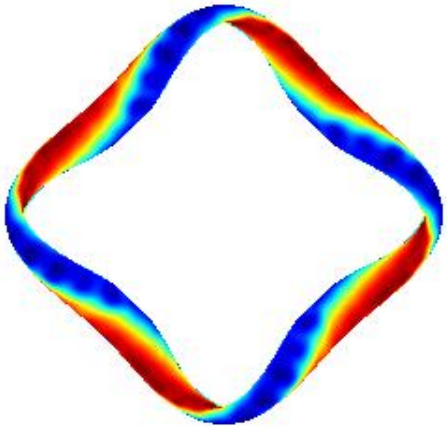
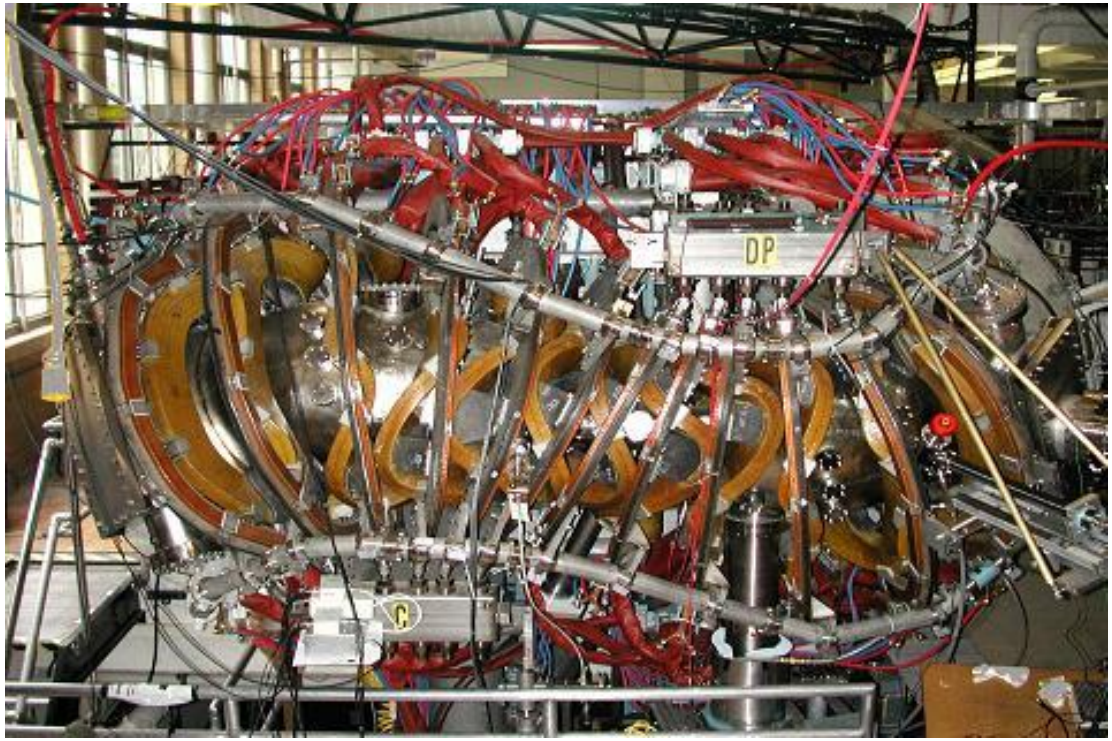
Measurements of Reynolds stress and its contribution to momentum balance in the HSX stellarator





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November 10, 2014

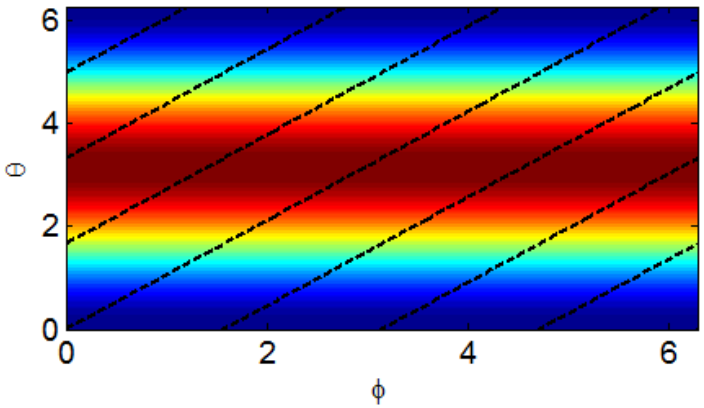
Thanks to the HSX team for their contributions to this work

Outline

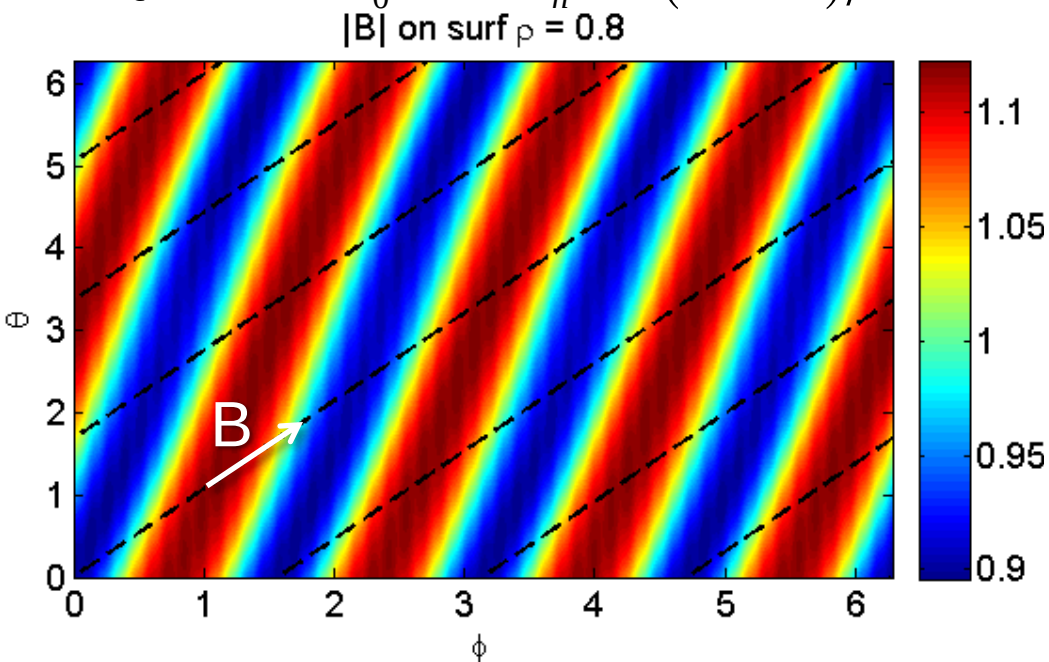
- Quasi-symmetry and the Helically Symmetric Experiment (HSX)
- What determines E_r in a stellarator approaching quasi-symmetry?
- Measured radial electric field and parallel flows differ from ambipolar solutions of neoclassical transport calculations in the edge
- Locally measured Reynolds stress profiles imply large flow drive
  - Multiple probe installations on a surface measure qualitatively different Reynolds stress
- Measurements in configurations with symmetry broken observe larger non-neoclassical flows than in QHS

HSX is the first stellarator optimized for quasi-symmetry

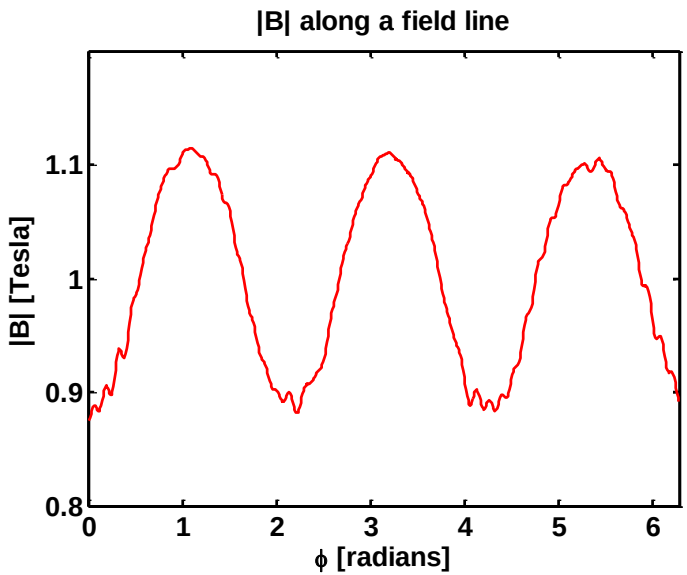
Tokamak: $B/B_0 = 1 - \varepsilon_t \cos \iota\phi$



QHS: $B/B_0 = 1 - \varepsilon_h \cos(n - m\iota)\phi$



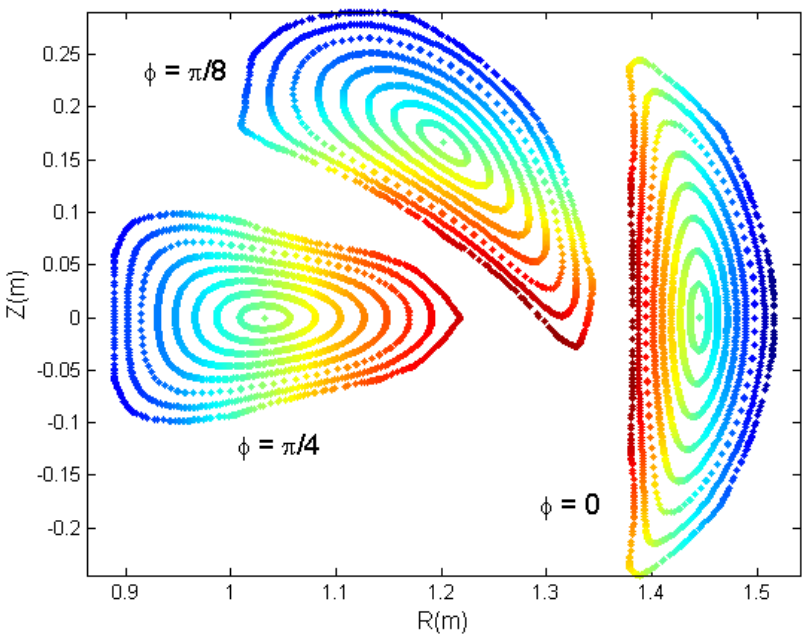
- **HSX has been optimized for quasi-helical symmetry: |B| symmetric in the helical direction (n=4, m=1)**
- **Designed to give tokamak-like viscosity and neoclassical transport properties**
- **HSX is not perfectly quasi-symmetric, there is still some finite amount of neoclassical viscosity and non-ambipolar transport**

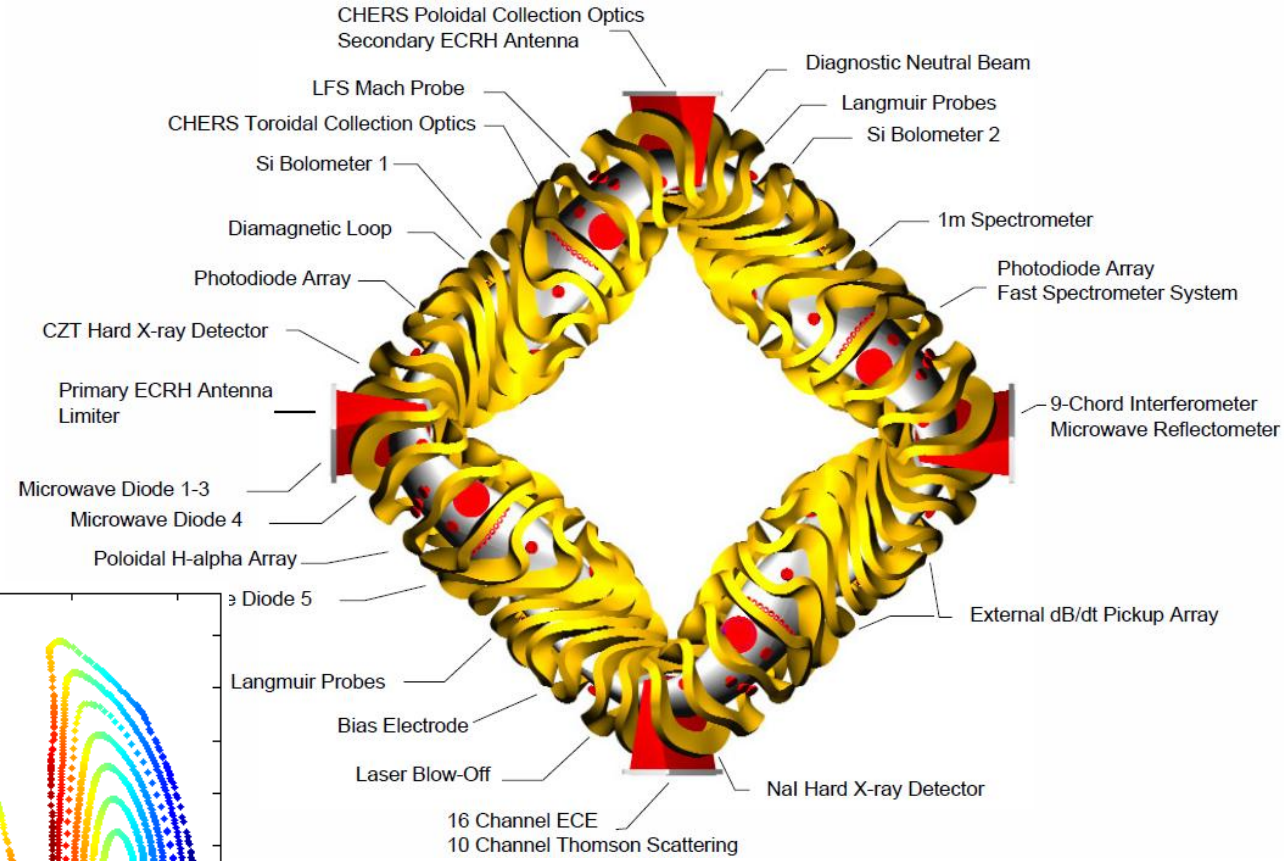


48 modular coils are designed to generate this optimized field in 4 field periods (only 6 unique coil types)

HSX Parameters

<R>	1.2m
<a>	0.12m
B_0	1.0 T
P_{ECRH}	2x 100 kW
ɨ	1.05→1.12
$T_{e,max}$	~2.5 keV
$T_{i,max}$	~60 eV
$n_{e,cutoff}$	10^{19} m^{-3}



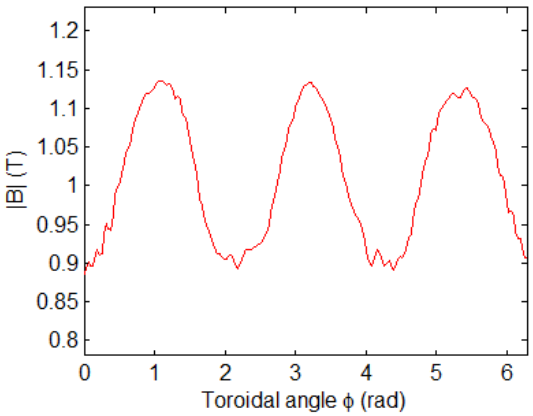


- **Gas puff fueling**
- **50 ms ECRH**
- **Boronized wall conditioning employed for maintaining density control**

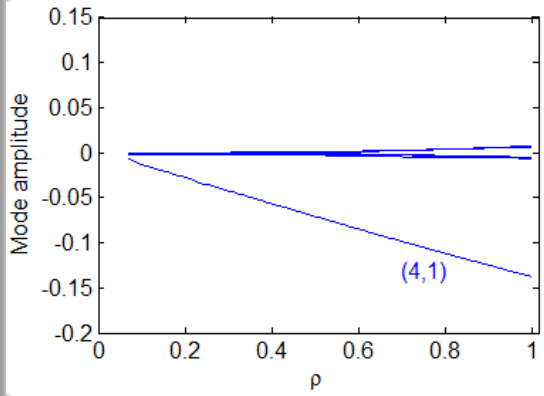
HSX can also operate in the "Mirror" configuration where the symmetry is intentionally broken

QHS

|B| along field line

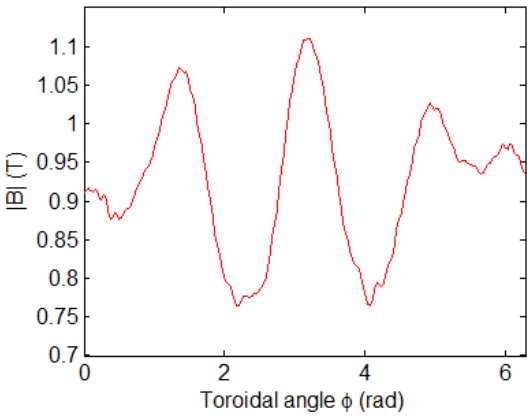


|B| Spectrum

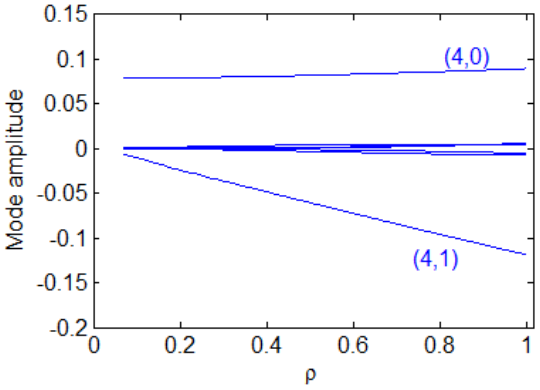


Mirror

|B| along field line

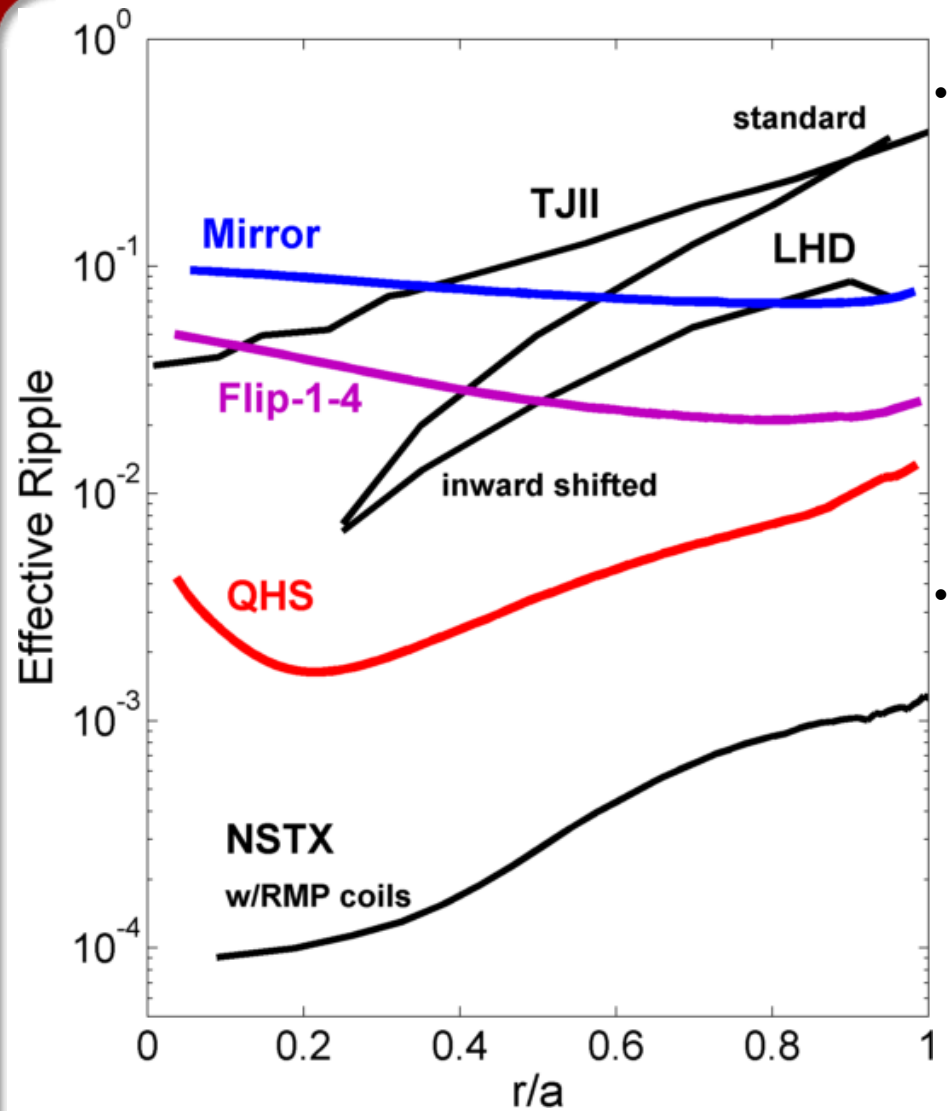


|B| Spectrum



- Energizing a set of planar auxiliary coils spoils the quasi-helical symmetry
- Other properties are similar (ι profile, well depth, plasma volume)
- Direct comparisons can be made between a quasi-helically symmetric and conventional stellarator

