=1$

Integration of 3D Magnetic Response and 3D Transport Models

Many Transport Codes Can Now Use M3D-C1 Output

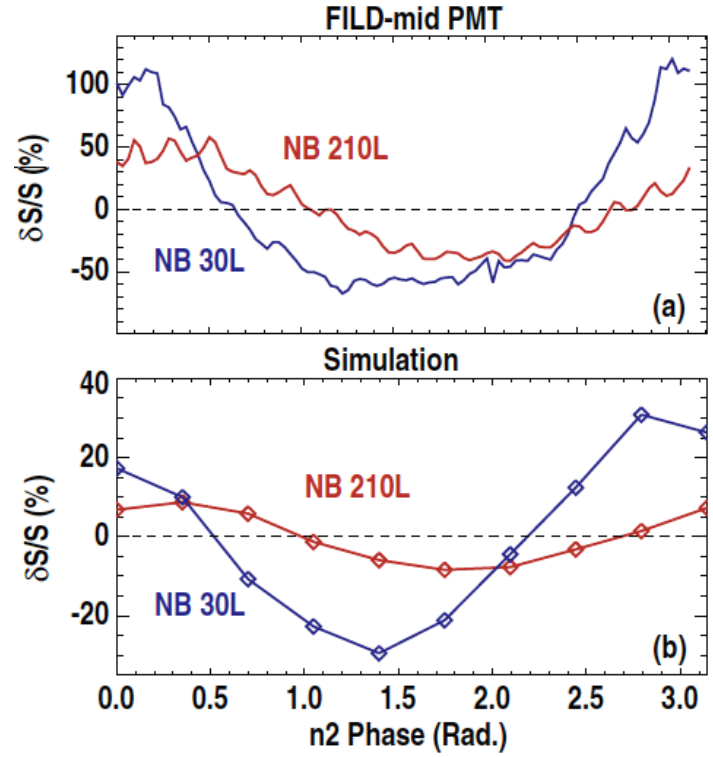
- **A software library serves as interface between transport codes and M3D-C1 output**
  - Transport code links to library, calls function to obtain $\mathbf{B}$ at any set of (R, φ, Z) coordinates
- **Software library is better solution than common data format**
  - No loss of information in interpolation
  - Minimal new coding in transport code
    - Doesn't need to know how to read or interpolated data
- **Many codes are now using this library to use M3D-C1 data**
  - SPIRAL, TRIP3D/SURFMN, MAFOT, EMC3-EIRENE, OMFIT, NewCcoords, MVZBeam, SXR synthetic diagnostic
- **Several other codes can use more specialized M3D-C1 output**
  - NEO3D, NTVTOK, RMPTran

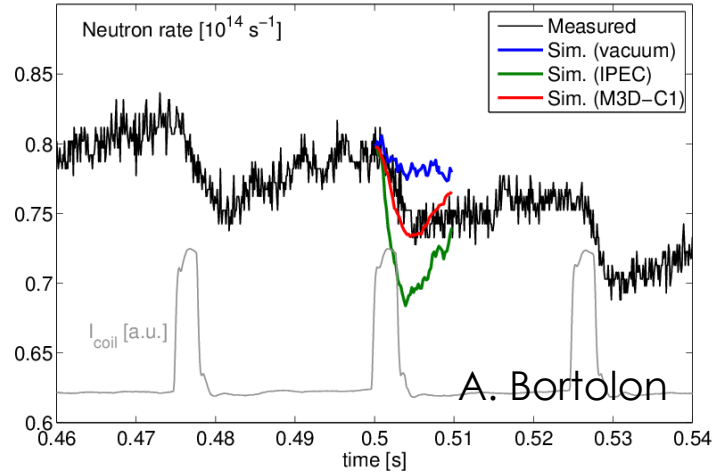
Effect of 3D Fields on Fast Ion Birth and Loss is Captured by Modeling

Van Zeeland, *et al*. *PPCF* **56** (2014) 015009

- **Fast ion loss is affected by 3D geometry**
  - Transport is enhanced by loss of symmetry
  - Beam ion birth is affected by changes to *n*, *T* profiles (both 2D and 3D)

- **SPIRAL with 3D fields from M3D-C1 reproduced empirical trends on DIII-D (Van Zeeland), NSTX (Bortolon), and ASDEX-U (Garcia-Muñoz)**





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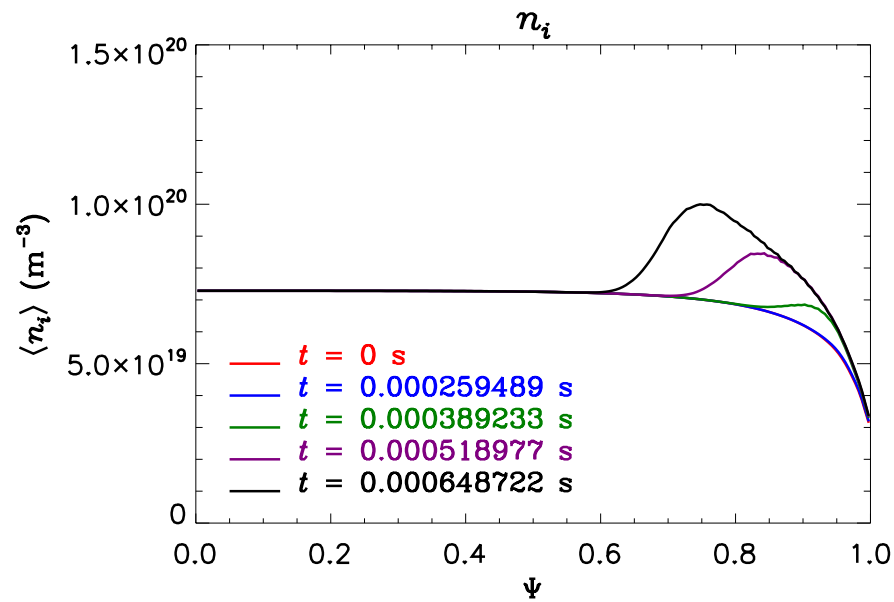
Pellet Modeling with M3D-C1

- **M3D-C1 has a crude fuelling-pellet model**
  - Pellet moves at constant velocity; ablation is a function of local n, T
  - Model can include local cooling from ablation, but only one ion species is modeled in M3D-C1 (thermal ions)



- **This model could be used to explore ELM pacing**
  - Easy way: axisymmetric nonlinear simulations with moving pellet source, checking periodically for peeling-ballooning stability
  - Hard way: non-axisymmetric nonlinear simulations with moving toroidally localized pellet source

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Summary

- **The 2014 JRT and other recent validation initiatives have pushed modeling capabilities and left us with a clearer understanding of where our models succeed and fail**

- **Linear, ideal-MHD models successfully describe low-beta 3D equilibria, away from locking thresholds**
  - Good progress is being made with high-beta equilibria (MARS-F)

- **New, clear experimental evidence that ELM suppression involves a nonlinear bifurcation, as with locking**
  - Quantitative predictive modeling will require coupled 3D / response transport modeling