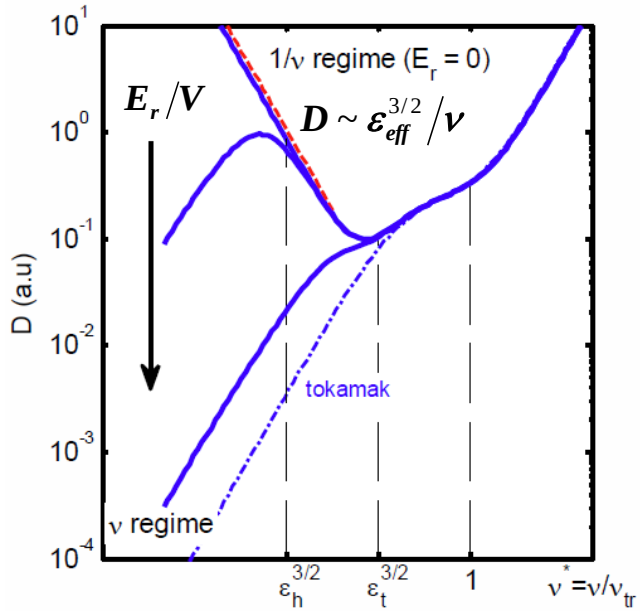
The level of symmetry breaking in HSX is somewhere between that of a conventional stellarator and a tokamak with RMP coils



- Effective ripple (ε_{eff}) is a measure of the neoclassical thermal particle losses at low collisionality, and is finite in all real magnetic confinement devices
  - Particle transport in the low collisionality regime scales with ~ $\varepsilon_{eff}^{3/2} / \nu$
  - Sources like RMP coils, TF ripple, field errors in tokamaks
- How symmetric is symmetric enough to meet the design goals of…
  - low thermal losses in low collisionality regime?
  - alpha particle confinement?
  - large flow velocities?

TJII ε_{eff}: Seiwald et. al, JCP 2008
LHD ε_{eff} (standard config): Okamara, EPS 2012
NSTX and ITER ε_{eff} calculations courtesy of John Canik and Don Spong, ORNL

With non-ambipolar neoclassical particle losses in stellarators, E_r and rotation are constrained



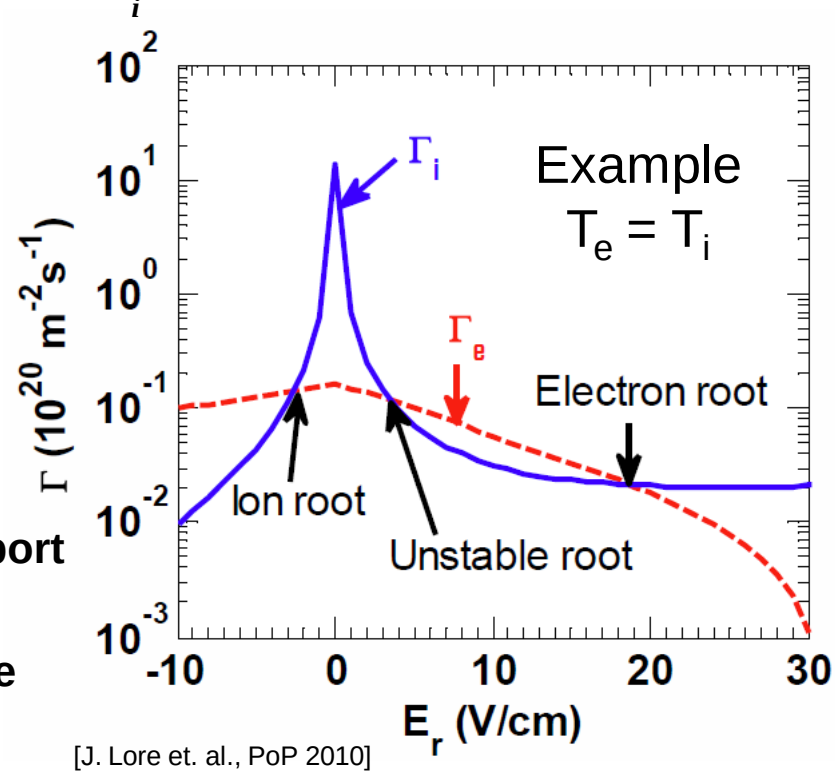
[J. Lore dissertation, 2010]

$$E_r = \frac{1}{Z_i e n_i} \nabla P_i - \vec{V}_i \times \vec{B}$$

- Ideal tokamaks and perfectly quasi-symmetric magnetic configurations have intrinsically ambipolar neoclassical transport
- Transport coefficients calculated using the Drift Kinetic Equation Solver (DKES)

- Un-optimized stellarators have large species-dependent particle losses due to magnetic ripple
- These non-ambipolar fluxes determine E_r
  - Equilibrium only exists at a few neoclassical ambipolar "roots" for each surface

$$\sum_i Z_i \Gamma_i(E_r) = \Gamma_e(E_r)$$

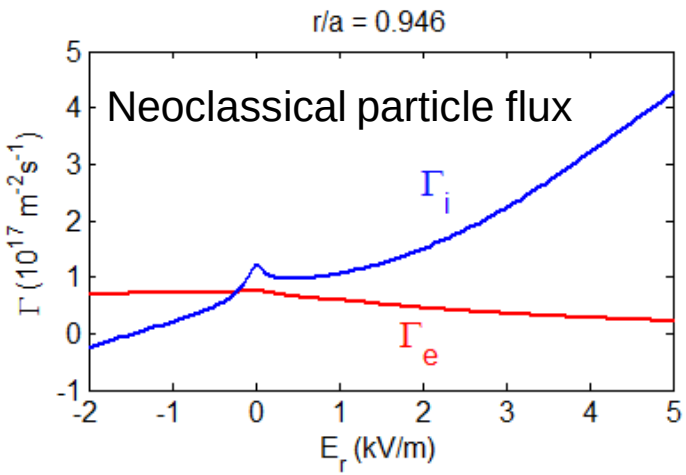


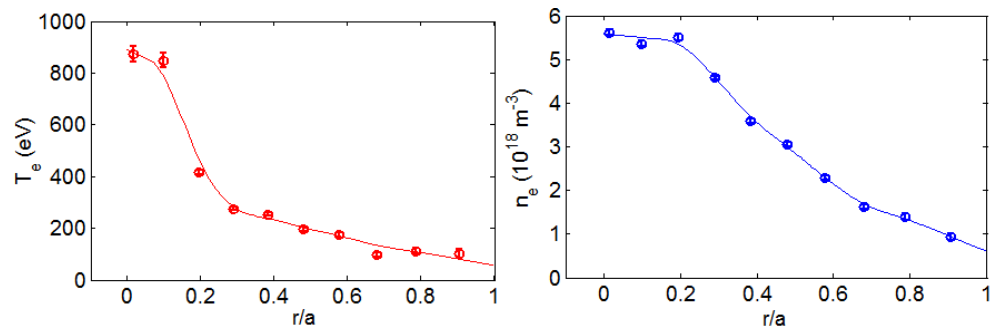
[J. Lore et. al., PoP 2010]

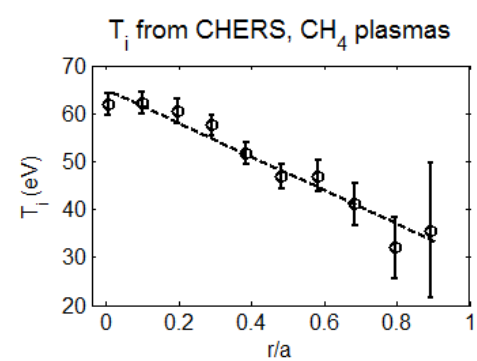
Neoclassical E_r profile prediction for HSX is determined by ambipolarity condition for particle flux using PENTA

$$\sum_i Z_i \Gamma_i(E_r) = \Gamma_e(E_r)$$

- **Transport coefficients are calculated using DKES for HSX, then PENTA** [Spong PoP 2005, Lore PoP 2010] **uses these and measured profiles to calculate fluxes while restoring momentum conservation**
  - **Momentum conservation important for parallel flows, only modifies calculated E_r slightly in HSX**
- **Vacuum VMEC equilibrium used as magnetic geometry for DKES**
- **All calculations here assume pure H plasmas (boronized walls)**
- **Only a single root is predicted for these plasmas (ion root solution only)**
  - **50 kW discharges, only making measurements in the edge**
- **No additional non-ambipolar losses (eg.: ECRH) or momentum sources are included in modeling**



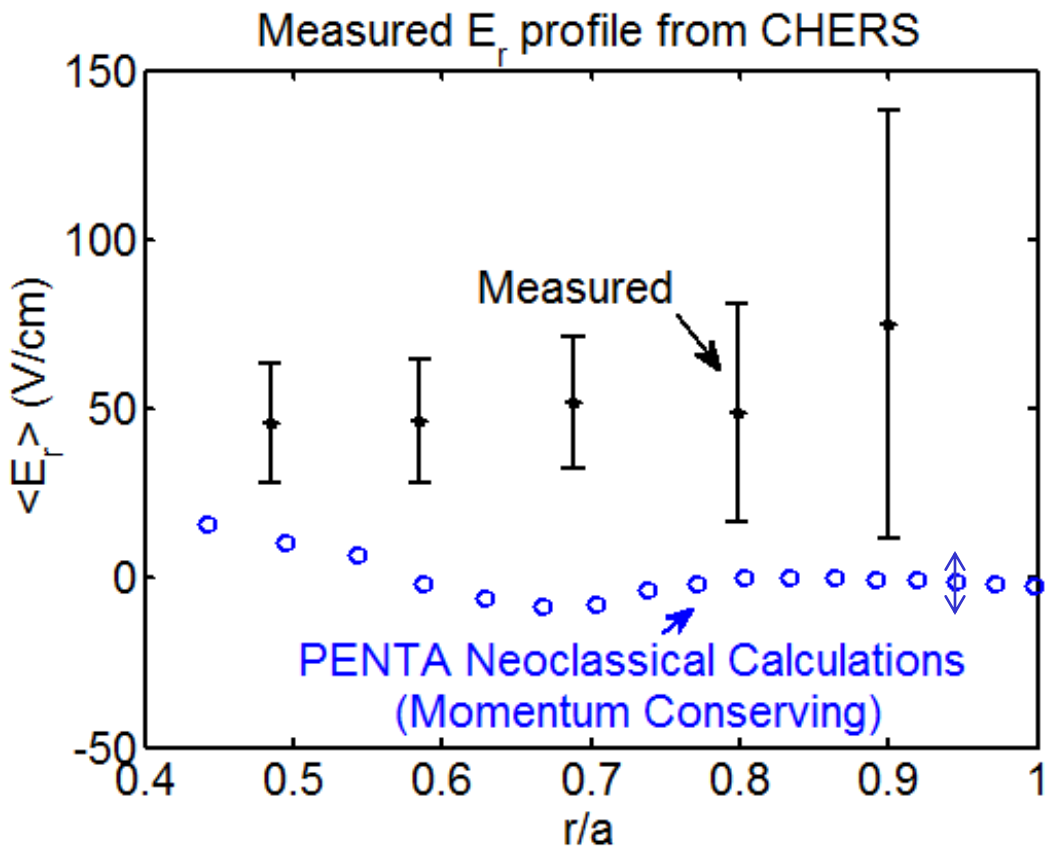




[T_i courtesy A. Briesemeister]

E_r measured in HSX using CHERS does not generally agree well with the values calculated by PENTA

- **CHERS measurements made in plasmas using CH_4 show that E_r differs from neoclassical calculations →**
- **Results of PENTA calculations is robust to changes to local profile inputs within errorbars of measurements**
  - **For typical 50 kW QHS edge parameters, E_r is consistently predicted to be ~ 0 or slightly negative**



[A. Briesemeister et al, PPCF 2013]

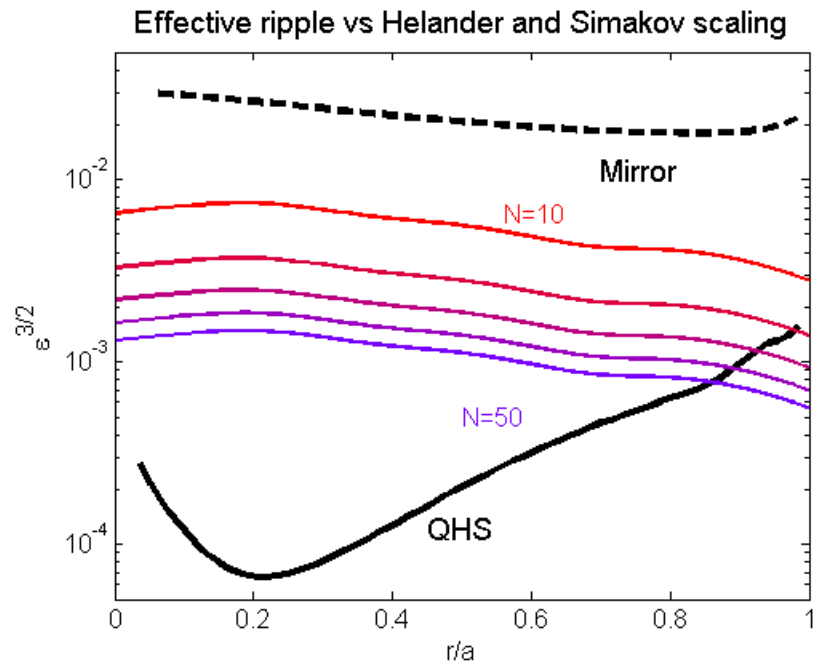
Investigate turbulent sources for determining E_r

Helander and Simakov scaling finds that collisionless plasmas in HSX should be constrained to neoclassical value of E_r at edge, not in core

- How much symmetry breaking is tolerable before neoclassical non-ambipolar transport dominates turbulence-driven Reynolds stress?
- For low collisionality plasma in 1/ν regime, a deviation from neoclassical ambipolarity can be maintained over a radial distance of N ion gyroradii if

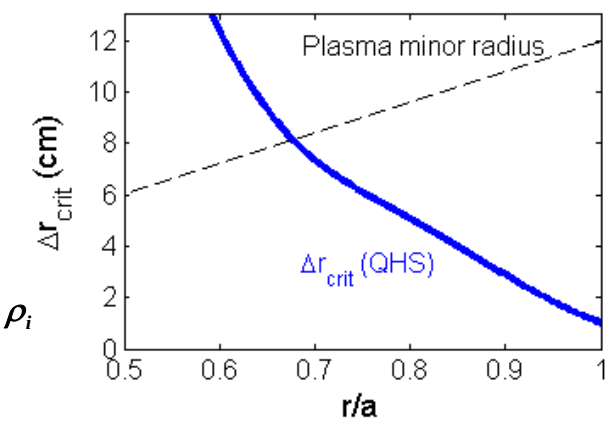
$$\varepsilon_h^{3/2} < \frac{L}{N\lambda_{MFP}}$$