- **Integrated 3D Response / Transport modeling is the focus moving forward**

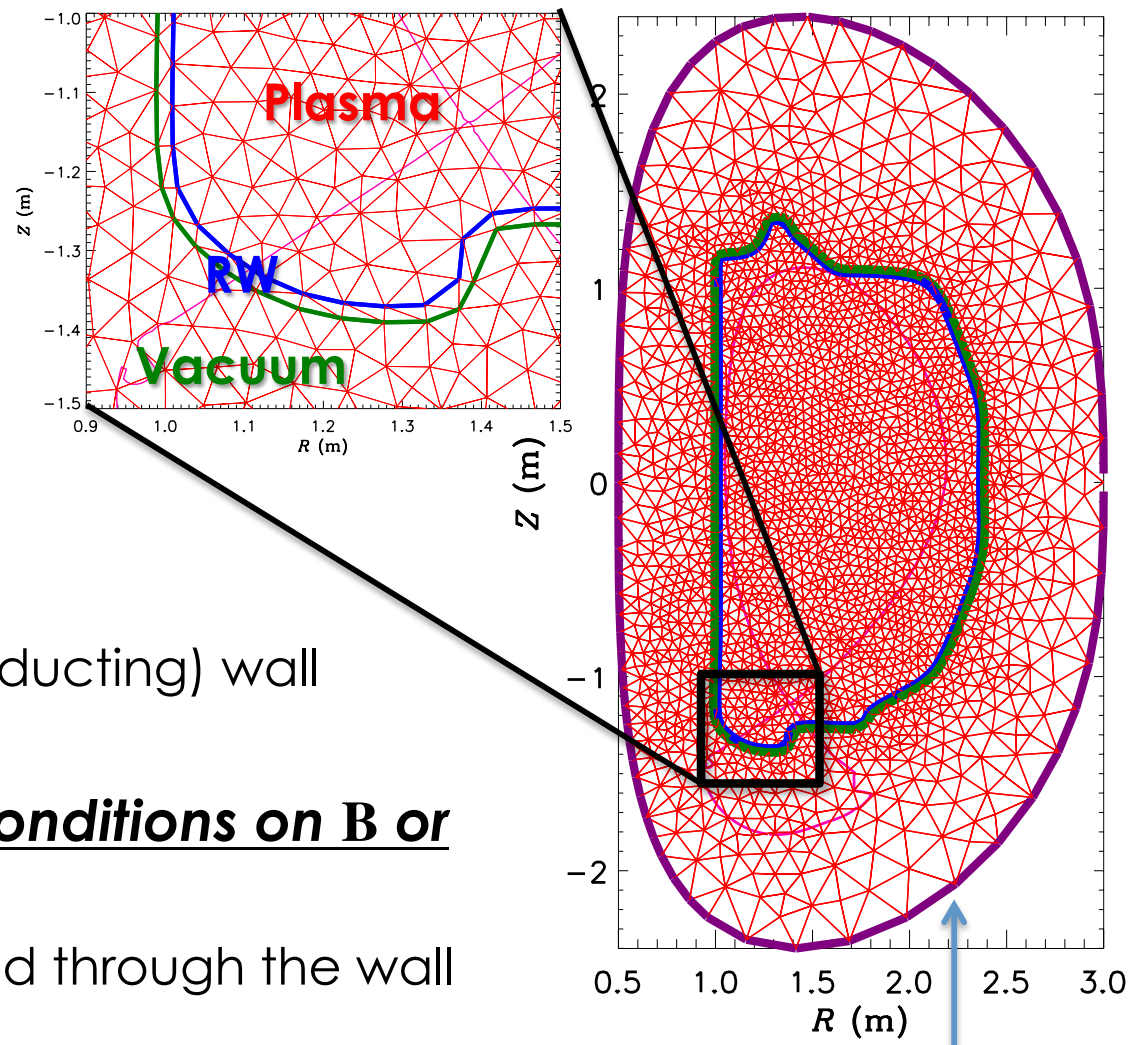
Disruption Modeling

New Resistive Wall Capability In M3D-C1 Includes Wall And Vacuum Regions In Simulation Domain

- **3 regions inside domain:**
  - Vacuum ($\mathbf{J} = 0$)
  - RW ($\mathbf{E} = \eta_W \mathbf{J}$)
  - Plasma (Extended MHD)

- **Boundary conditions:**
  - $\mathbf{v}$, p, n set at inner wall
  - $\mathbf{B}$ set at outer (superconducting) wall

- ***<u>There are no boundary conditions on B or J at the resistive wall</u>***
  - Current can flow into and through the wall



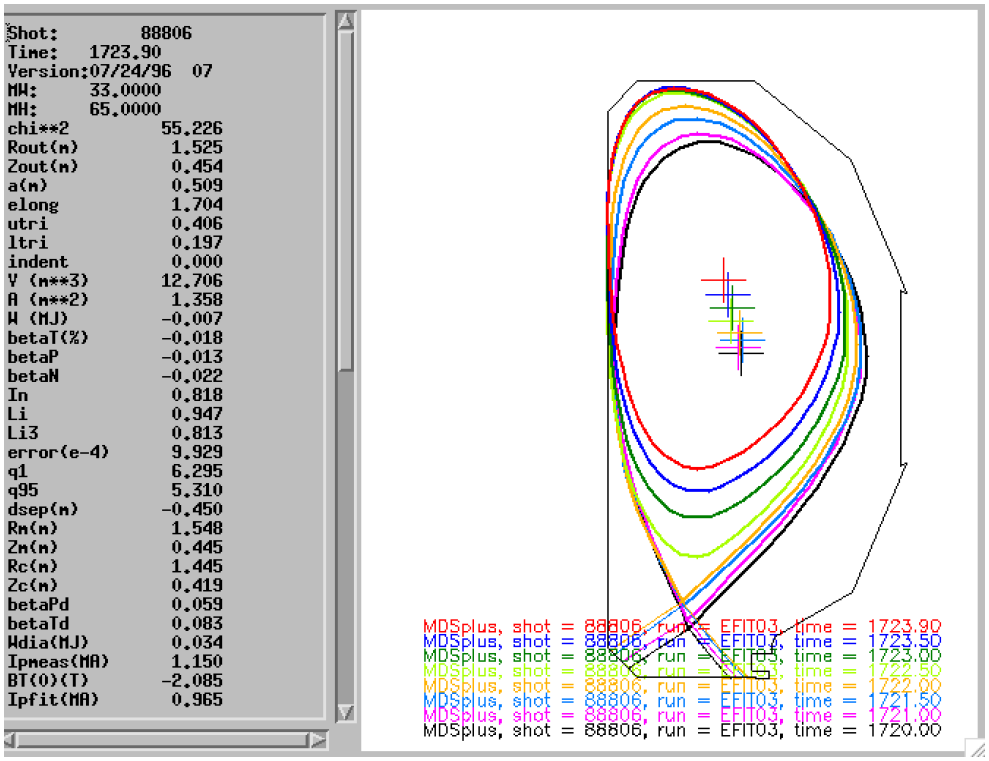
$$\mathbf{B} = \mathbf{B}_{plasma}(t) + \mathbf{B}_{coils}$$

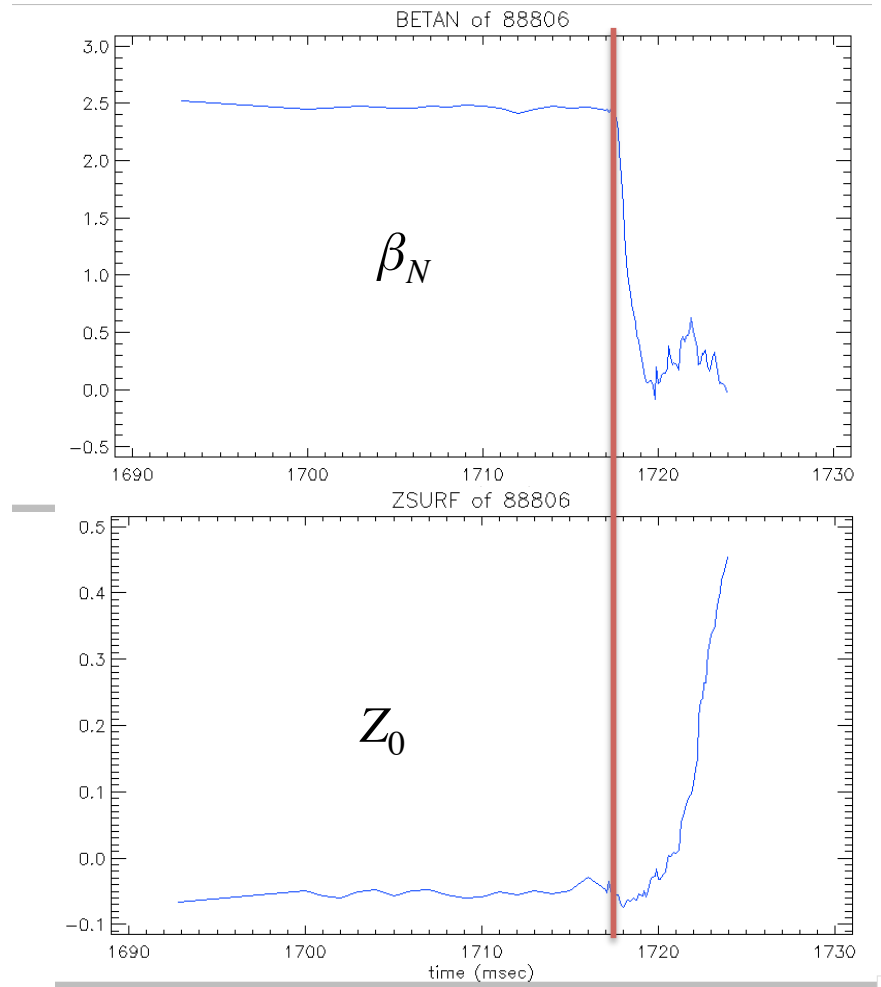
$$\mathbf{J}_{coils} = 0$$

Superconducting Wall

Nonlinear Calculation Recovers $n = 0$ Instability In DIII-D VDE Discharge

- **DIII-D discharge 088806 disrupted due to "killer pellet"**
  - Vertical stability was lost shortly after thermal quench
  - VDE timescale ~3 ms





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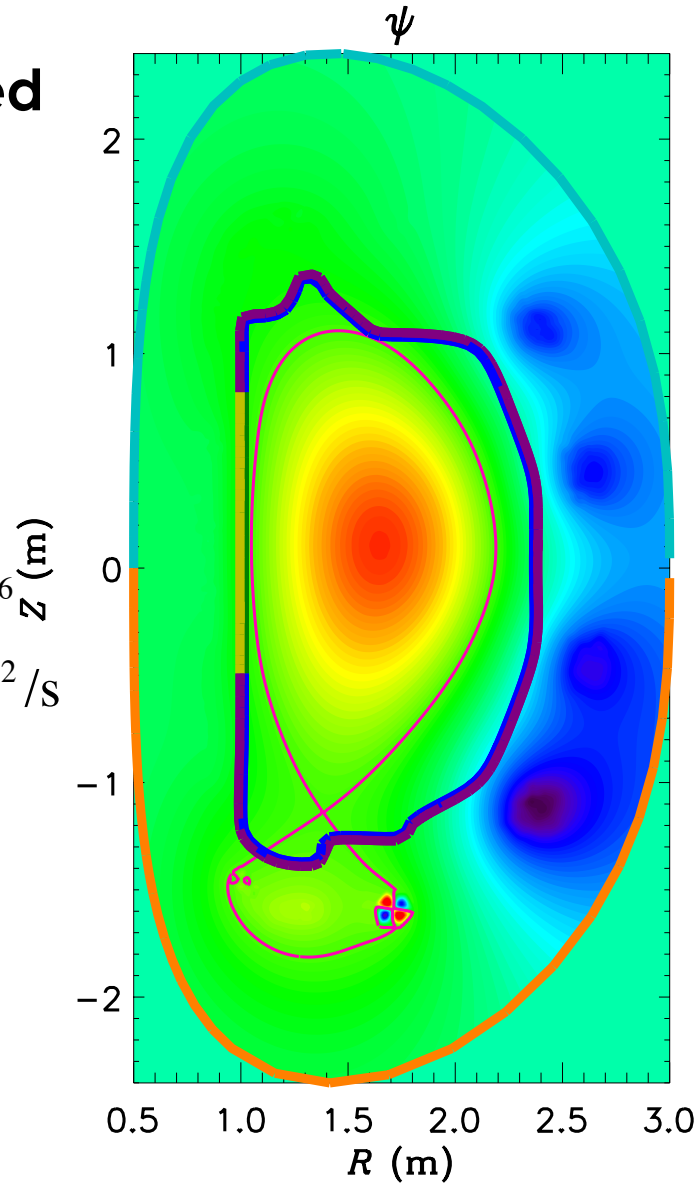
Nonlinear Calculation Initialized From EFIT Reconstruction

- **M3D-C1 is initialized using the reconstructed equilibrium just before TQ ($t = 1720$ ms)**
  - Equilibrium is re-solved on M3D-C1 grid
- **Nonlinear $n = 0$ calculation uses fairly realistic plasma parameters**
  - Spitzer resistivity: $S_0 \approx 6.8\times10^7$
  - Anisotropic thermal conductivity: $\chi_{\parallel}/\chi_{\perp} = 10^6$
  - Anomalous perp. transport: $100 < \chi_{\perp} < 800\ \mathrm{m}^2/\mathrm{s}$
- **RW approximates first wall, not vacuum vessel here; using "modern" first wall, different from old experiment**



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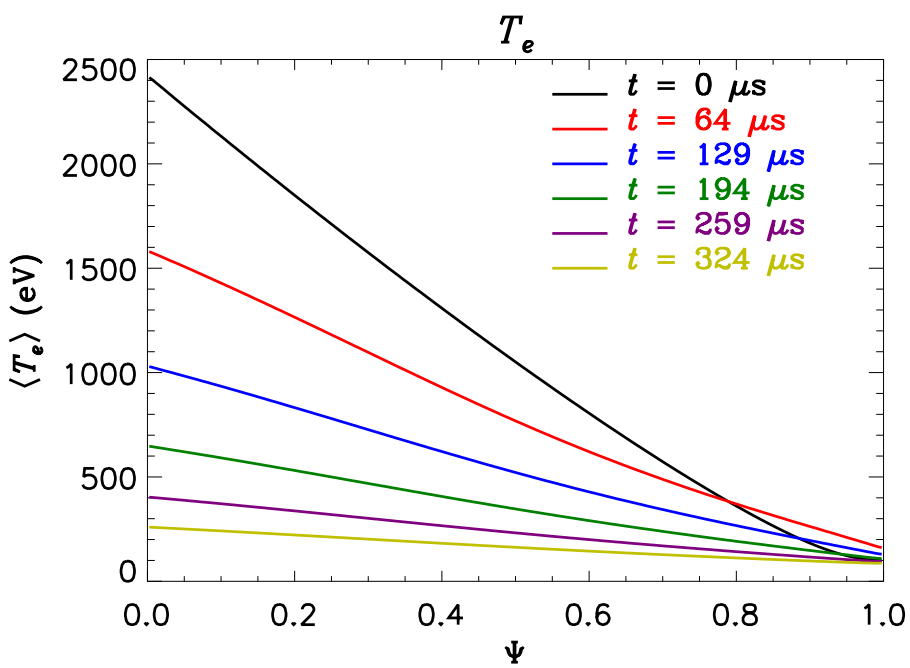
Simulations Include Thermal Quench Stage

- **Thermal Quench happens on ~$100\ \mu s$ timescale, due to large perpendicular thermal conductivity**
  - Not meant to be physically realistic! We are interested in CQ phase.



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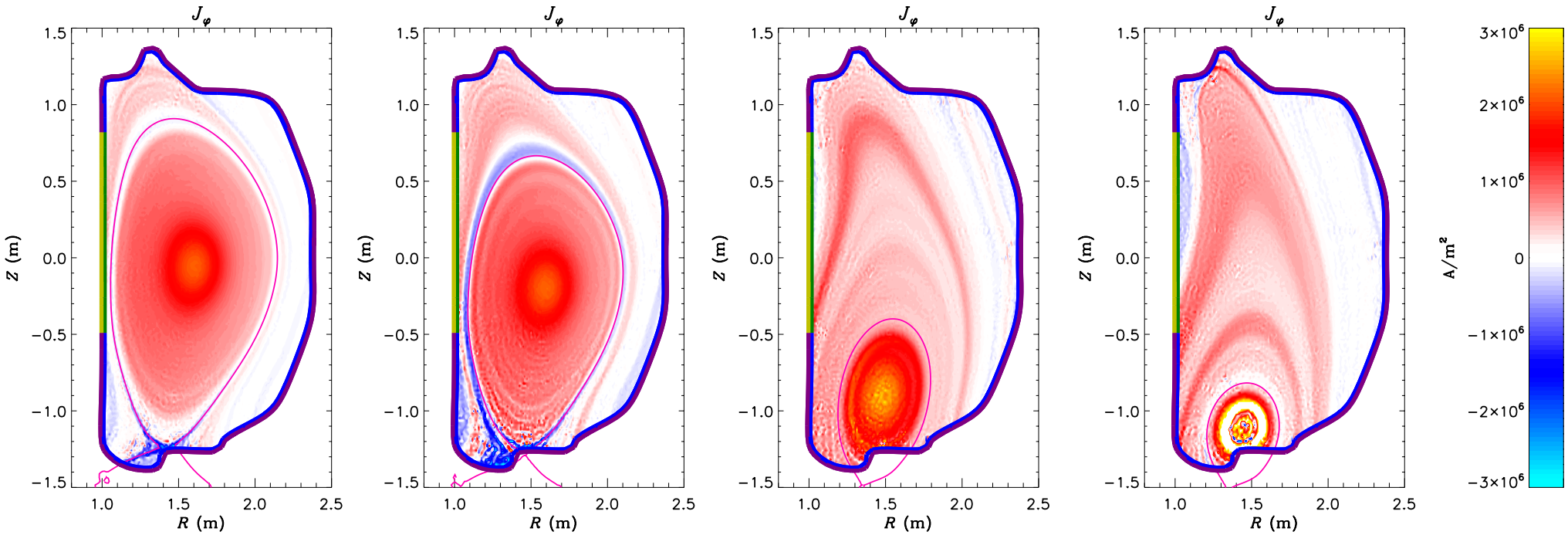
Calculation Shows Vertical Displacement Into Lower Divertor

- **Both co-I_P and counter-I_P currents are seen in the open field-line region**

- **Plasma always moves to lower divertor, unlike in experiment**
  - Maybe due to different wall configuration?