- This prediction finds that HSX should be constrained to neoclassical rotation at edge, rotation in core may be determined by non-neoclassical processes
- Assumes 1/ν ions, which doesn't apply to these HSX plasmas (measured T_i put H ions squarely in plateau regime)
  - Model represents an upper bound on allowable ripple for HSX



$$r_{crit} = N_{crit}\rho_i$$

$$r_{crit} = \frac{L}{\varepsilon_h^{3/2}\lambda_{MFP}}\rho_i$$

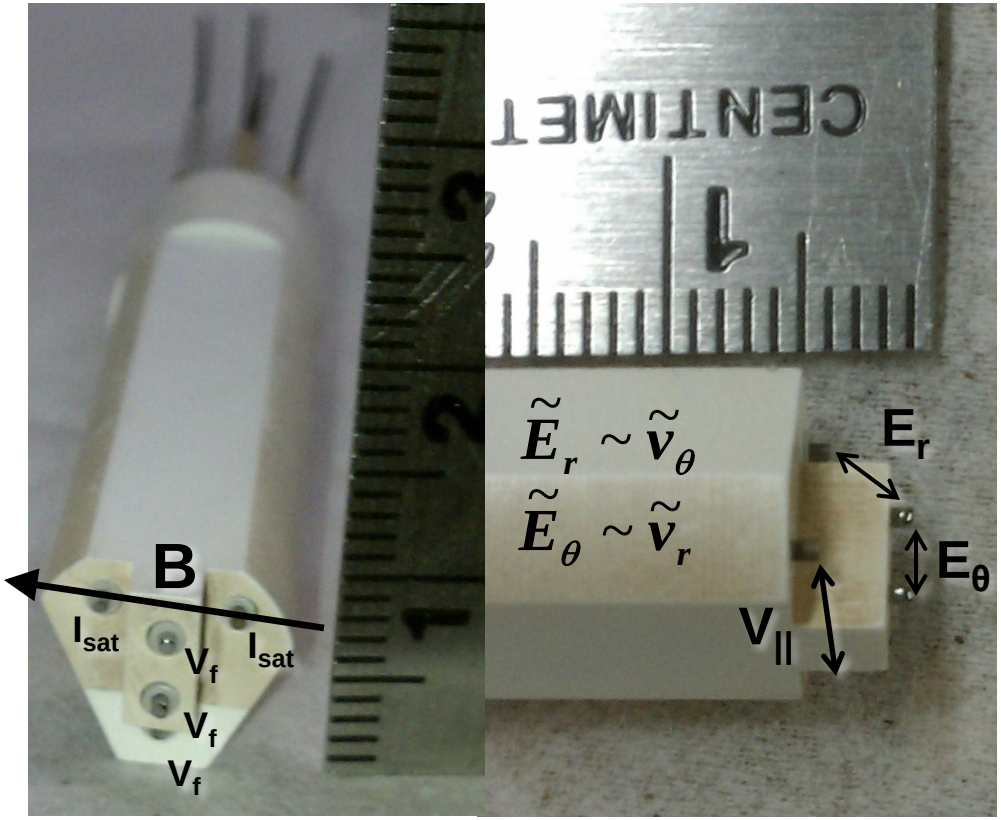


[P. Helander and A.N. Simakov, PRL 2008]

Langmuir probes installed to measure local fluctuating plasma parameters for Reynolds stress studies

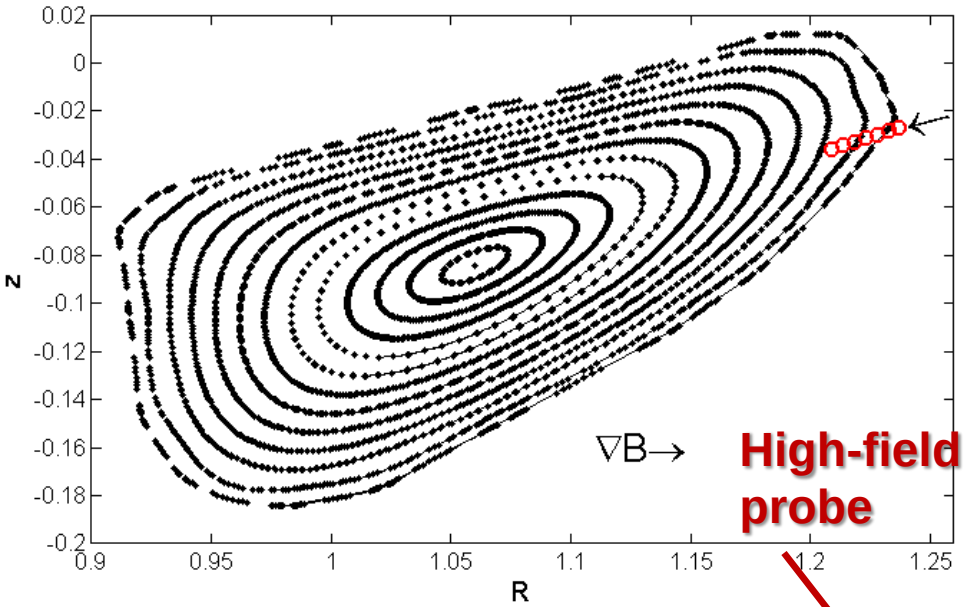
- Tungsten tips shielded by bulk BN, V_f and I_{sat} signals digitized at 2.5 MHz
- Radial profiles taken on a shot-by-shot basis
  - Discharges are very repeatable
- Differential V_f fluctuations are assumed to be potential fluctuations
  - Fluctuating T_e not accounted for
- Fluctuating v_r and v_θ quantities for Reynolds stress assumed from measured E_r and E_θ fluctuations, respectively

$$\tilde{v}_r \tilde{v}_\theta = -\frac{\tilde{E}_\theta \tilde{E}_r}{B^2}$$

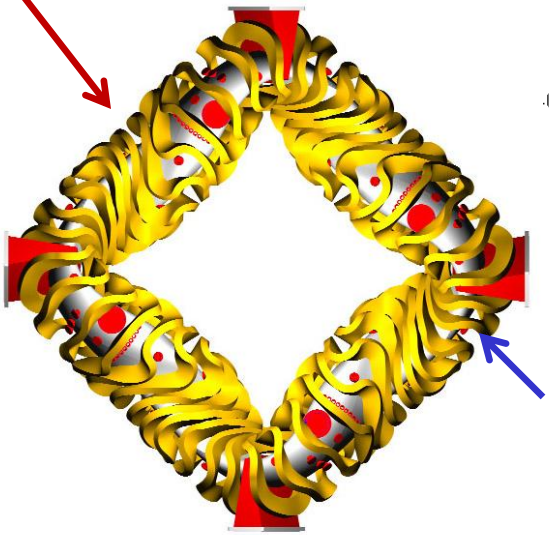


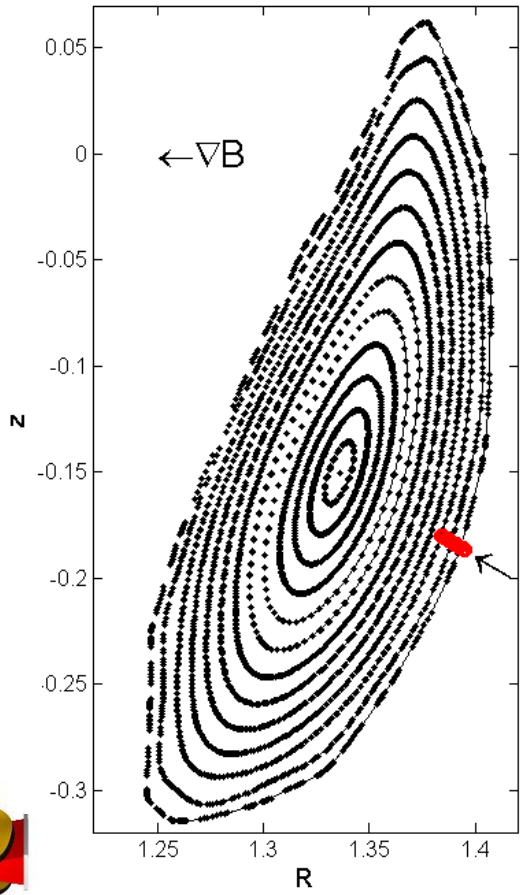
$$\tilde{E}_r = \frac{\tilde{V}_{f,2} - \tilde{V}_{f,3}}{\Delta r} \qquad \tilde{E}_\theta = \frac{\tilde{V}_{f,2} - \tilde{V}_{f,1}}{\Delta x_\theta}$$

Two probe installations to investigate how local Reynolds stress measurements vary on a flux surface



High-field side probe

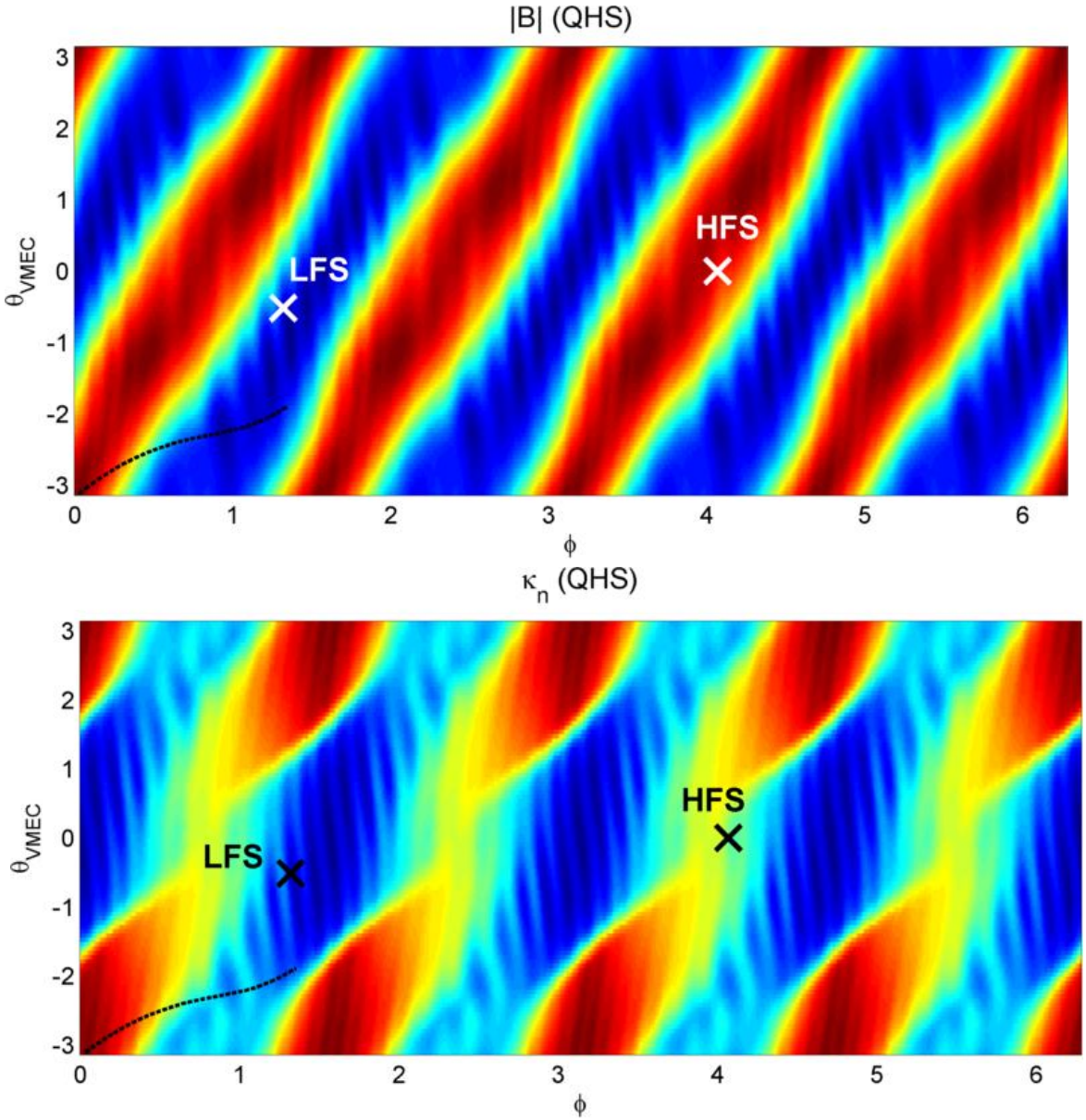




Low-field side probe

- **Probes located in regions of different magnetic field strength, curvature, and flux expansion on a flux surface**
- **Compare measurements at two locations with different local magnetic geometries**
  - **Flux surface average RS necessary for momentum balance**

Two probe installations to investigate how local Reynolds stress measurements vary on a flux surface

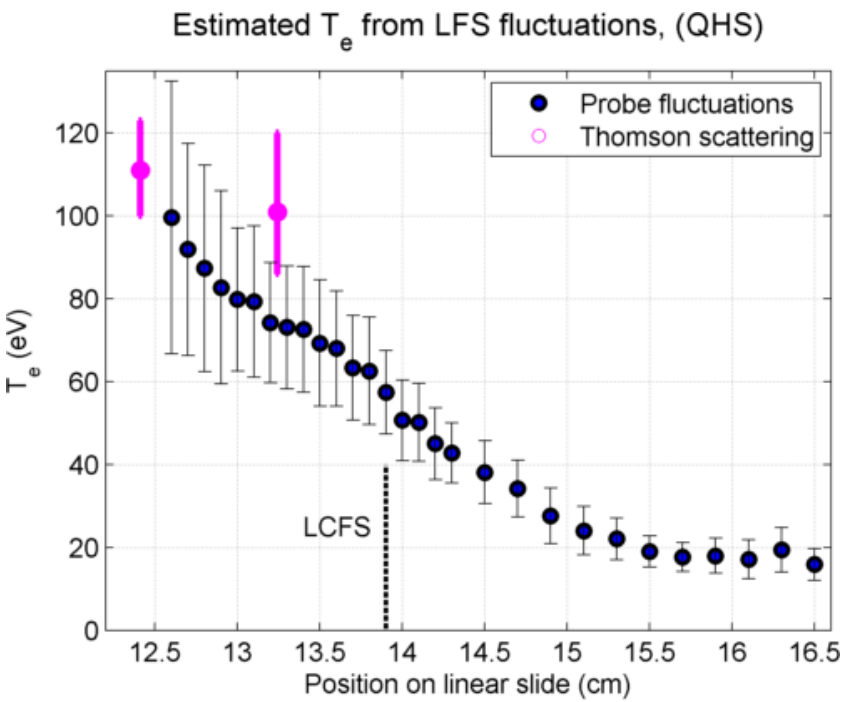


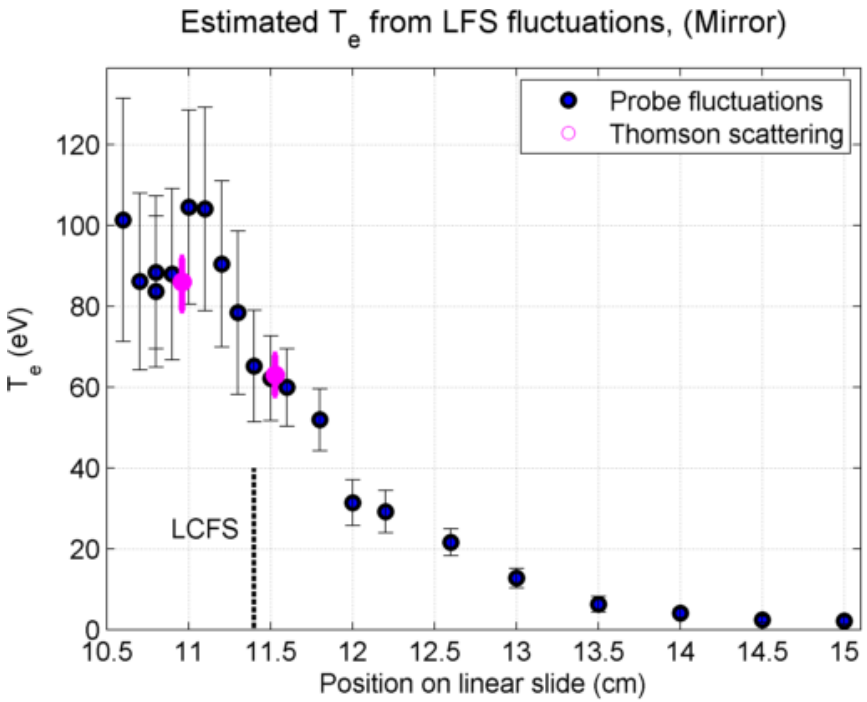
Radial electric field is calculated from gradient of floating potential measurement and an estimate of T_e from fluctuations

$$\Phi_{fl} = \Phi_p - \mu \frac{T_e}{e} \qquad \frac{\tilde{n}}{n} = e^{e\tilde{\phi}/T_e} - 1$$

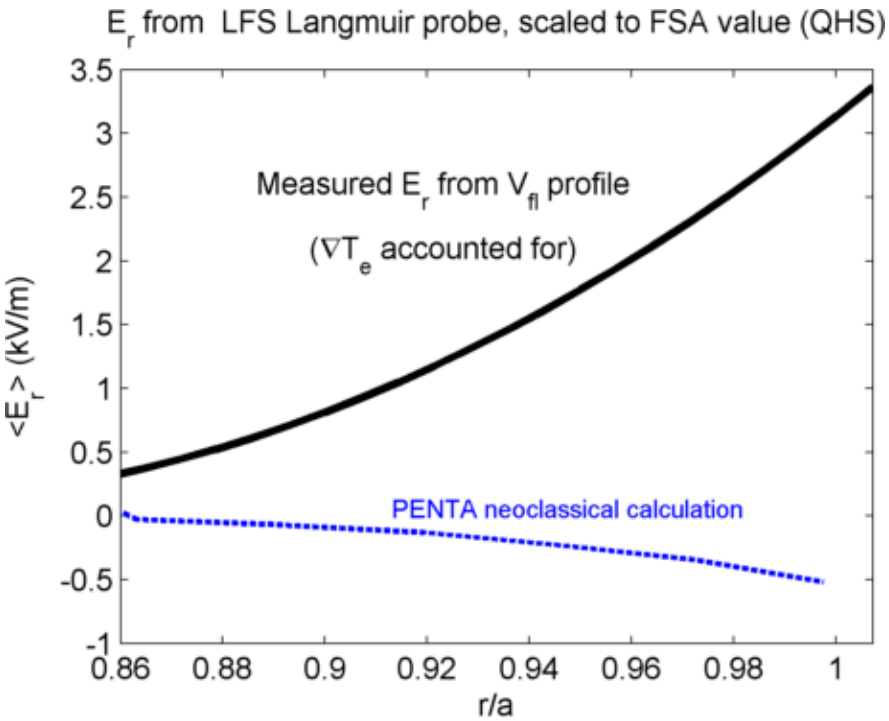
$$\frac{T_e}{e} \approx \frac{\tilde{\phi}}{\ln\left(\frac{\tilde{n}}{n} + 1\right)}$$

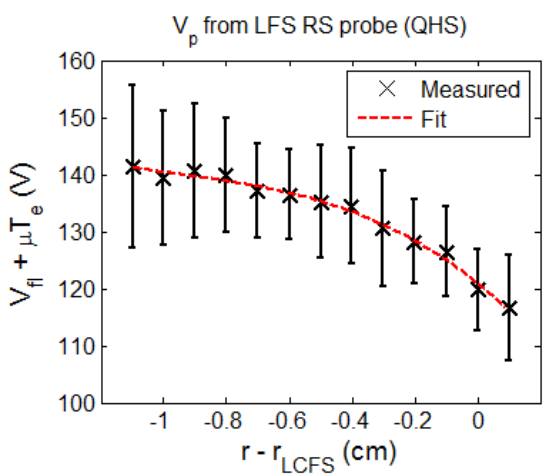
- Assuming adiabatic electron response and negligible T_e fluctuations, T_e estimate can be extracted from V_{fl}, I_{sat} fluctuations
  - Fluctuations are large, $\tilde{n}/n \approx 20\%$
- Calculating T_e in this way agrees well with Thomson scattering in the edge, has better spatial localization and tracks small shot-to-shot changes in plasma