$t = 2.27$ ms $t = 2.59$ ms $t = 2.92$ ms $t = 3.24$ ms

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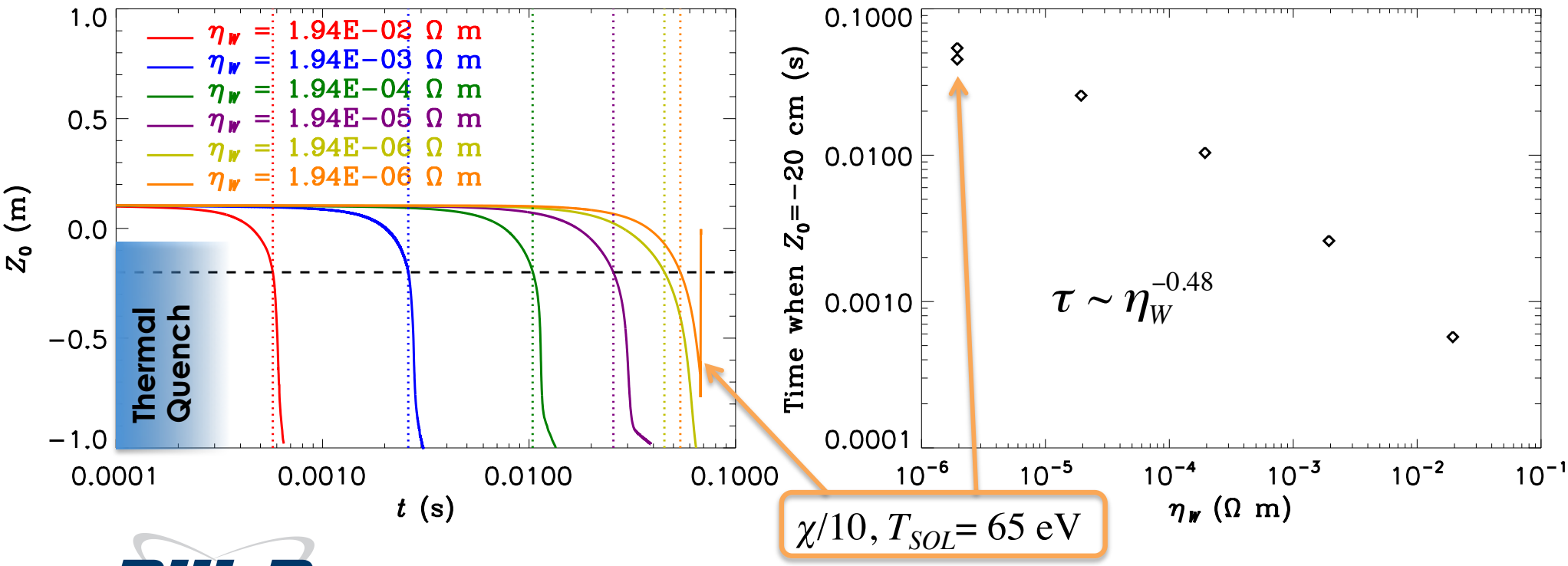
Timescale of VDE Scales Inversely with $(\eta_W)^{1/2}$

- **Given wall thickness** $\delta = 2$ cm **and a poloidal scale length** $d =$ 50 cm, **resistive wall diffusion times range from** ~6.5 ms **to** ~0.65 μs

- **VDE timescale is longer than resistive wall time**
  - Doesn't seem strongly affected by T_{SOL}; need more cases

$$\tau_W = \frac{\mu_0 d \delta}{\eta_W}$$



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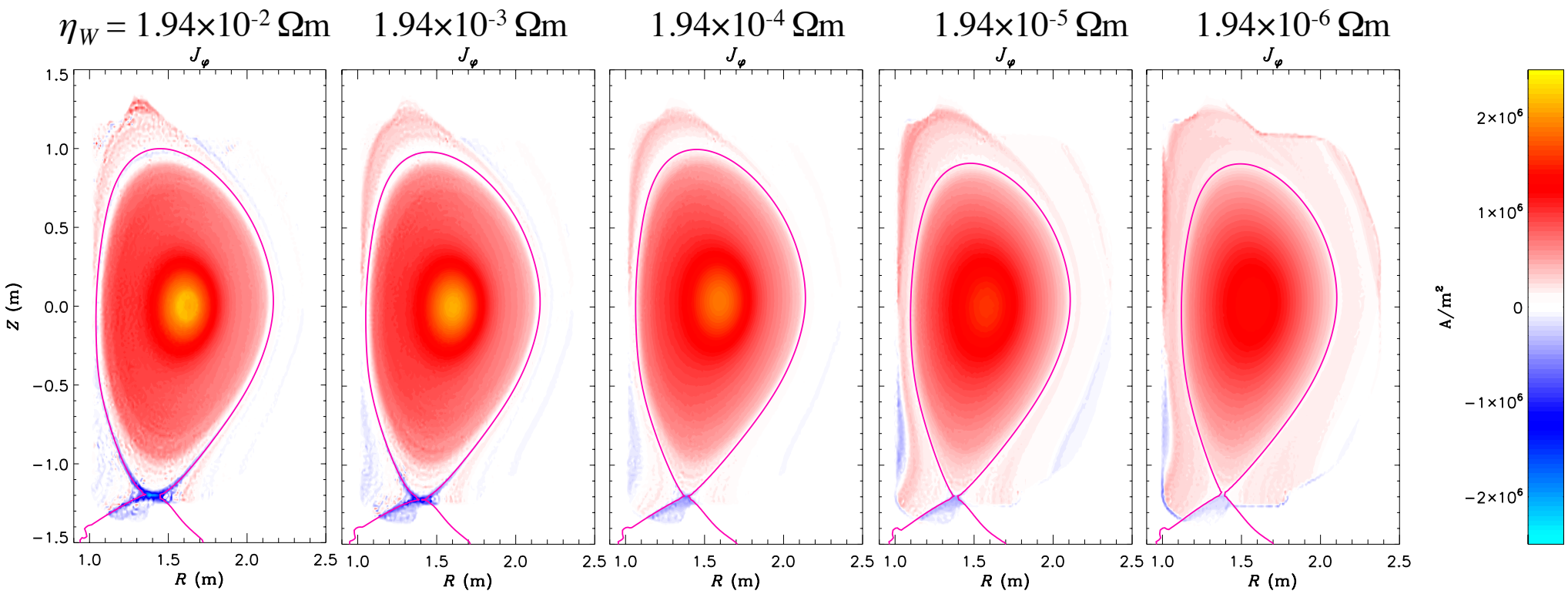
Currents in Wall and Open Field-Line Region Change with η_W

- At early stage of VDE, currents in the wall are stronger at lower η_W
- Counter-I_P currents are significantly stronger at higher η_W



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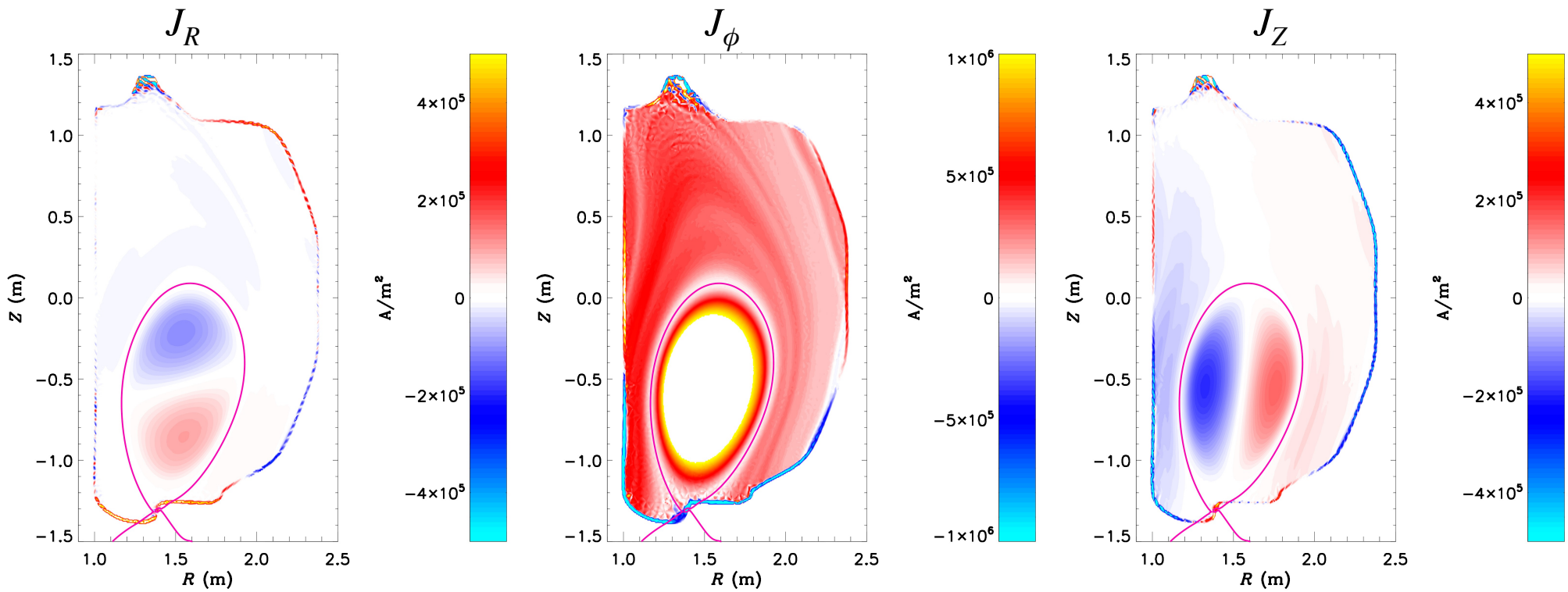
Wall Currents are Mostly Inductive