Radial electric field measured by probes deviates from PENTA calculations using neoclassical ambipolarity constraint





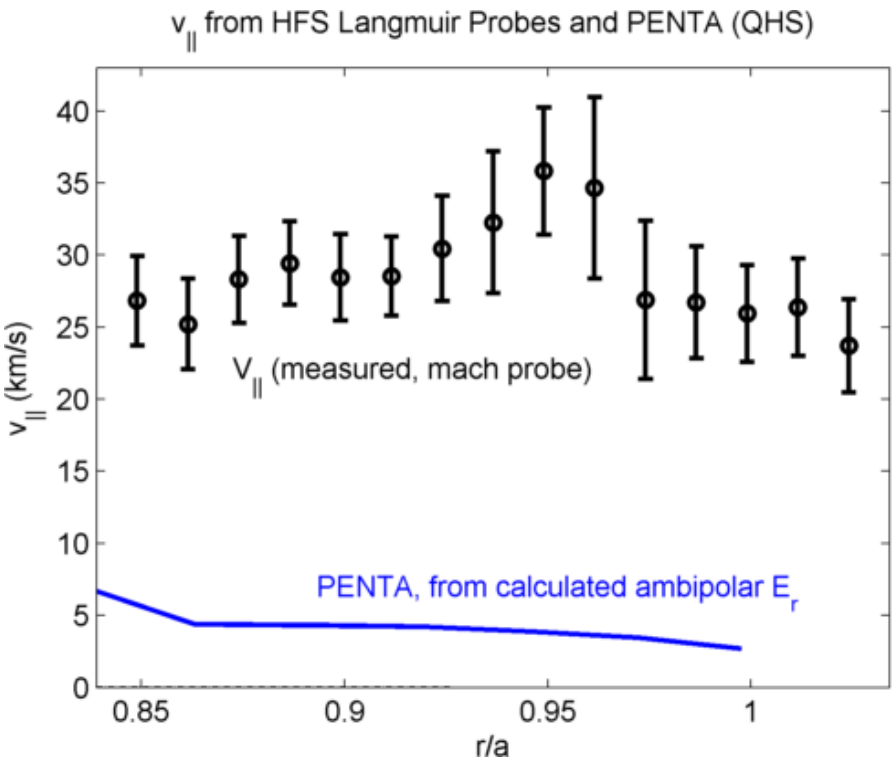
- **E_r calculated from gradient of shot-by-shot floating potential profile**
- **T_e gradient is accounted for based on estimate of T_e from fluctuating quantities**

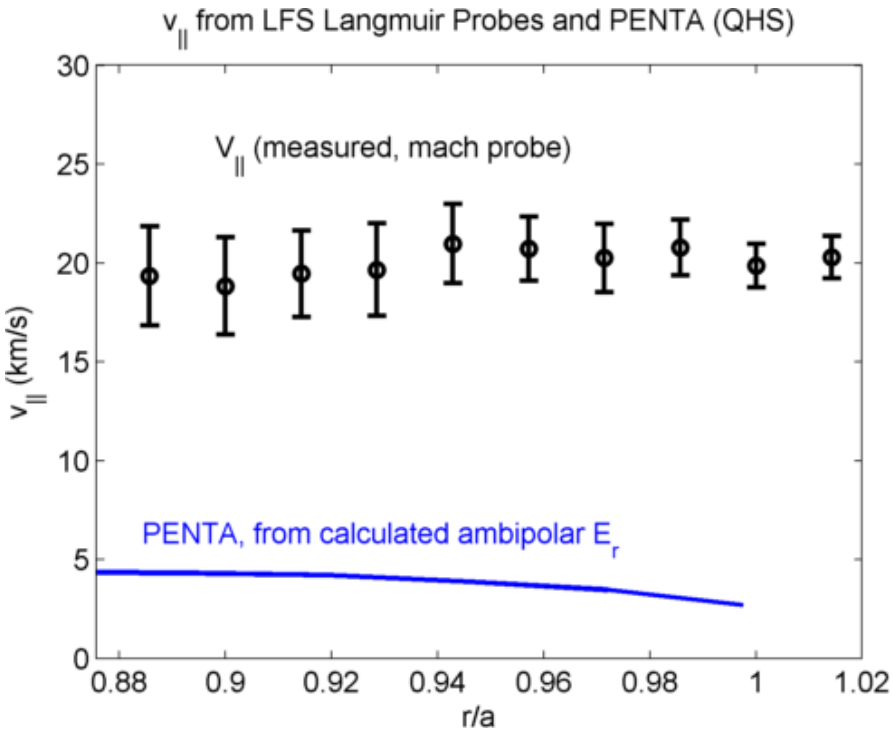
$$V_{fl} = \Phi_p - \mu \frac{T_e}{e}$$

- **Neoclassical calculations have small errorbars relative to measured deviations of E_r for r/a>0.9**
  - **Changes to input parameters (T_e, T_i, n_e) within measurement errorbars make only minor changes to calculated E_r (<~0.5 kV/m)**

Measurement of $V_{||}$ is another indication that E_r and flows are not consistent with neoclassical calculations

- **Edge $V_{||}$ measured using mach probe**
- **Measured $V_{||}$ is significantly larger than value predicted by PENTA for ambipolar E_r solution**





- **Small island near r/a≈0.95 at HFS probe location**
- **Pfirsch-Schluter ion flows account for different measured $V_{||,i}$ at two probe locations**

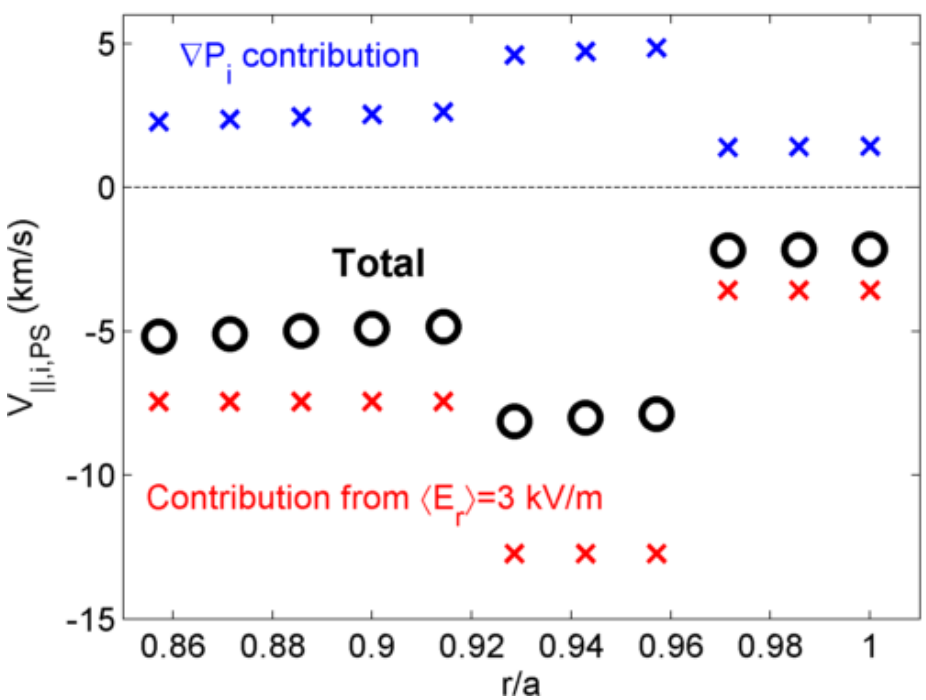
Calculated Pfirsch-Schluter flows account for measured difference in $V_{||,i}$ at two probe locations

- **Measured parallel ion flows include contribution of Pfirsch-Schluter flows**
  - **Both E_r and ∇P_i terms are relevant**

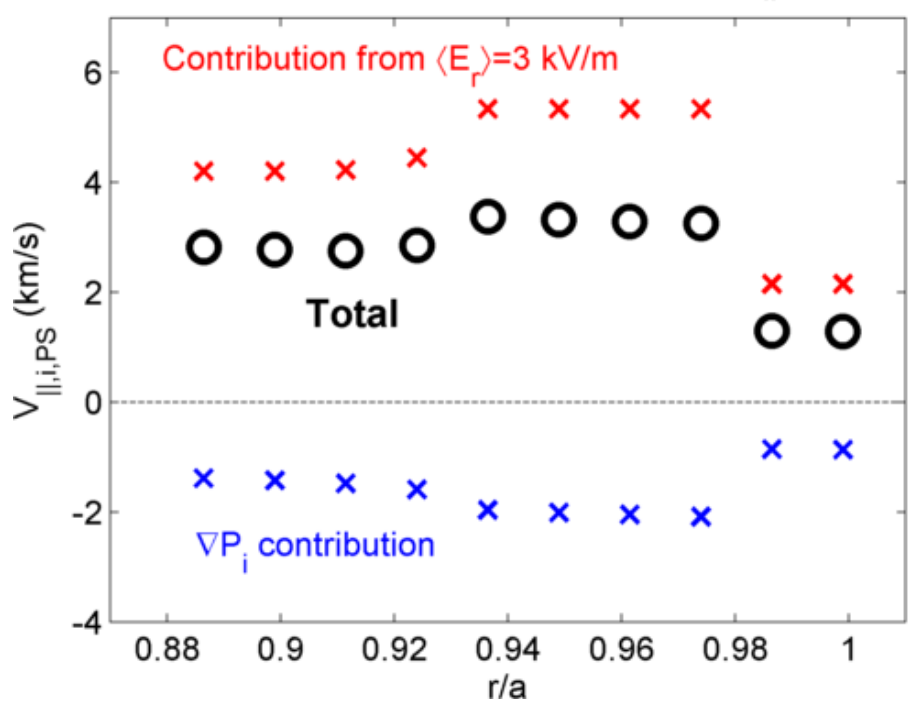
$$h = c\frac{B_\theta}{B^\zeta B^2}$$

$$V_{||,i,PS} = \left(E_r + \frac{1}{en_i}\nabla p_i\right) h\mathbf{B}$$

LFS Pfirsch-Schluter contribution to $V_{||,i}$ (QHS)



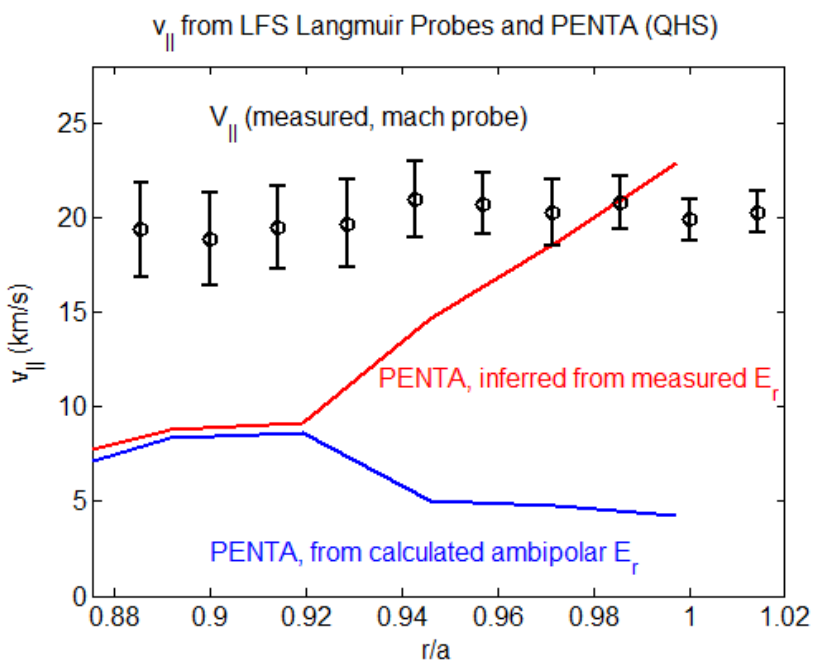
HFS Pfirsch-Schluter contribution to $V_{||,i}$ (QHS)

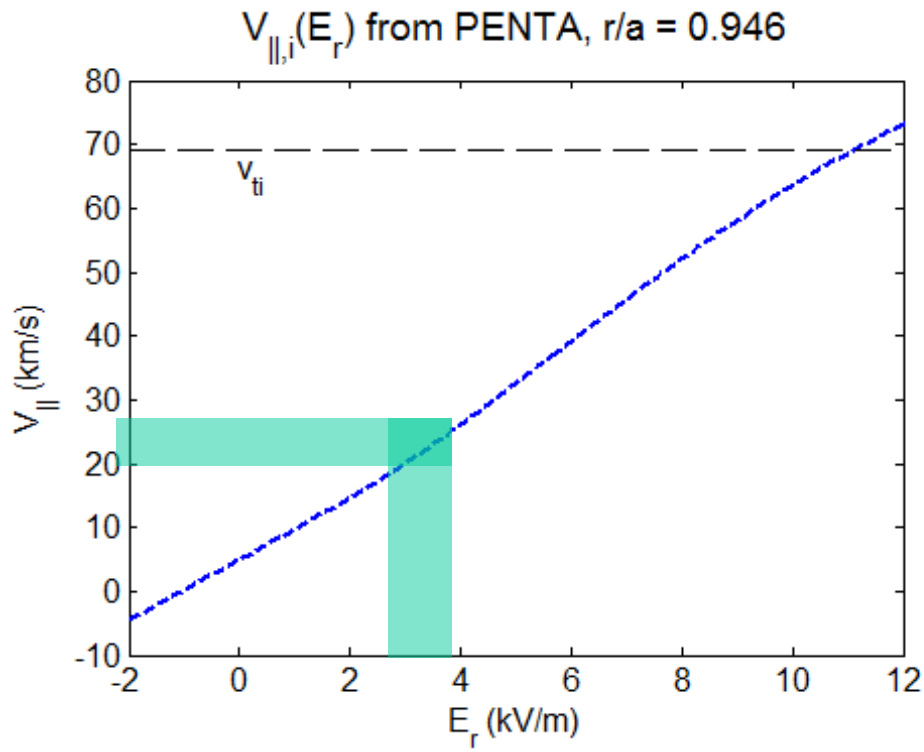


Measured parallel flows consistent with larger E_r than measured using shot-by-shot V_{fl} profile

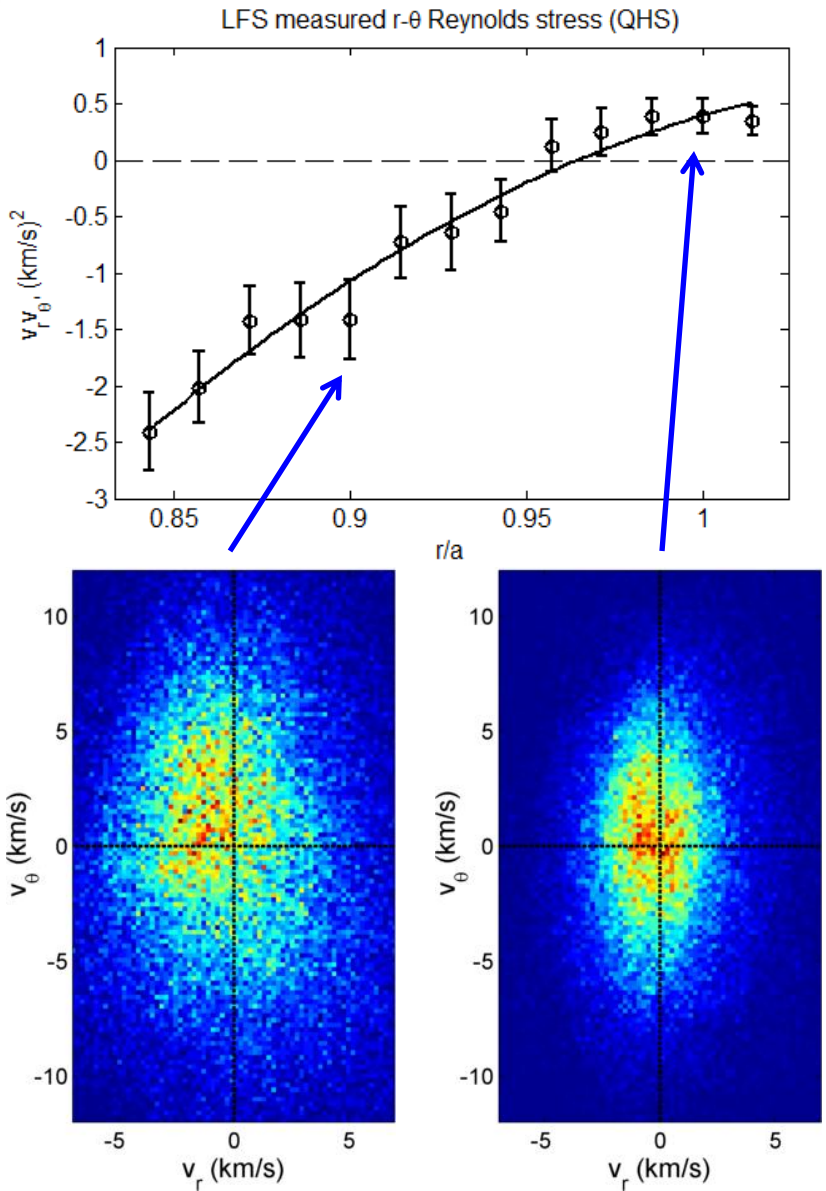
- $V_{||}$ can be calculated from known E_r using PENTA
  - CHERS measurements in previous experiments have shown that flows follow direction of symmetry
    [A. Briesemeister et. al., CPP 2010]

- Measurements of $V_{||}$ are consistent with higher E_r than measured using V_{fl} method





Langmuir probes measure radial gradient in time-averaged Reynolds stress profile, implying flow drive



- **Flow drive is proportional to Reynolds stress gradient**

$$\left(\frac{\partial v_\theta}{\partial t}\right)_{RS} \propto \frac{\partial}{\partial r}\langle \tilde{v}_r \tilde{v}_\theta \rangle$$

- **PDF of velocity fluctuations (inferred from V_f measurements) show negative correlation between instantaneous v_r and v_θ fluctuations at interior measurement points**
  - **Fluctuations are more isotropic near the last closed flux surface**

- **Measurement needs to be compared to the calculated poloidal viscosity to determine resulting rotation and E_r**
  - **Magnitude can be compared to actual deviation from neoclassical ambipolar calculation to see if flow drive is relevant**

Estimate of neoclassical poloidal viscosity is calculated to find contribution of measured Reynolds stress to flows and E_r

$$\frac{\partial}{\partial t}\langle v_\theta\rangle = 0 = -\frac{\partial}{\partial r}\left(\langle \tilde{v}_r \tilde{v}_\theta\rangle\right) - \mu_\theta \langle v_\theta\rangle - \nu_{in}\langle v_\theta\rangle + \tau_{ext}$$

$$\langle v_\theta\rangle_{RS} = -\left(\frac{1}{\mu_\theta + \nu_{in}}\right)\frac{\partial}{\partial r}\left(\langle \tilde{v}_r \tilde{v}_\theta\rangle\right)$$

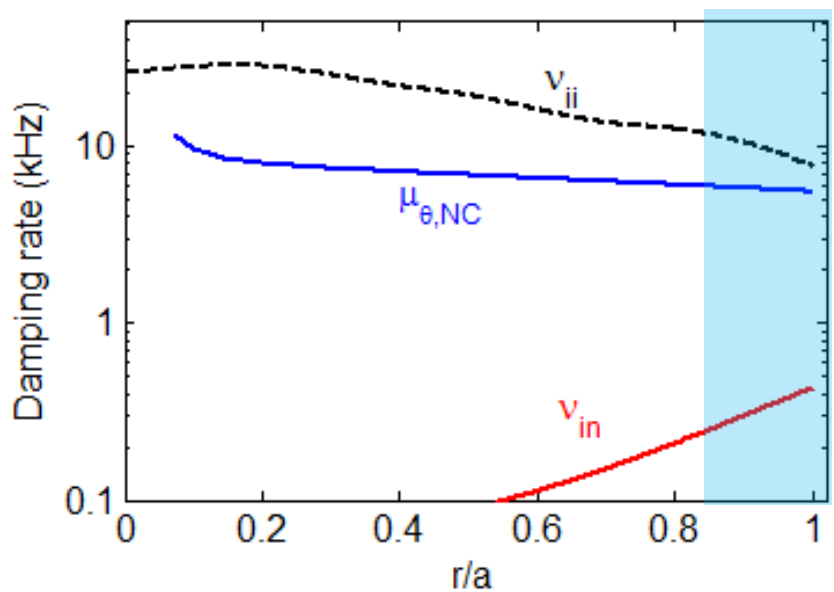
- **Estimate the purely poloidal component of neoclassical viscosity for poloidal momentum balance**
  - **Coupling of E_r and $V_{||}$ not accounted for in these calculations**

- **Neutral density based on measurements from H-alpha arrays and calculations from DEGAS neutral gas code**

$$\upsilon_{in} \approx N_n 10^{-14} T_i^{0.318}$$

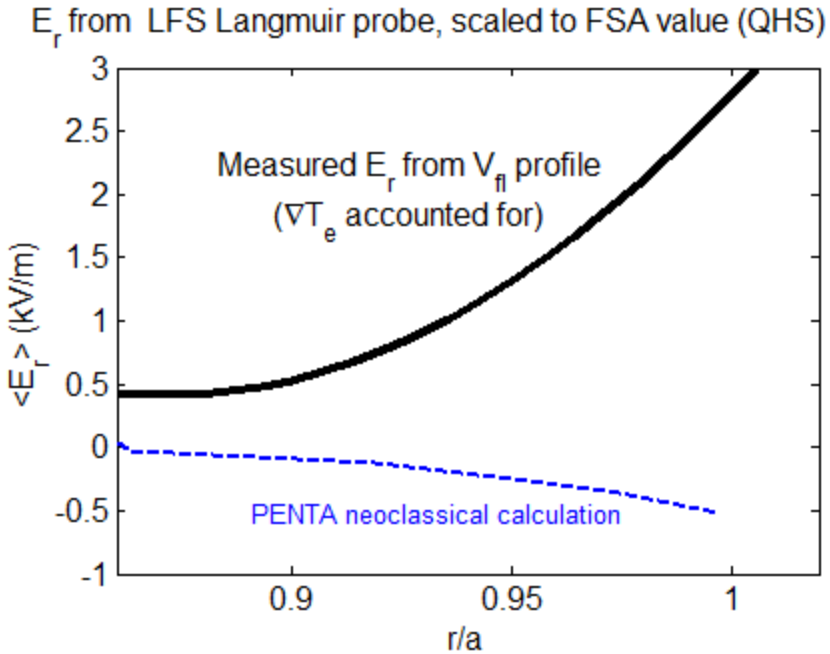
$$\mu_\theta = \frac{1}{4\pi^{1/2}}\frac{v_{ti} R_0}{r^2}\sum_{n,m}\frac{m^2 b_{n,m}}{|n - \iota m|}$$

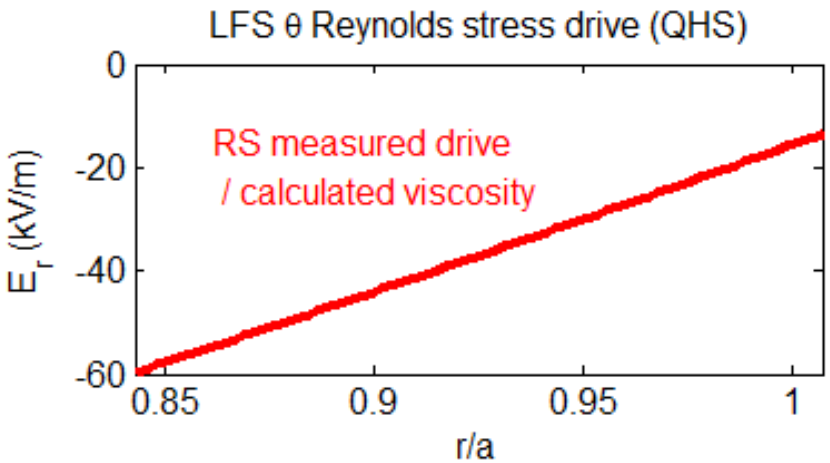
[M. Coronado and H. Wobig, *Phys. Fluids B* 1986]



- **Neutral damping is small but non-negligible at the edge where probe measurements are made**

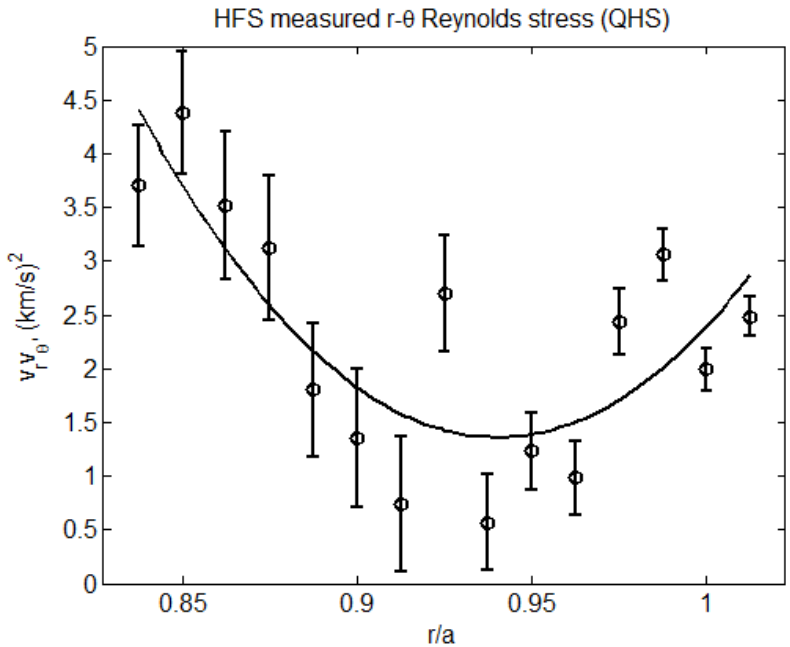
Large Reynolds stress flow drive measured in edge directed in opposite direction compared to observed flow





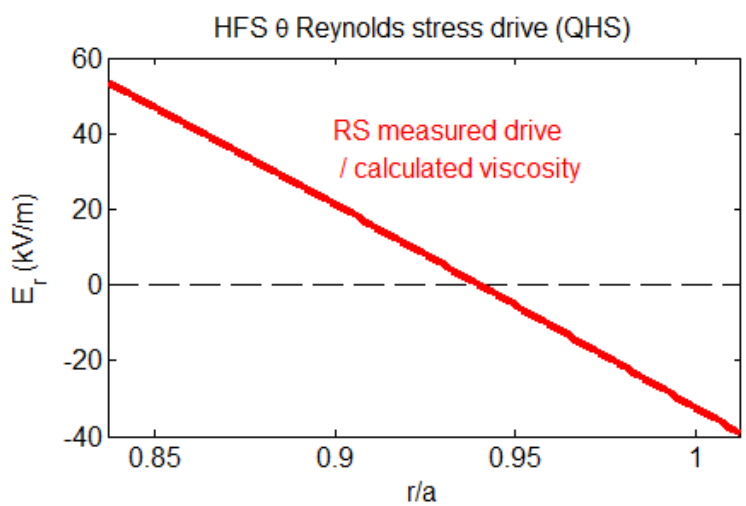
- **Large Reynolds stress gradient exists in same radial region as measured deviation of E_r and $V_{||}$**
- **Using viscosity estimate, perpendicular flow and E_r resulting from measured Reynolds stress can be calculated**
  - **Assumes that measurement can be extrapolated to a flux surface average**
- **Reynolds stress measurements imply flow drive much larger than observed flows**
  - **Single point measurement (should be flux surface average)**
  - **T_e fluctuations observed through V_f may lead to over-estimation of fluctuating φ** [Gennrich & Kendl, PPCF 2012]

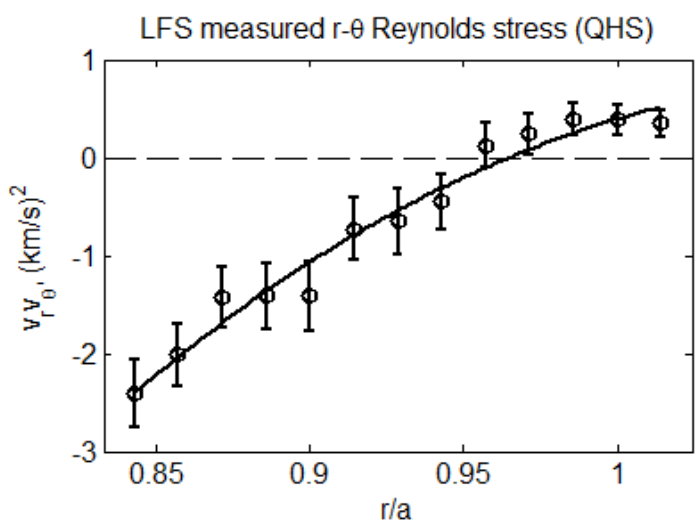
High field side probe measures a Reynolds stress profile qualitatively different to measurements at low field side probe



- **Reynolds stress measured on high-field side is generally larger magnitude, but local flux expansion is larger, so drive term would be similar if extrapolated to flux surface average**

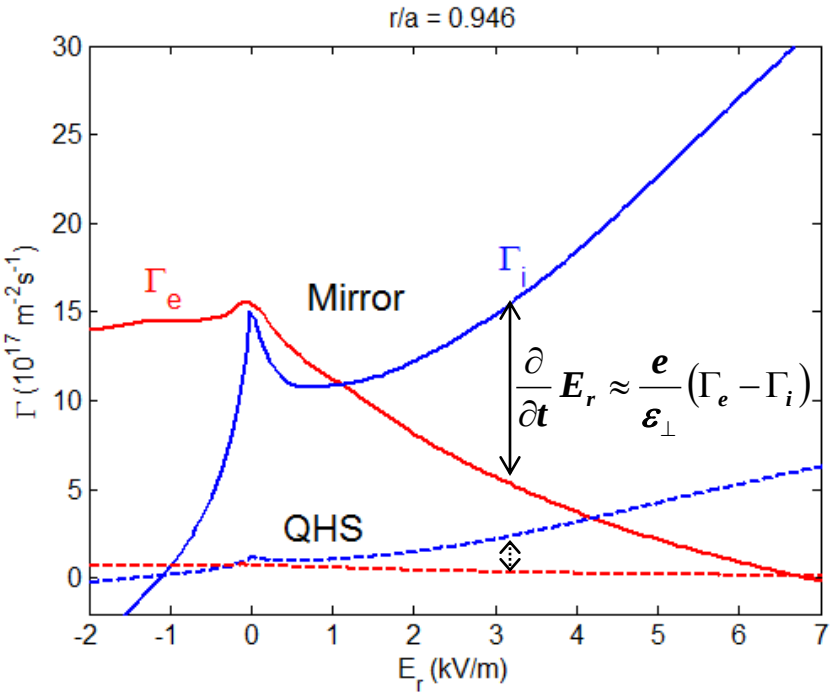
- **Transforming the measured Reynolds stress from 2 locations on a surface to a flux surface averaged value probably requires gyrokinetic simulations**

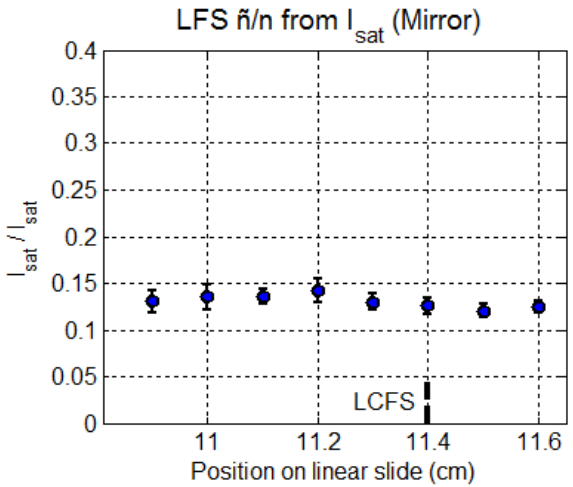


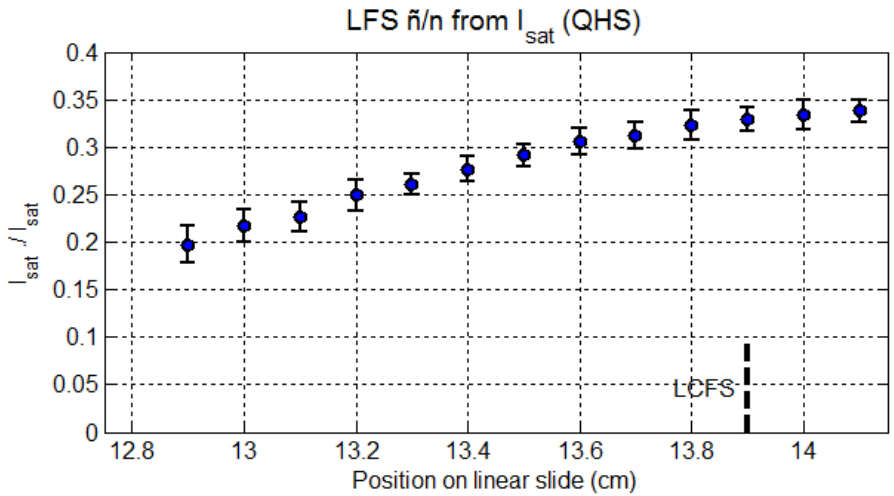


Expectation is that flows and E_r in Mirror configuration should match more closely with neoclassical calculations

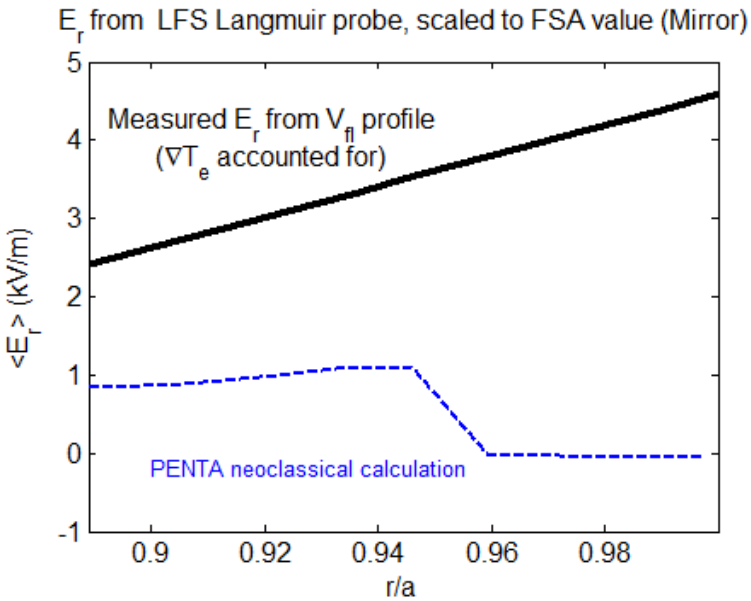
- **No longer a direction of symmetry with reduced neoclassical viscosity**
- **Much more neoclassical radial current for non-ambipolar E_r solution**
- **Normalized fluctuations are smaller in Mirror than in QHS**
  - **Reynolds stress would be expected to scale with ~(ñ/n)²**

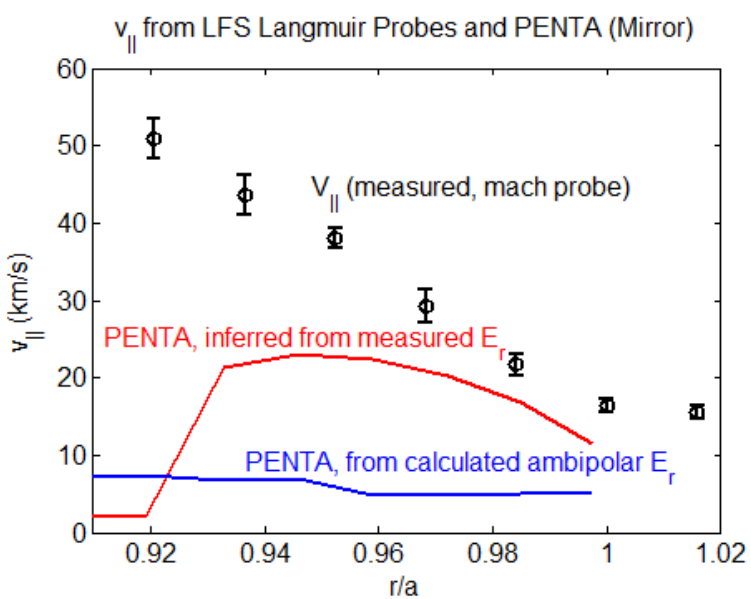




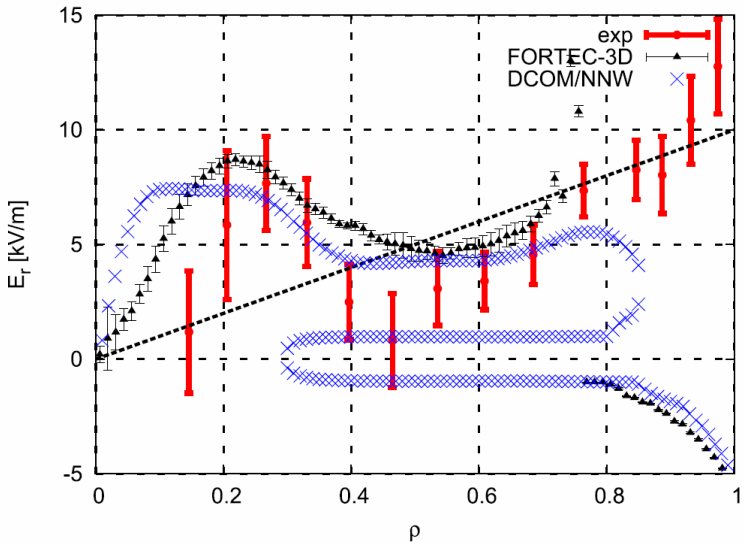


Large deviation from neoclassical calculations observed in Mirror configuration despite similar Reynolds stress drive