- **Currents are also present in the open field-line region**
  - Magnitude may be an artifact of high T_e in the open field-line region
  - Current flows from plasma to wall to ensure $\nabla \cdot \mathbf{J} = 0$
- **Wall currents are consistent with excluding poloidal flux**



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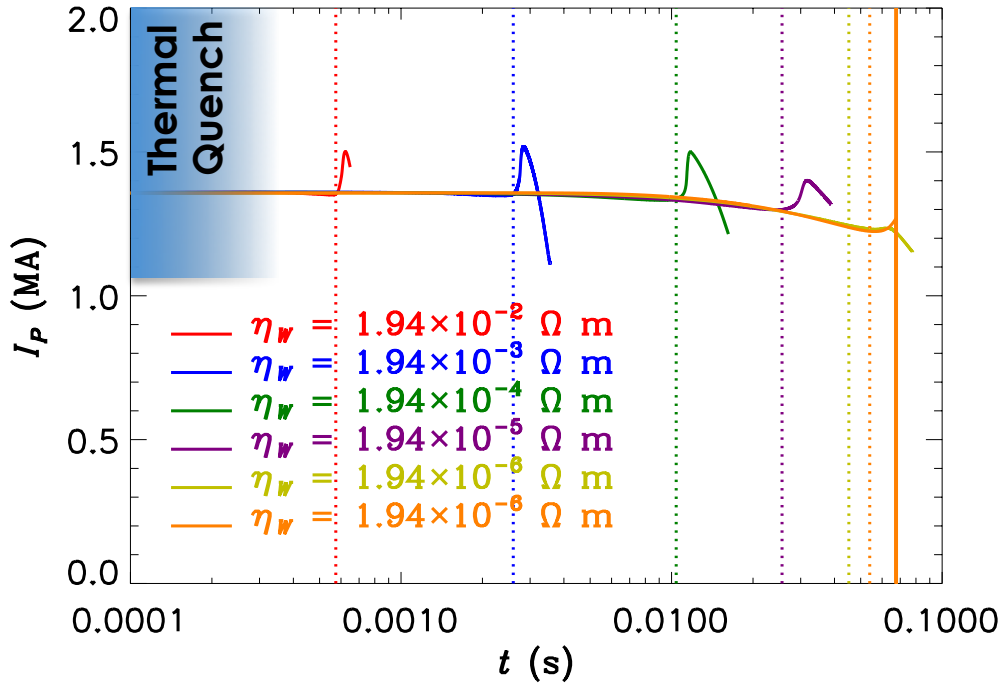
Current Spikes Observed Before Current Quench; Associated with Vertical Motion of Plasma

- **Current spike onset is correlated with vertical motion of plasma, unlike TQ**



- "I_P" **here only includes all toroidal current in the plasma region, but not in the resistive wall**
- **Spike is significantly diminished when** $\eta_W < \eta_{SOL}$

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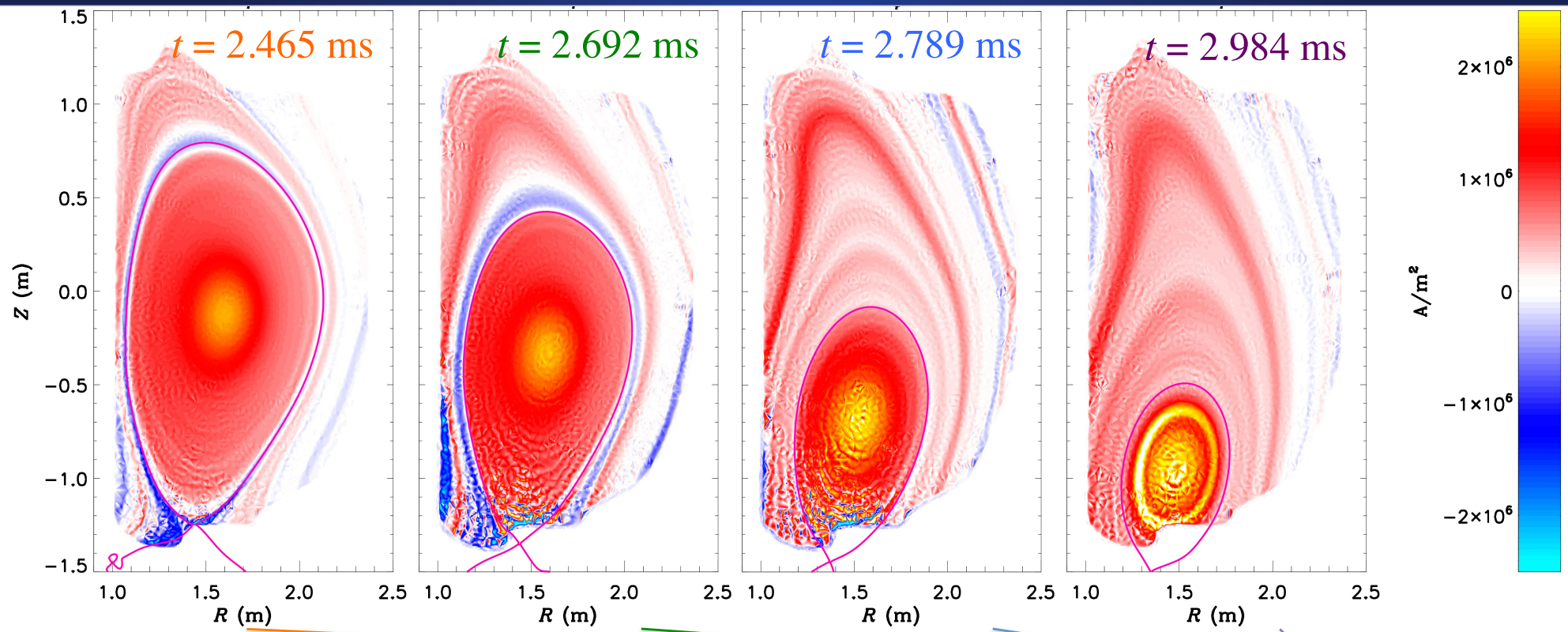
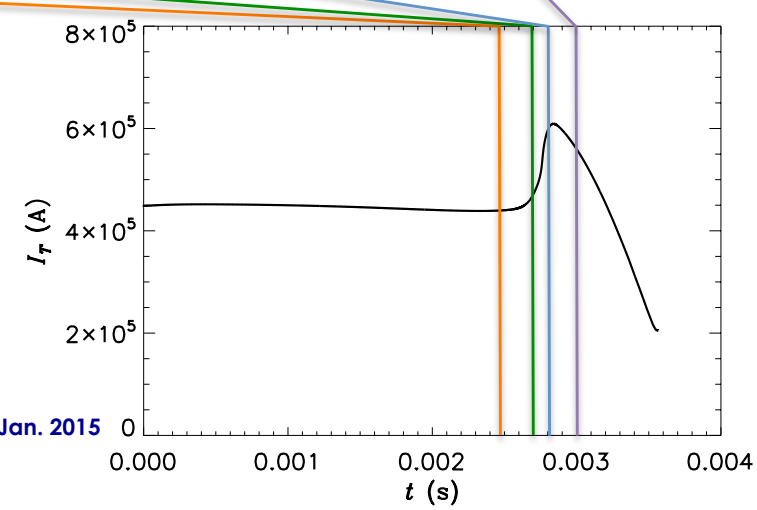
Current Spike is Associated With Loss of Counter-I_P Current In Open Field-Line Region



$$\eta_W = 1.94\times10^{-3}\ \Omega\ \mathrm{m}$$



- **Plasma undergoes rapid contraction during current spike**

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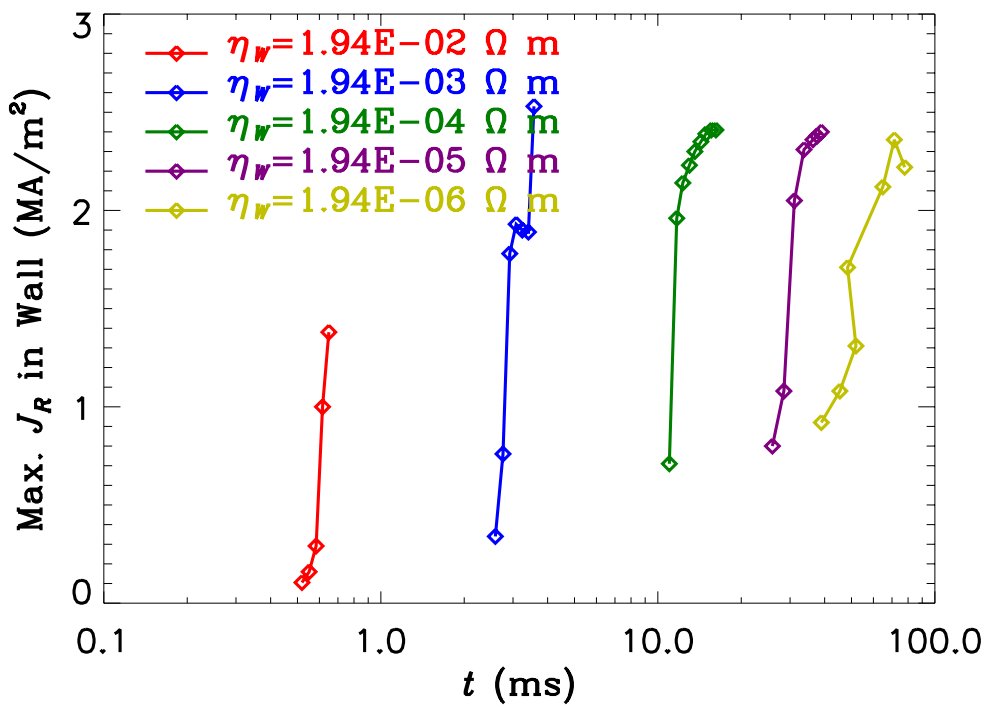
Max Poloidal Current in Wall Depends Weakly on η_W