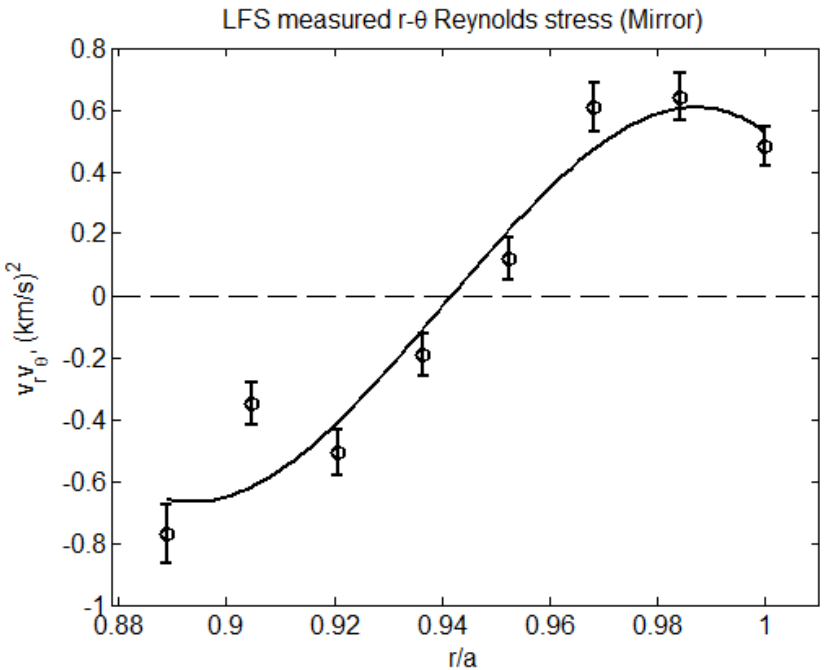


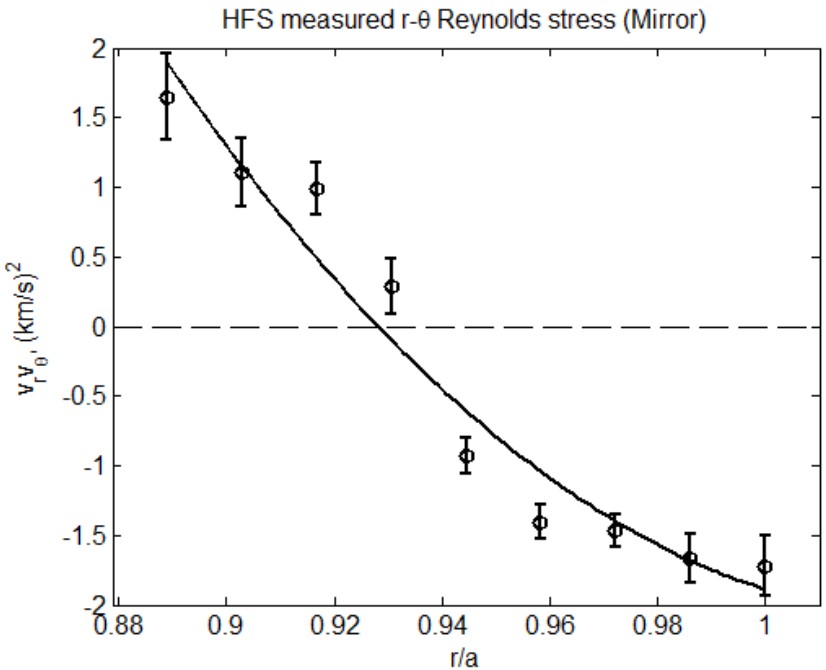


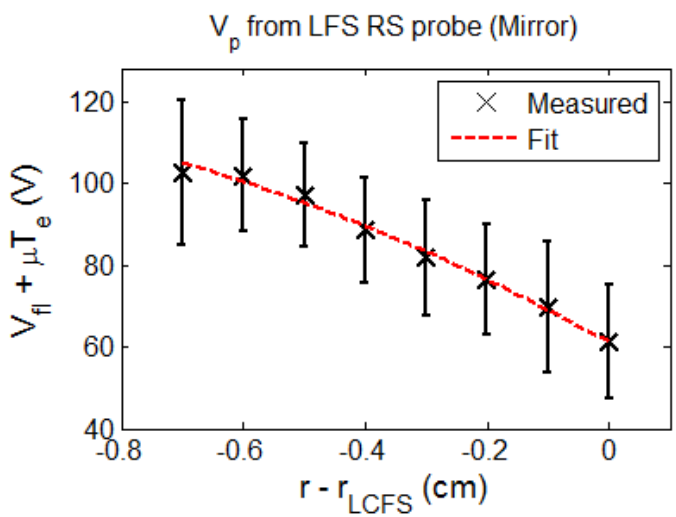
- **Despite breaking of quasi-symmetry, larger deviation of E_r and $V_{||}$ from neoclassical calculations is measured in Mirror configuration than QHS**
- **Physics of edge transport may not be captured by DKES/PENTA modelling**
  - **Non-diffusive particle transport may be significant (collisionless loss orbits)**
  - **LHD also measures edge E_r much larger than what is calculated by NC codes near last closed flux surface**



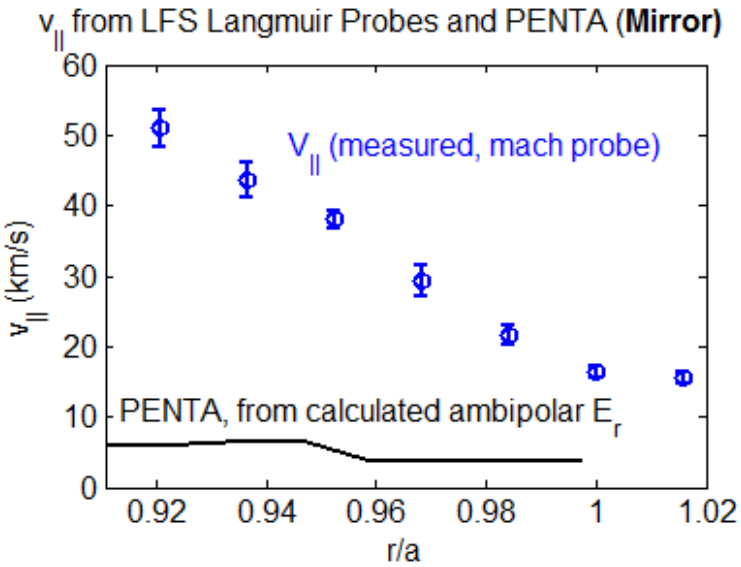
Measured Reynolds stress gradient in Mirror is not significantly larger than in QHS, does not explain additional rotation

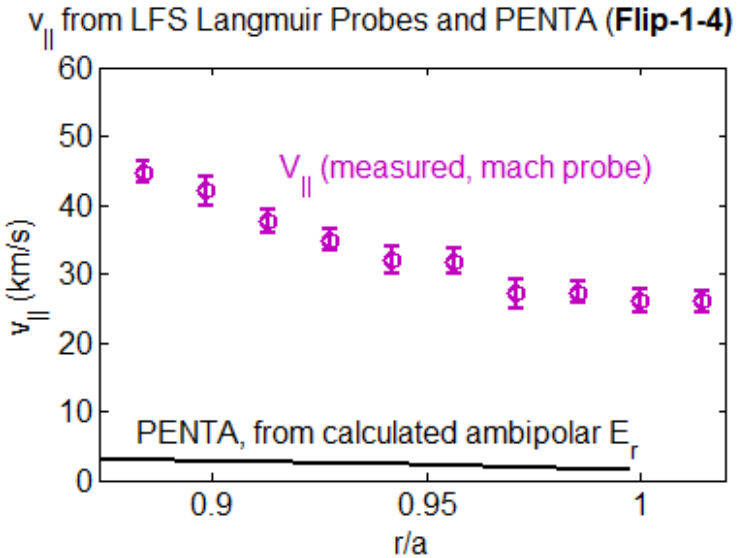




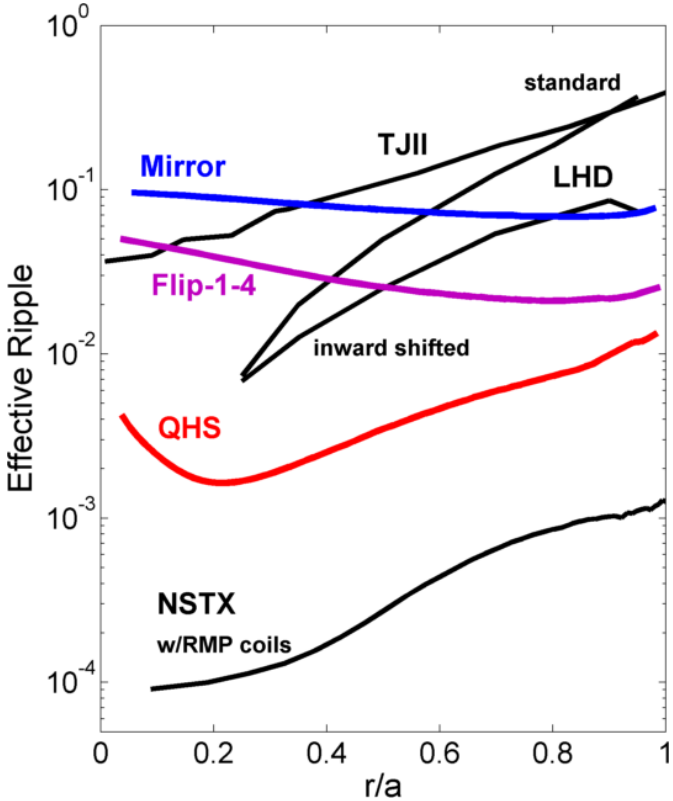


Smaller flows observed in Flip-1-4 configuration with moderate effective ripple than in Mirror configuration





- **Flip-1-4 edge effective ripple between that of QHS and Mirror configurations**
- **Parallel flows and E_r smaller than Mirror configuration inside plasma**



Summary

- The measured radial electric field and parallel ion flow in the edge of QHS configuration of HSX deviates from the purely neoclassical values calculated using PENTA

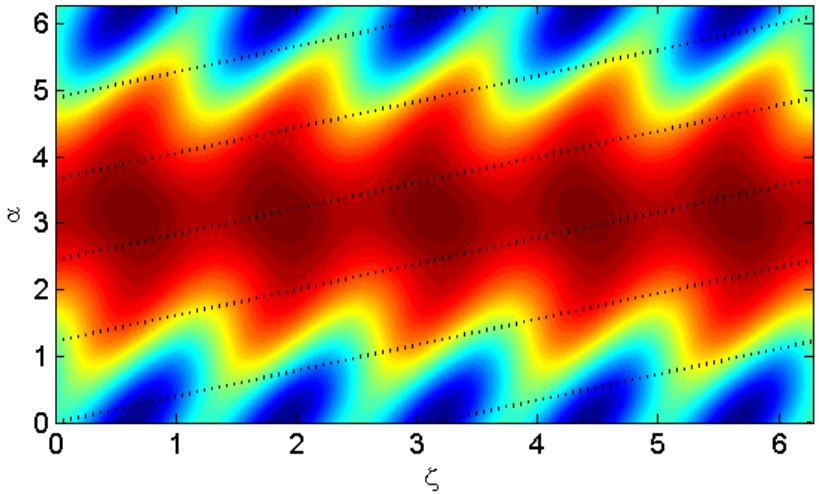
- Fluctuations measured using multiple Langmuir probes indicate a large Reynolds stress gradient, but local values vary significantly on a surface
  - Flux surface average needed for inclusion in momentum balance
  - Unable to determine causal link between Reynolds stress fluctuations and observed flows based on two points on a surface

- Contrary to expectations, measurements in configurations with symmetry broken do not match more closely with neoclassically calculated flows
  - Edge flows larger as ripple is increased, despite reduction in fluctuations
  - Similar result as LHD experiments, where edge E_r is more positive than what is calculated using neoclassical codes

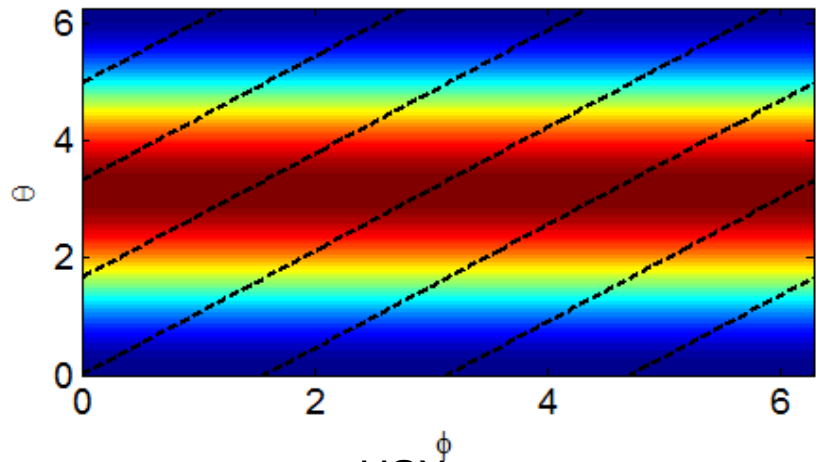
Extra slides

Non-quasi-symmetric stellarators have no direction of symmetry in |B|, which leads to large viscosity in all directions

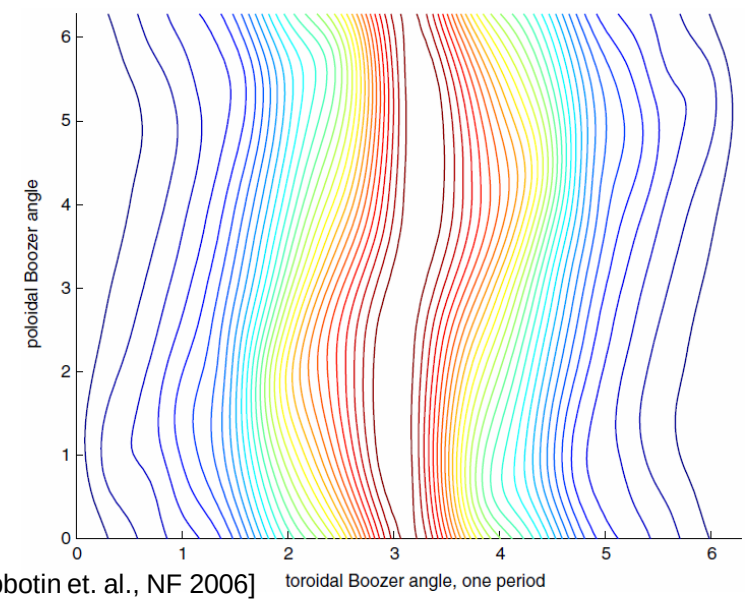
Conventional stellarator WEGA
|B| on a flux surface



Axisymmetric tokamak |B| on a flux surface

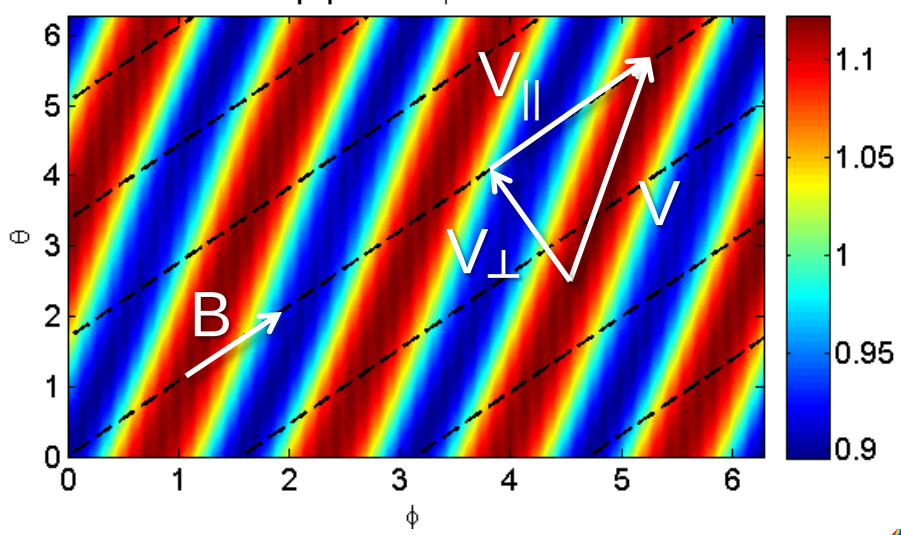


W7-X

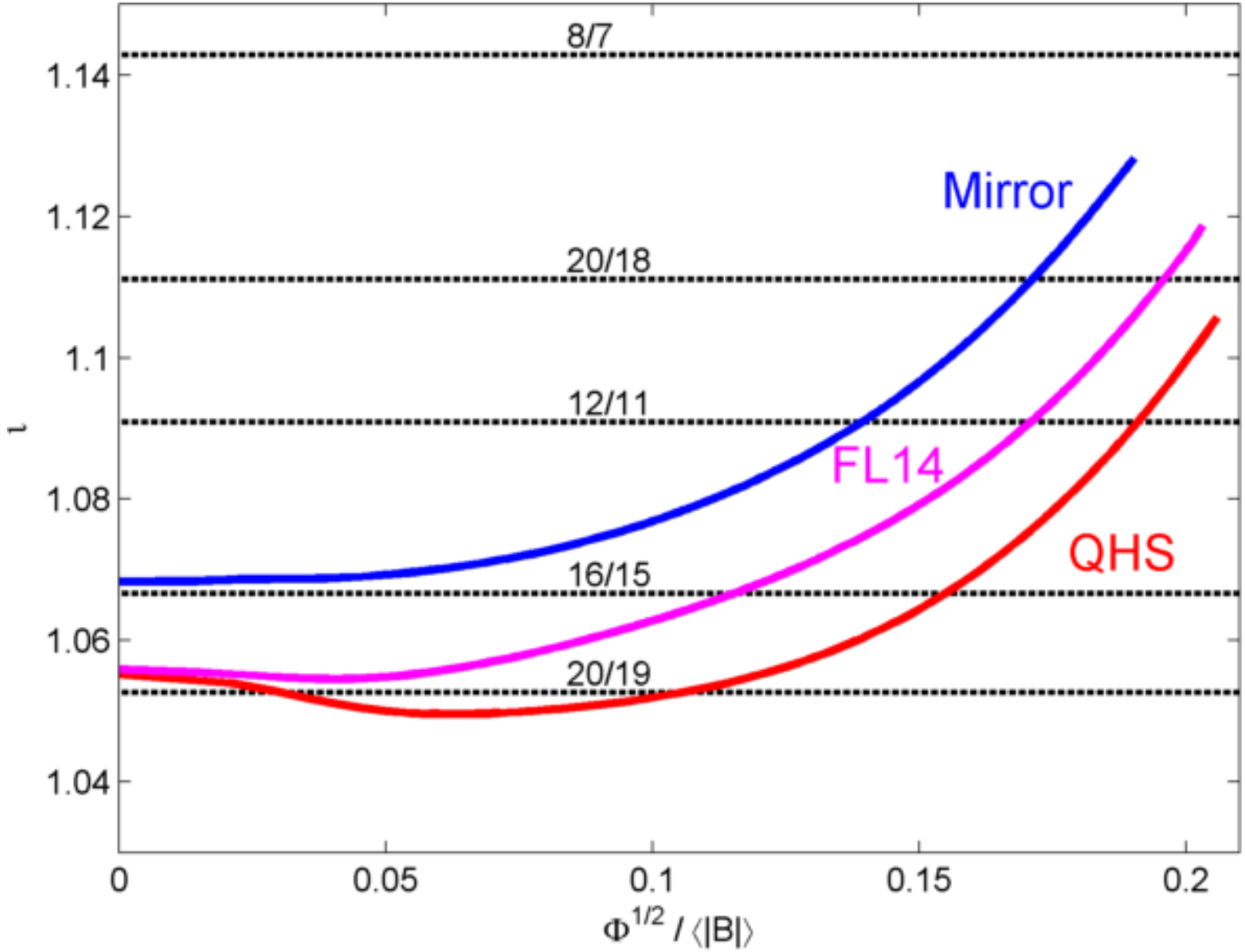


[A. A. Subbotin et. al., NF 2006]