- **Maximum J_R occurs during current quench, when plasma is limited by lower divertor**
- **Maximum J_R is roughly** $2–2.5\ \mathrm{MA/m^2}$ **in this case**
  - Corresponds to $F_Z \sim 500\ \mathrm{kN}$ over ~$50\ \mathrm{cm}$ of the lower divertor
- **Impulse to vessel depends on η_W because time scale changes**



Summary of VDE Modeling

- **Resistive wall model in M3D-C1 seems to be working properly in 2D**
  - Axisymmetric nonlinear & complex linear
  - Realistic transport parameters and timescales

- **VDE calculations show how response currents flow in plasma and vessel**
  - Timescale of VDE scales roughly as $(\eta_W)^{1/2}$; different from growth rate
  - Maximum current & force in vessel is weakly dependent on η_W → impulse decreases with η_W
  - A spike in the total current in the plasma region before the CQ is associated with the plasma contacting the wall; gets smaller when $\eta_W < \eta_{SOL}$

- **Next step is 3D modeling of VDE**
  - As plasma shrinks, current rises and becomes kink-unstable → non-axisymmetric forces and additional transport

Extra Slides

M3D-C1 Frequently Asked Questions

- **Q: Is M3D-C1 a version of M3D?**
  - **A: No!** There are some similar design principles (flux / potential representation of $\mathbf{v}$ and $\mathbf{B}$), but codes are totally separate.

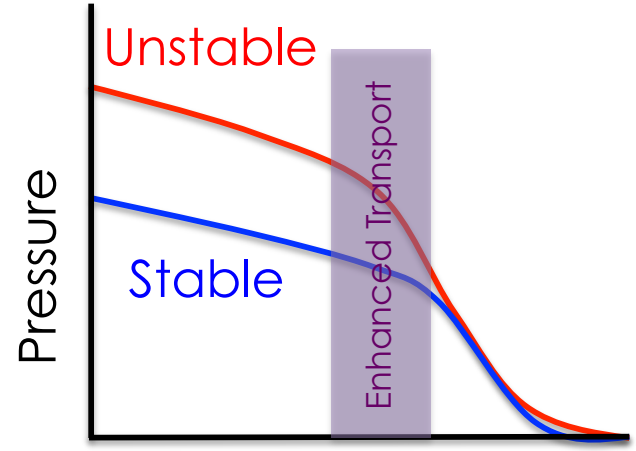
- **Q: Can M3D-C1 simulate stellarators?**
  - **A: Yes, in principle.** There is no restriction on the plasma geometry. The mesh and walls must be axisymmetric (for now)

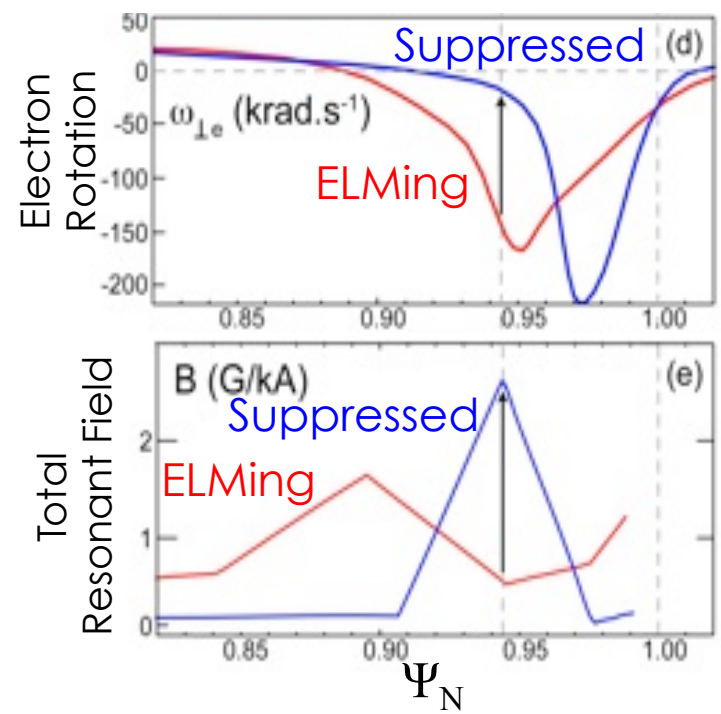
- **Q: How is M3D-C1 different from [insert code name here]?**
  - A: M3D-C1 is most similar to NIMROD. M3D-C1 is the only code that has been used to calculate two-fluid perturbed equilibria.

ELM Suppression

EPED Model Suggests ELM Suppression Requires Enhanced Transport Localized at Pedestal Top

- **EPED Model of pedestal structure:**
  - Gradient determined by local KBM stability
  - Width grows until global PB stability threshold is reached (ELM)
- **Implies model of ELM suppression**
  - Something stops widening of pedestal before threshold
  - Requires enhanced transport at $\Psi \approx 96–97\%$
- **Predictive modeling needs model of RMP effect on transport**
  - Enhanced classical transport?
  - Change to KBM stability?
  - Stochasticity? (DIII-D experiments show evidence of locking/penetration)





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ELM Suppression Correlates with Calculated Changes to Magnetic Topology from 3D Fields

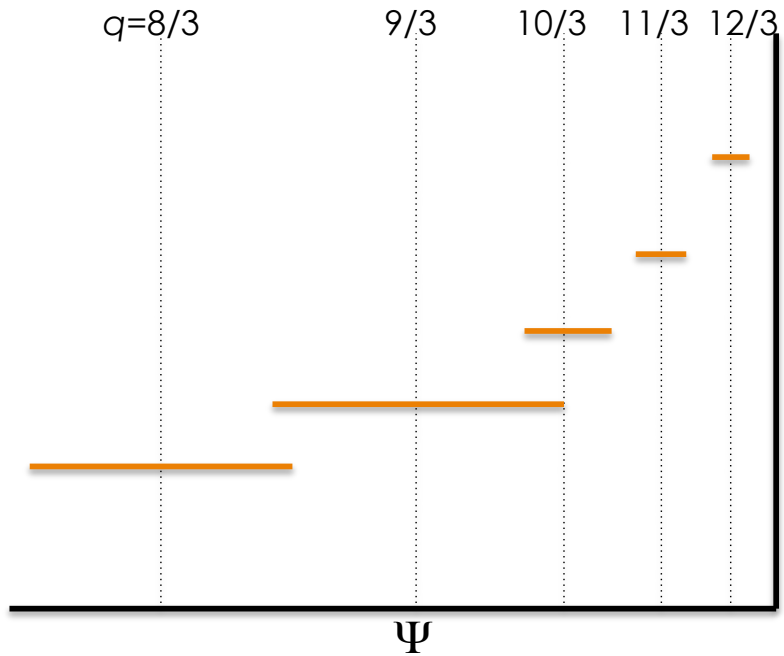
- **In a set of DIII-D discharges, it was found that the Vacuum Island Overlap Width (VIOW) correlates with ELM suppression***
- **Actual mechanism of ELM suppression is more complicated than VIOW**
  - VIOW ignores plasma response (kinking, screening)
  - Pedestal is probably not stochastic, so "IOW" may not be physical
- **We have revisited this correlation study, now considering plasma response and localized measures of stochasticity**
- **Effectiveness of ITER ELM suppression coils are evaluated on the basis of these findings**

*Fenstermacher *Phys. Plasmas* **15**, 056122 (2008)

"Local Chirikov" Value Gives Indication of Localized Stochasticity

- **Island width is estimated using pitch-resonant field at each mode-rational surface**

$$w(\Psi_m) = \frac{2}{\pi}\sqrt{\frac{\psi_{mn} q}{\psi' q'}}$$



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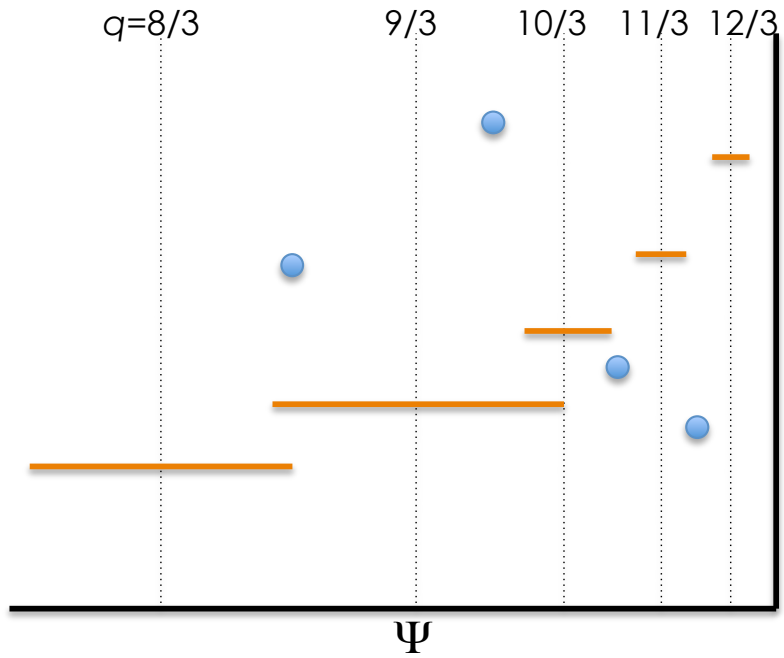
$$\sigma\left(\frac{\Psi_{m+1} + \Psi_m}{2}\right) = \frac{1}{2}\frac{w(\Psi_{m+1}) + w(\Psi_m)}{\Psi_{m+1} - \Psi_m}$$



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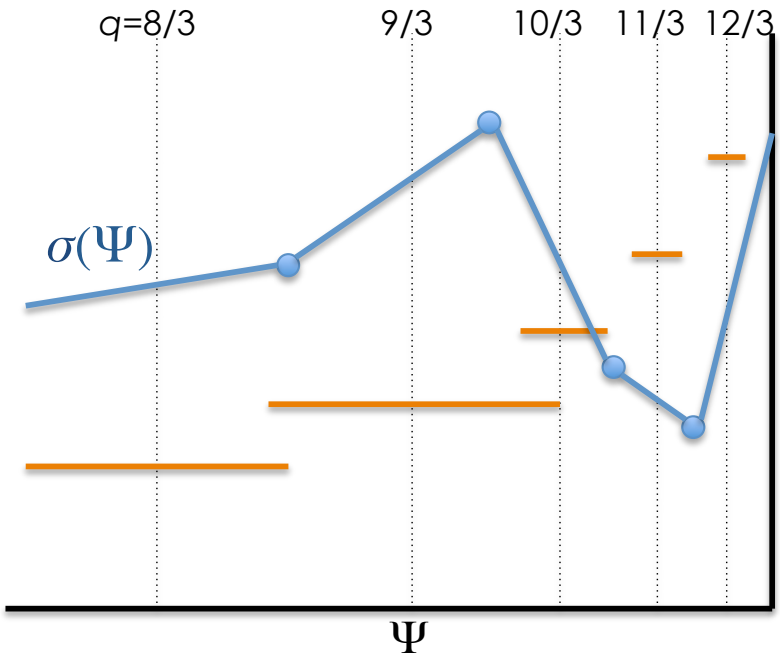

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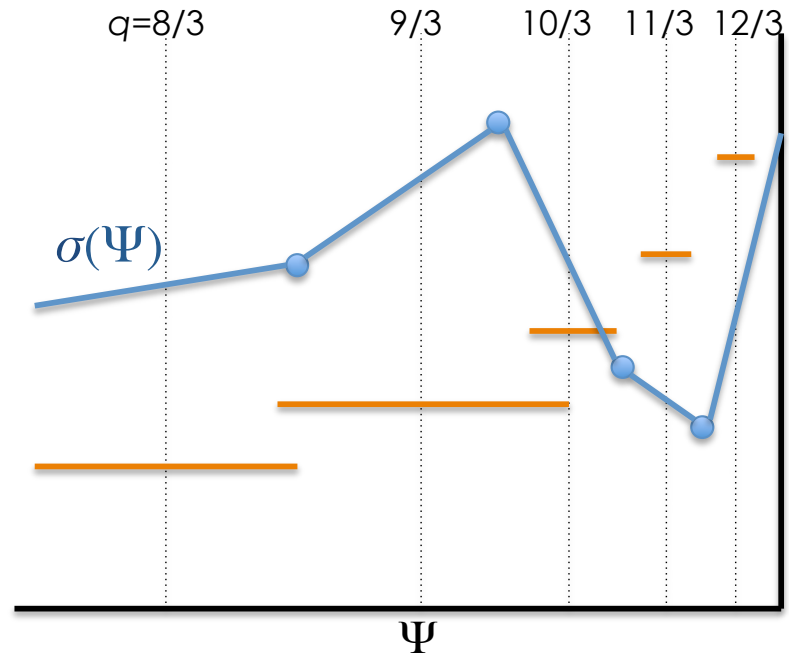
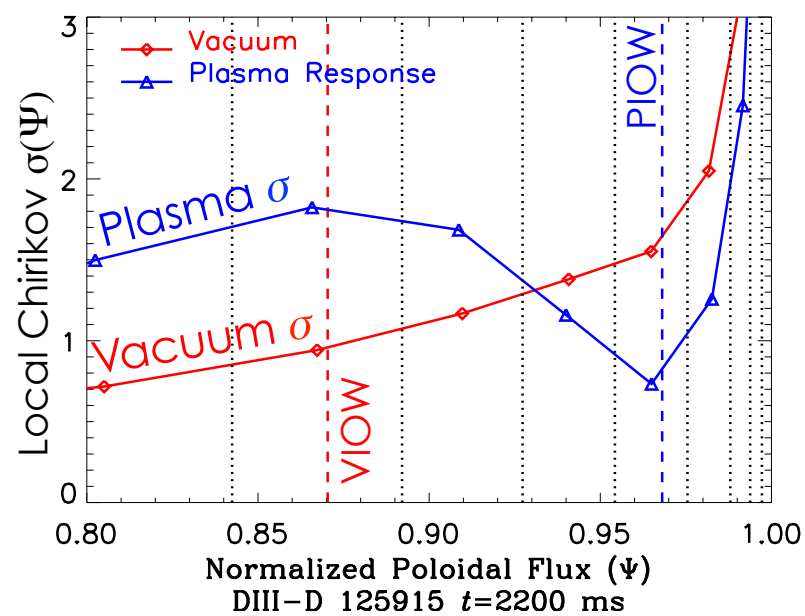
- **"Local Chirikov" value is defined by linear interpolation of these values**

- **Plasma response reduces σ in the pedestal**





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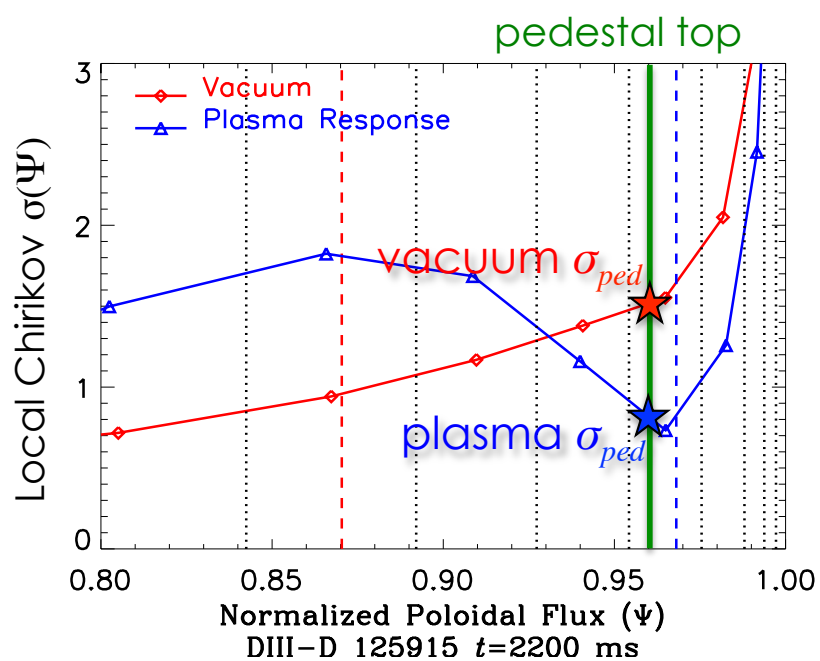
Local Chirikov Correlates Better Than IOW; Plasma Response Doesn't Always Improve Correlation

- **Unlike IOW, σ parameter is a function of Ψ**
  - σ correlates best when evaluated at the top of the pedestal ($\Psi \approx 96$–97%): "σ_{ped}"
- **IOW and $\sigma(\Psi)$ calculation for 162 DIII-D equilibria from 13 discharges, with and without plasma response**