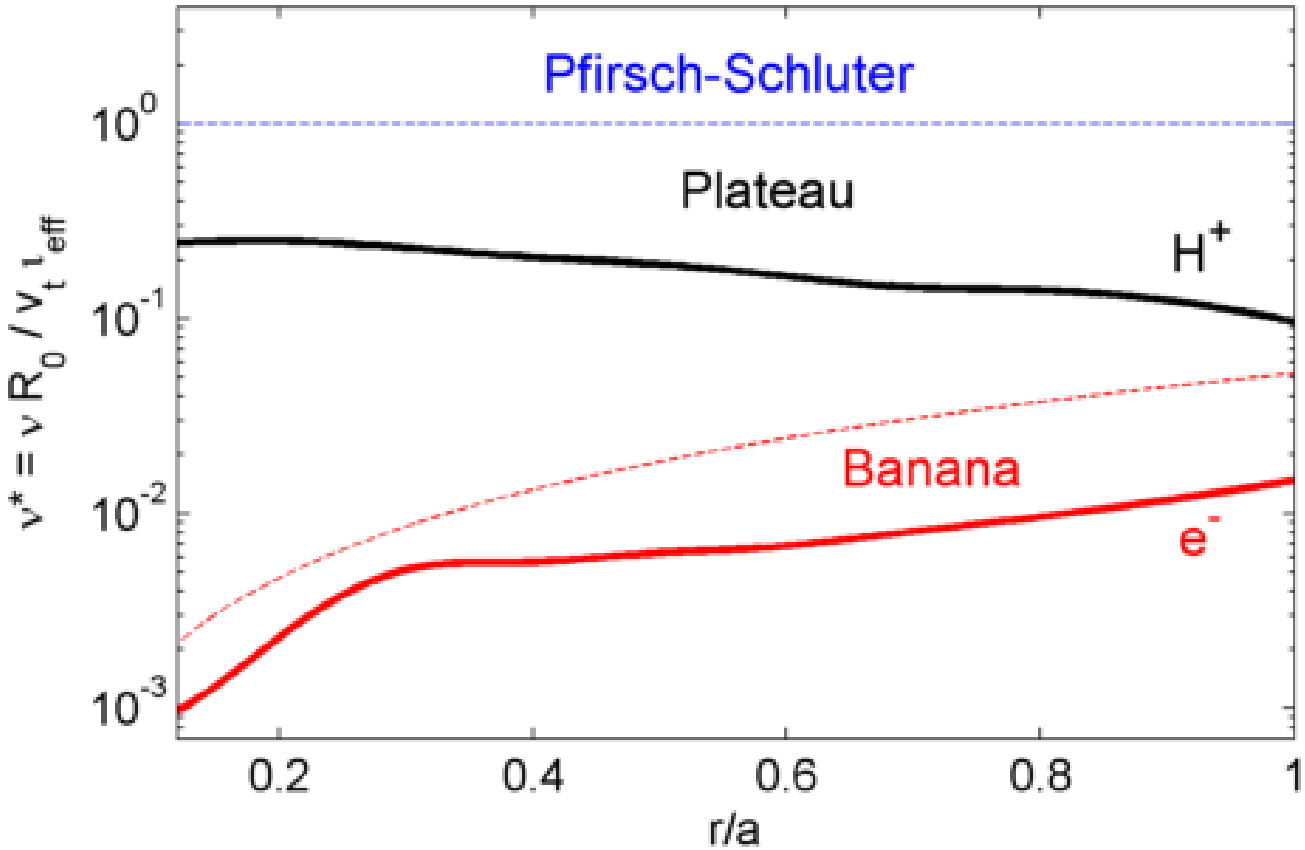
HSX
|B| on surf ρ = 0.8



There are no large islands or low order rational surfaces in the confinement region in any of the configurations used here



Electrons are in low collisionality regime and ions are in plateau regime throughout plasma



$$\upsilon_j^* = \frac{\upsilon_j R}{v_t l_{eff}}$$

For sufficiently optimized configurations, the Reynolds stress may compete with neoclassical viscosity to determine E_r

- **Momentum equation summed over all species with no externally injected momentum:**

$$\vec{J}\times\vec{B}-\nabla p-\frac{\partial\left(\rho\vec{V}\right)}{\partial t}=\nabla\cdot\left(\rho\vec{V}\vec{V}+\vec{\vec{\pi}}\right)$$

Reynolds stress Viscosity

- **Decompose J and B into equilibrium and fluctuating components,**

$$\vec{J}=\vec{J}_0+\vec{J}_1 \qquad \vec{B}=\vec{B}_0+\vec{B}_1 \qquad \left(\vec{J}_0\times\vec{B}_0=\nabla p_0\right)$$

- **Scalar product in the direction of symmetry and average over a flux surface,** $\langle\cdots\rangle$ **, gives**

$$\cancel{\frac{\partial\left\langle\rho\vec{V}\cdot\hat{e}_{sym}\right\rangle}{\partial t}}=\left\langle\nabla\cdot\left(\rho\vec{V}\vec{V}+\vec{\vec{\pi}}+\cancel{\vec{\vec{M}}}\right)\cdot\hat{e}_{sym}\right\rangle$$

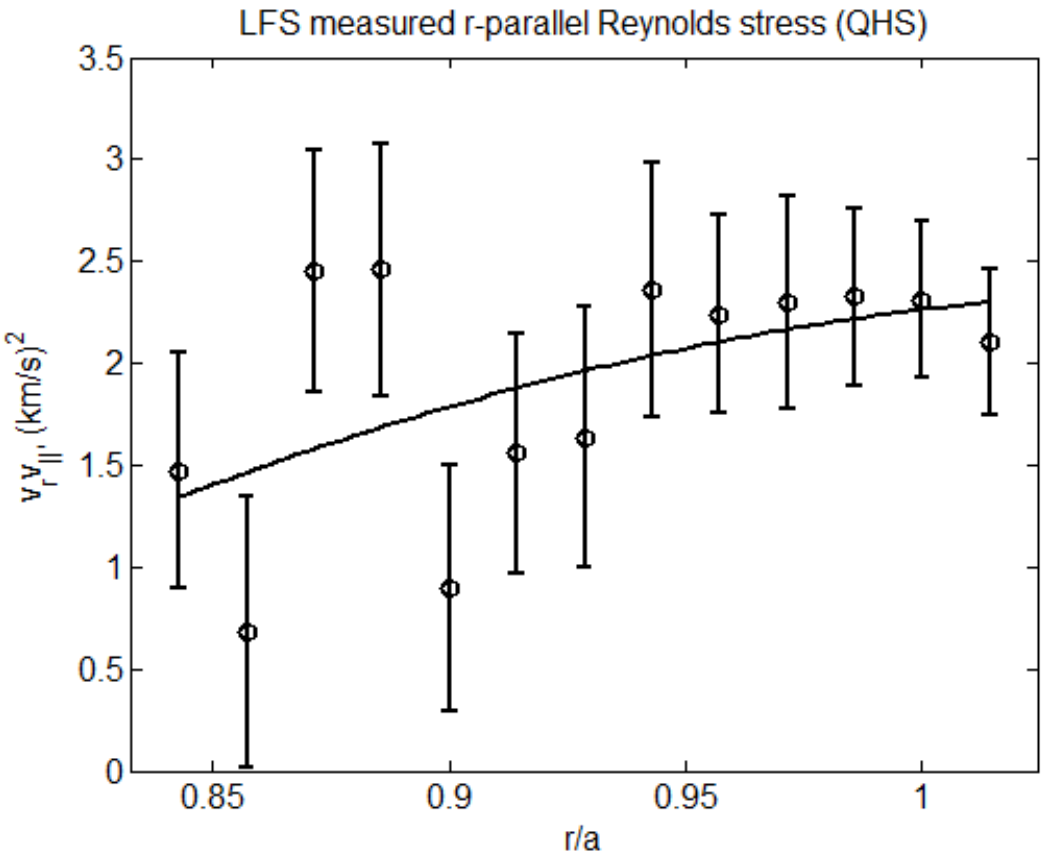
- **In equilibrium,**

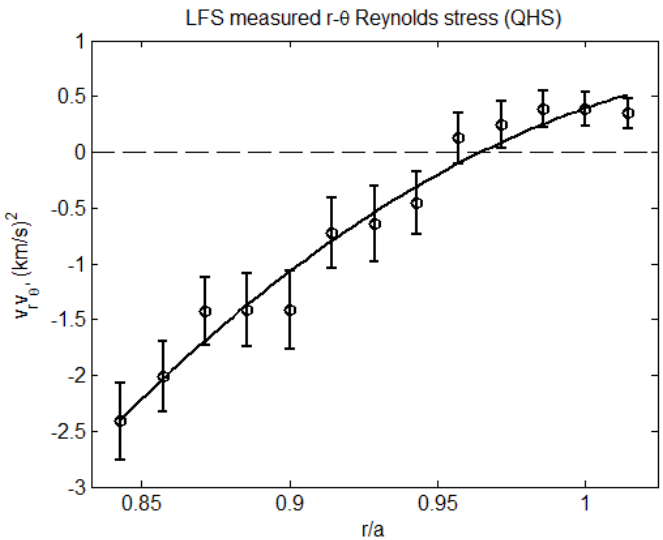
$$\left\langle\nabla\cdot\left(\rho\vec{V}\vec{V}+\vec{\vec{\pi}}\right)\cdot\hat{e}_{sym}\right\rangle=0$$

Reynolds stress can contribute (or dominate) if π_{NC} is small enough in some direction

Measured parallel Reynolds stress gradient is smaller and less consistent than bi-normal component

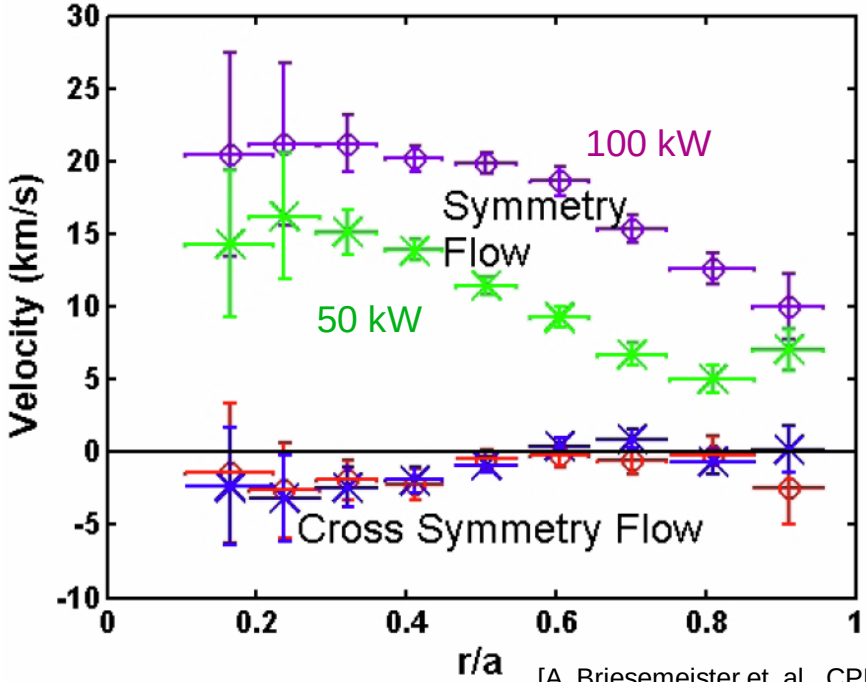
$$\langle \nabla \cdot (\tilde{\mathbf{V}}\tilde{\mathbf{V}}) \rangle_{probes} = \hat{b} \left\langle \frac{\partial}{\partial \rho} (\tilde{v}_\rho \tilde{v}_{\|}) \right\rangle + \hat{\theta} \left\langle \frac{\partial}{\partial \rho} (\tilde{v}_\rho \tilde{v}_\theta) \right\rangle$$





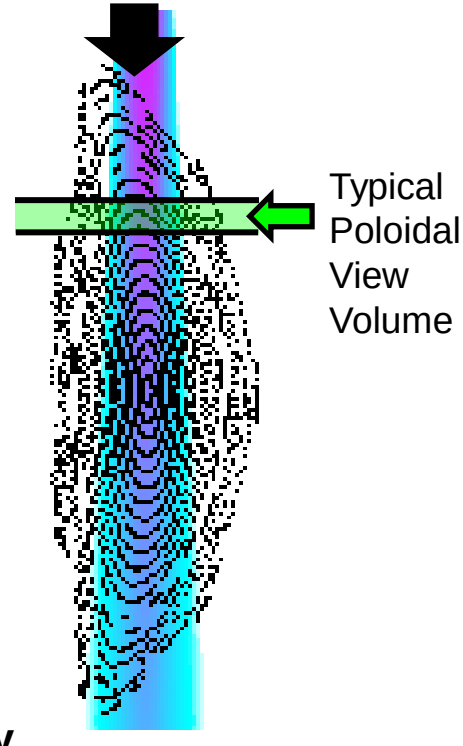
Using the CHERS diagnostic, large intrinsic flows have been measured to follow the direction of symmetry

Velocity Along and Across the Symmetry Direction



[A. Briesemeister et. al., CPP 2010]

Neutral Beam



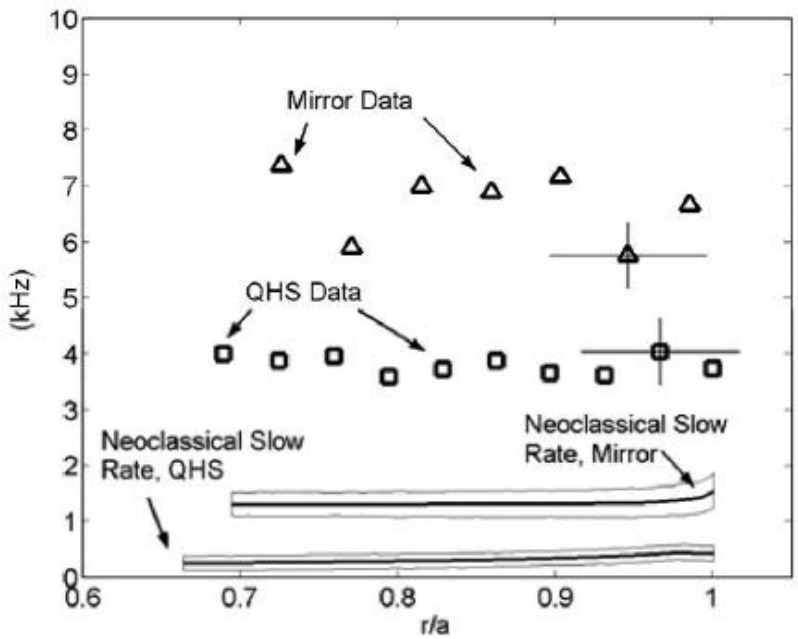
[courtesy A. Briesemeister]

- **30 keV diagnostic neutral beam launches H neutrals tangentially**
- **2 intersecting views measure the Doppler shift of light emitted from carbon impurity species stimulated by the beam**
- **Transforming to the local plasma coordinate system shows that flows are predominantly in the direction of symmetry**

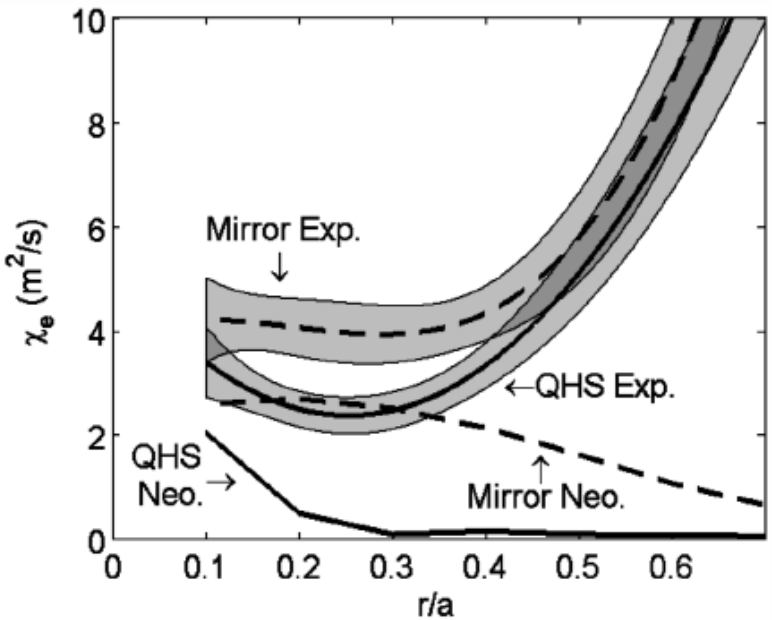
Some of the predicted beneficial effects on confinement due to quasi-symmetry have been demonstrated experimentally by HSX