Metric	Threshold	Accuracy
Vacuum IOW	12.7%	63%
Plasma IOW	6.4%	70%
Vacuum σ_{ped}	1.55	89%
Plasma σ_{ped}	0.90	73%



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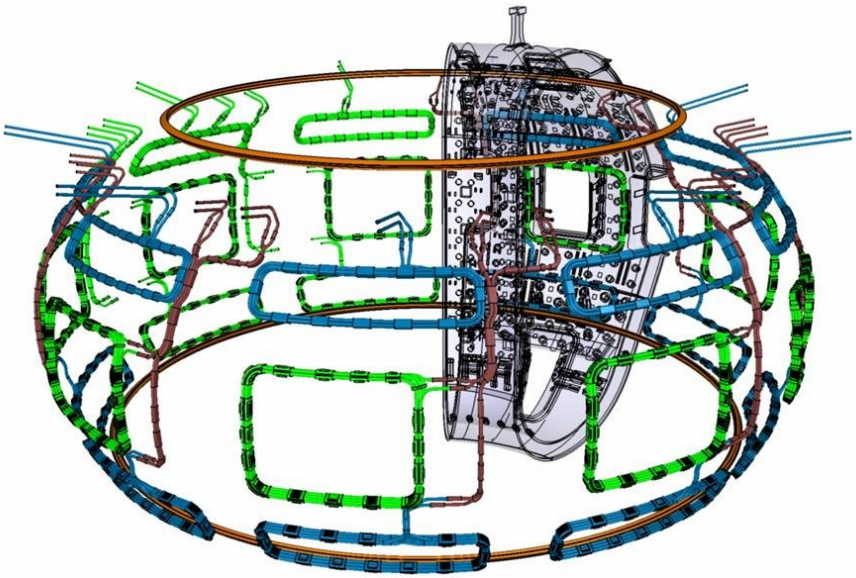
New Metrics Correlate Better, But Are Not Predictive

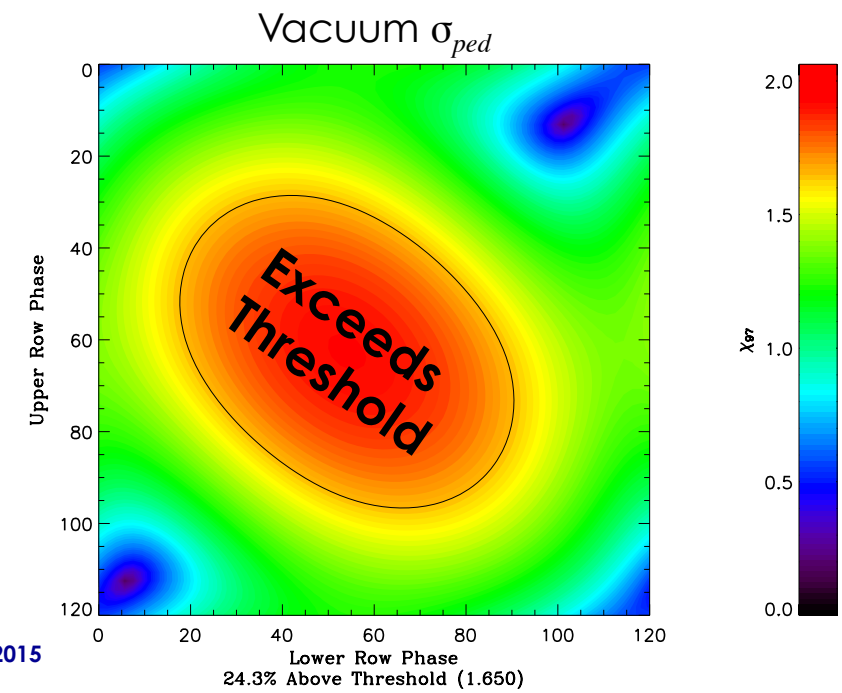
- **Only the *n* = 3 component is considered here**
  - Strong evidence that sidebands can be important (Orlov)
  - Fenstermacher (2008) VIOW definition includes sidebands
- **Plasma response conflates cause / effect of suppression**
  - ELM suppression changes *n* = 0 profiles, which changes response
  - Nonlinearities in experiments suggest feedback loop
- **Linear response misses some important physics**
  - Amplification of islands implies nonlinear effects important (locking)
  - IOW and σ are imperfect indicators of enhanced transport

Suppression Correlation Metrics Have been Applied to Several ITER Scenarios

- **Metrics have been calculated for 7 ITER scenarios, *n* = 2–4**
  - 15 MA Q_{DT}=10 T_{ped}=3.8, 4.4, 5.0, and 6.3 keV
  - 12 MA Hybrid
  - 10 MA Ramp-Up
  - 9 MA



- **IOW and σ_{ped} calculated as a function of the phase of the upper and lower coil rows (relative to center row)**



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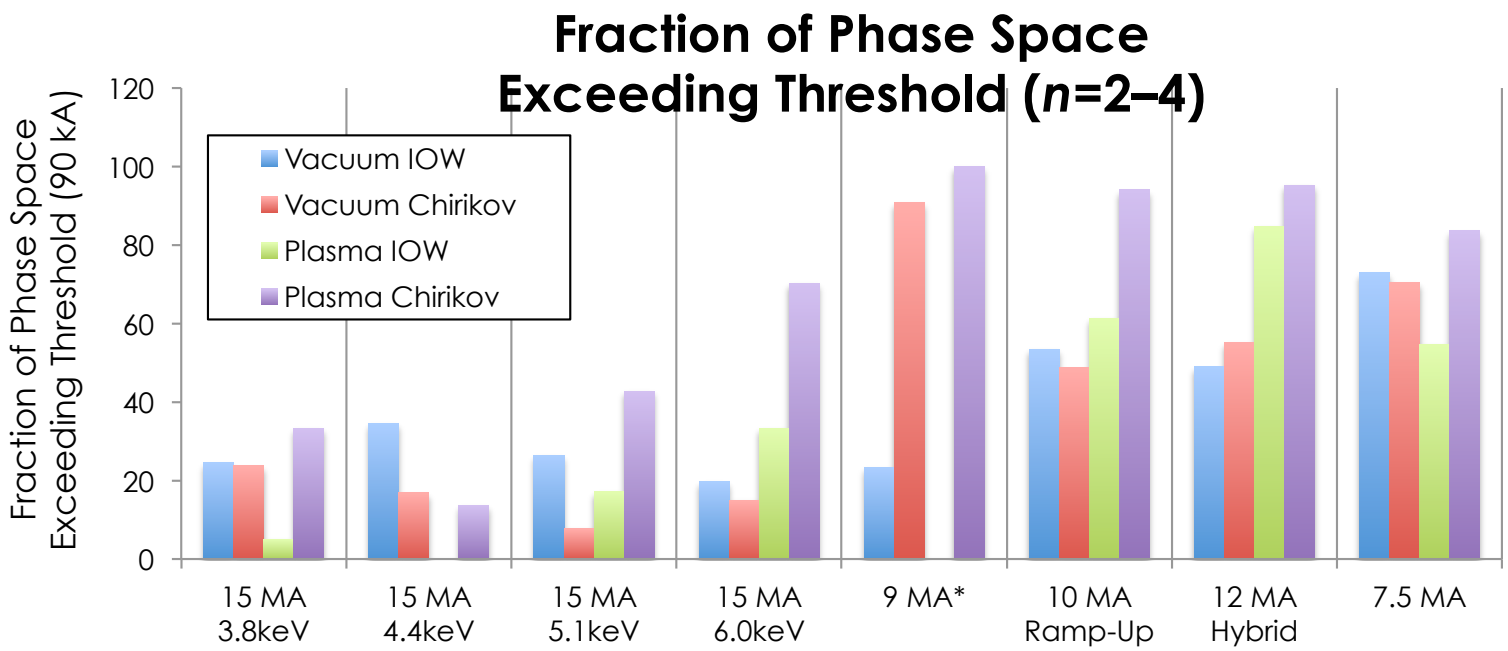
Suppression Threshold of Three of Four Metrics can be Achieved for All ITER Scenarios



*only n=4 considered for 9 MA scenario

- **Thresholds for three of four metrics can be satisfied for all scenarios**
  - Plasma IOW cannot be satisfied for 2/8 scenarios
- **Metrics tend to agree on optimal coil phases; generally find easier suppression at higher *n***

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Summary of ELM Suppression Correlation Modeling

- **Applying new understanding of pedestal evolution and perturbed 3D equilibria yields improved ELM suppression metrics**
  - Local measure of stochasticity at pedestal top appears to correlate better than vacuum island overlap width
  - Still imperfect (don't recover q_{95} window)

- **Three of four metrics can be satisfied for all ITER scenarios**

- **For truly predictive models, better understanding of transport in 3D geometry is needed**