- **Neoclassical direct-loss orbits and large excursions from magnetic surfaces are reduced**
  - **Reduced neoclassical particle and energy transport**
  - **Reduced non-ambipolar particle transport**

- **Reduced flow damping in the direction of symmetry**



[S. P. Gerhardt et. al., PRL 2005]

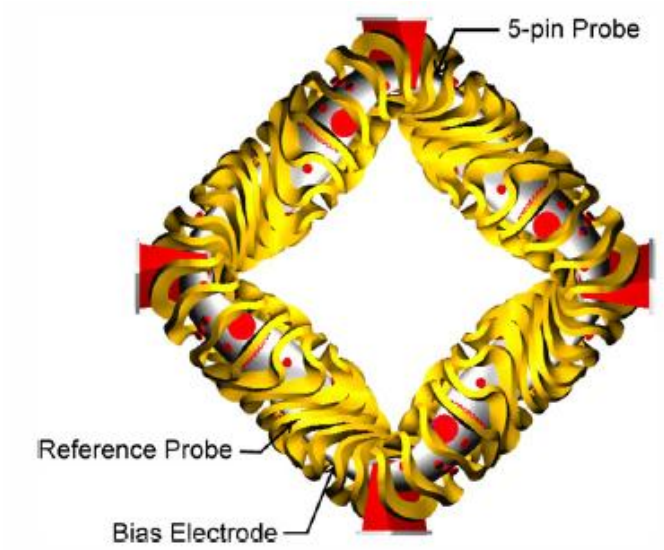


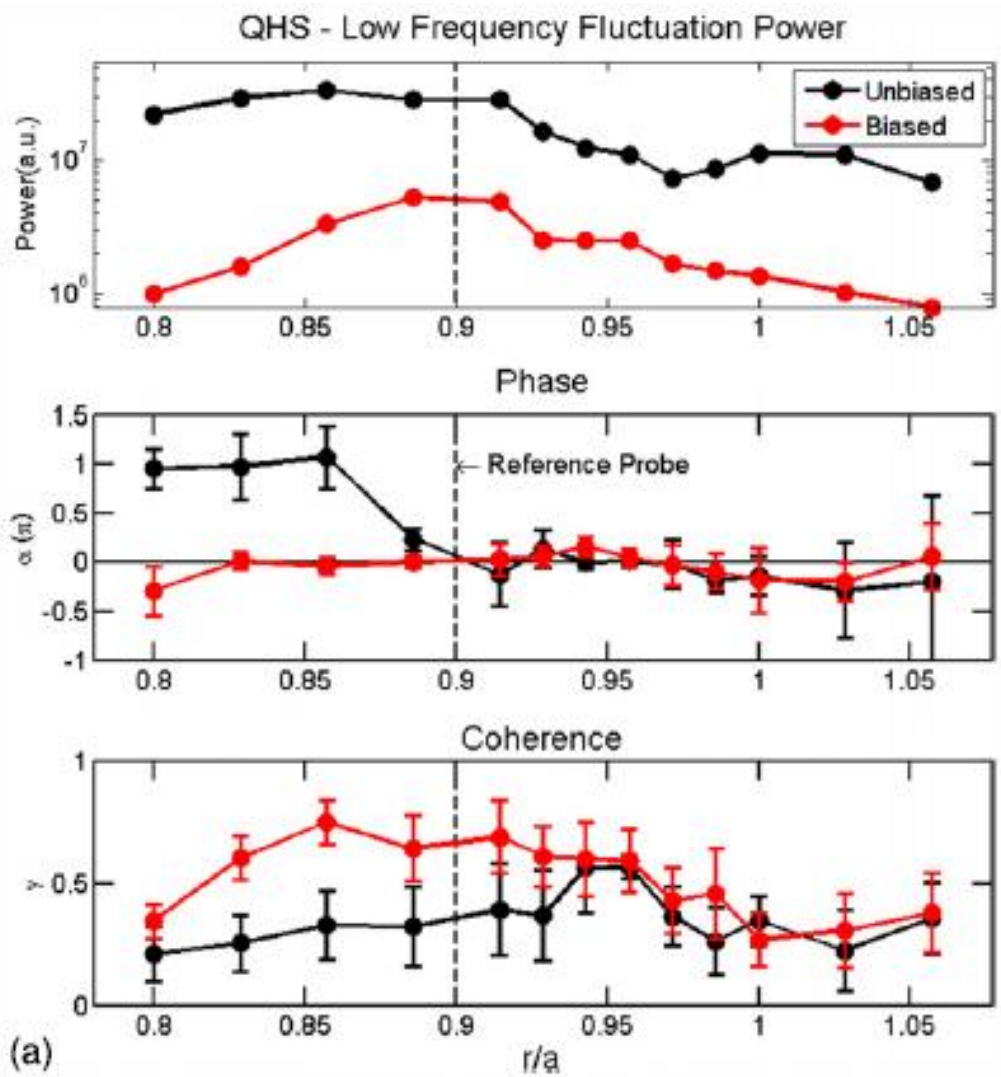
[J. M. Canik et. al., PRL 2007]

- **Once neoclassical terms have been reduced enough, turbulence becomes dominant mechanism for transport and flow damping**

Long-range potential correlations and bicoherence of E_θ fluctuations observed in region of high E_r during bias

- **Long-range correlations of potential fluctuations observed when bias applied (reference probe on HFS, toroidally separated)**
- **Phase of low-frequency fluctuations (<10 kHz) goes to zero, coherence increases across wide region**





[R. S. Wilcox et. al., *Nuclear Fusion* 51, 083048 (2012)]

Long-range potential correlations and bicoherence of E_θ fluctuations observed in region of high E_r during bias

- **Bias probe inserted to r/a=0.8, biased at end of discharge**
- **Langmuir probe measurements show broad-band bicoherence of E_θ flucutations in region of high imposed E_r**
- **No signs of zonal flows without biasing**

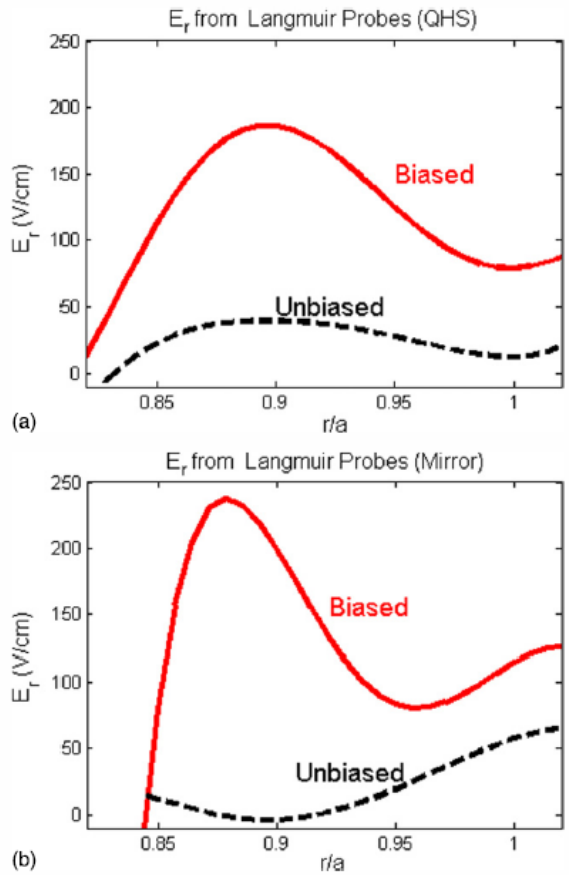


Figure 4. Radial electric field profiles in the (*a*) QHS and (*b*) mirror configurations from Langmuir probes. These profiles are calculated by fitting a curve of the shot-by-shot floating potential profile and taking the spatial derivative.

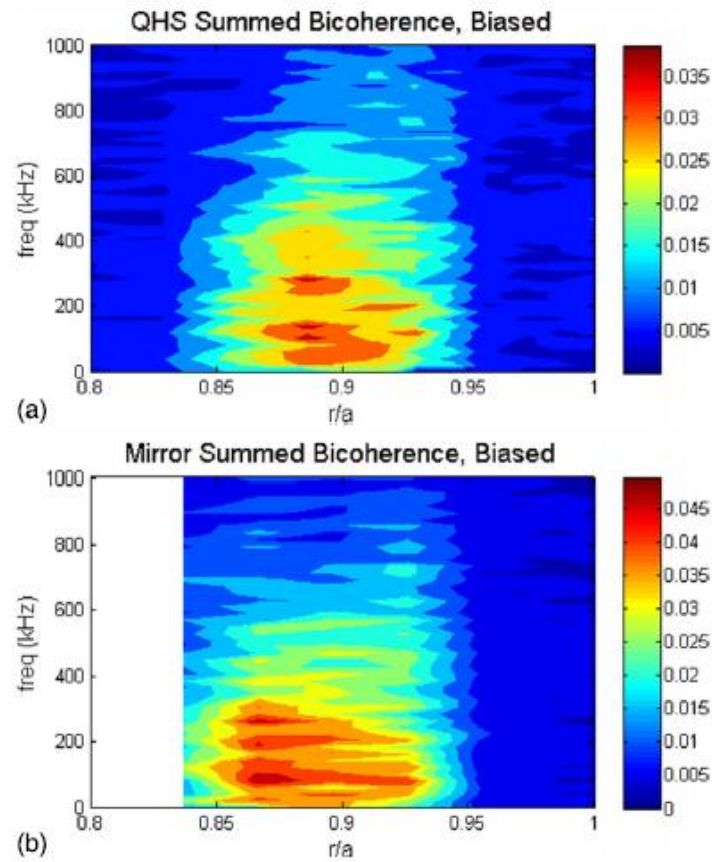
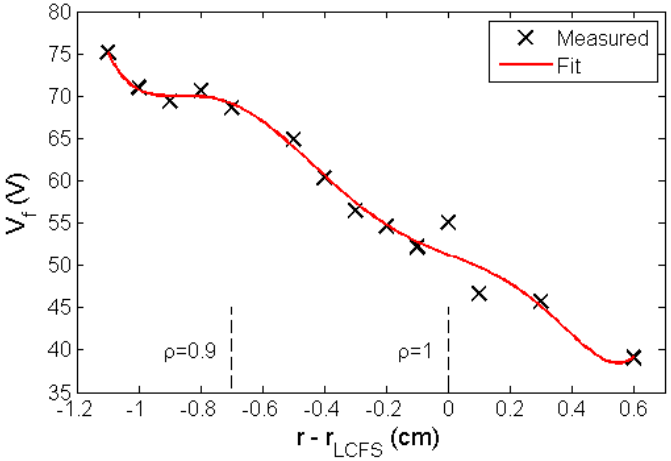


Figure 6. Summed bicoherence as a function of minor radius and frequency during biasing, in the (*a*) QHS and (*b*) mirror configurations. The scales are different between the two plots to demonstrate the similar radial extent and frequency range of the bicoherence. The unbiased times produced insignificant bicoherence relative to the noise level.

[R. S. Wilcox et. al., *Nuclear Fusion* 51, 083048 (2012)]

E_r measured by radial scan of floating potential signal, which also has electron temperature dependence



- E_r measured using fit of shot-by-shot Langmuir probe V_{fl} profile
  - Local T_e gradient not accounted for in measured E_r plots
- For low T_e, physical ∇T_e would result in more-positive E_r, larger deviation from NC ambipolarity
- Non-linearity above ~70 eV due to secondary electron emission, γ_e
- E_r measurements from probe may not be reliable above T_e~100 eV

$$\Phi_{fl} = \Phi_p - \mu \frac{T_e}{e}$$

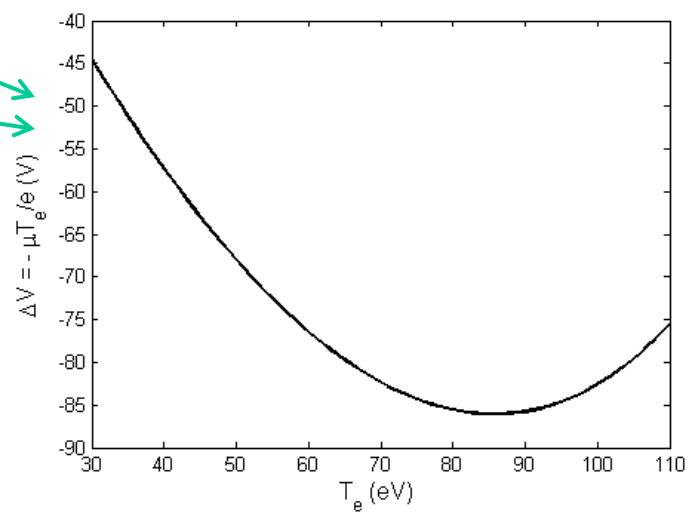
$$\mu = \ln\left(\frac{(1-\gamma_e)}{(1+\gamma_i)} \frac{A_e}{A_i} \sqrt{\frac{T_e}{T_e + T_i}} \sqrt{\frac{m_i}{2\pi m_e}} \right)$$

[R. Schrittwieser, PPCF 2002]

$$E_r = -\nabla\left(\Phi_{fl} + \mu \frac{T_e}{e} \right)$$

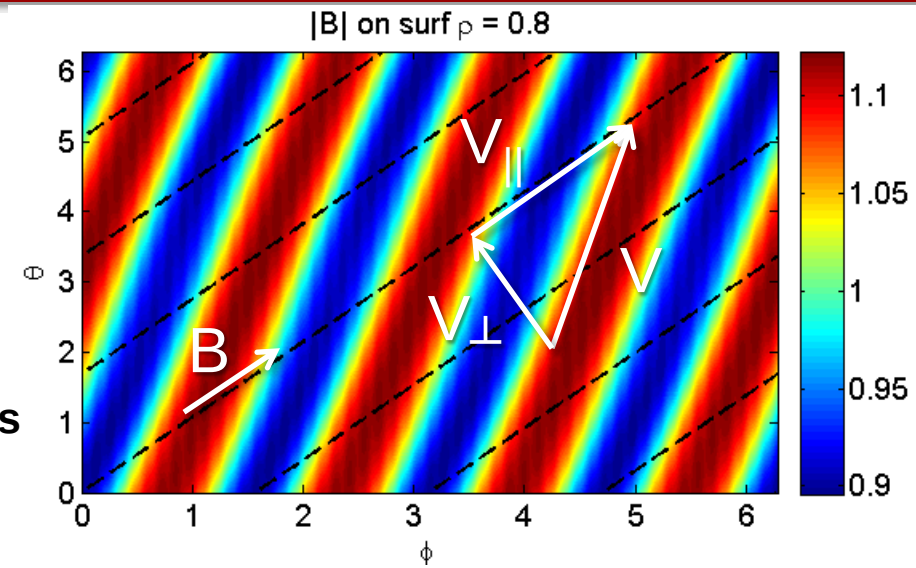
From ~70-100 eV, offset has small effect on E_r measurement (gradient small)

Contribution of T_e to V_{fl} measurement

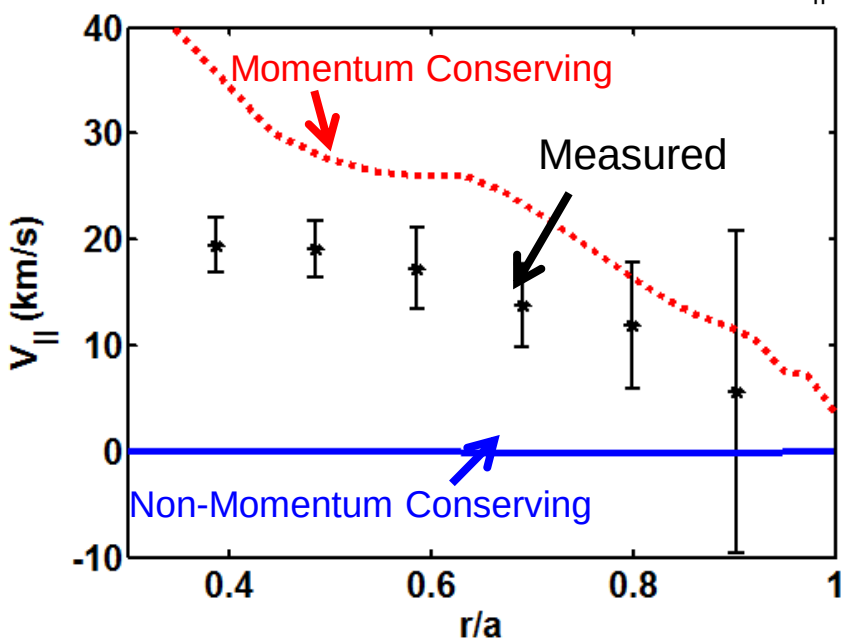


Momentum conservation is important for calculation of parallel flows in a quasi-symmetric configuration

- **Parallel flows develop so that total flow is in the direction of reduced flow damping (direction of symmetry)**
- **DKES uses a non-momentum conserving collision operator**
  - **This is acceptable in a configuration with large flow damping in all directions**



CHERS Measured and Calculated $V_{||}$



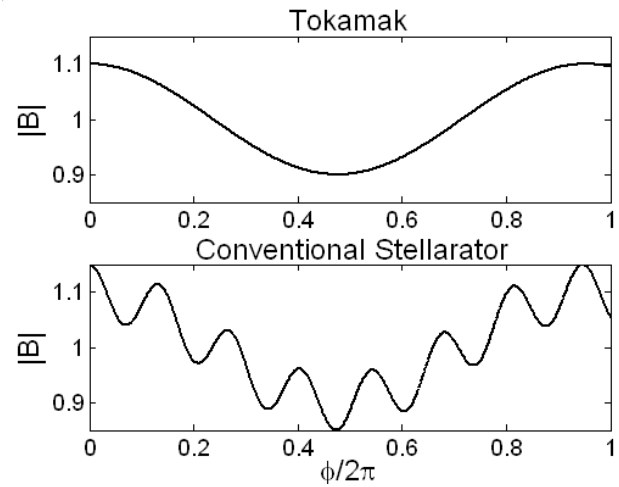
- **PENTA code** [Spong PoP 2005, Lore PoP 2010] **takes calculated DKES transport coefficients, calculates flows with correction for momentum conservation**

- **Momentum conserving calculations get close to measured parallel flows from CHERS**
  - **DKES calculations without momentum conservation predict no parallel ion flow**

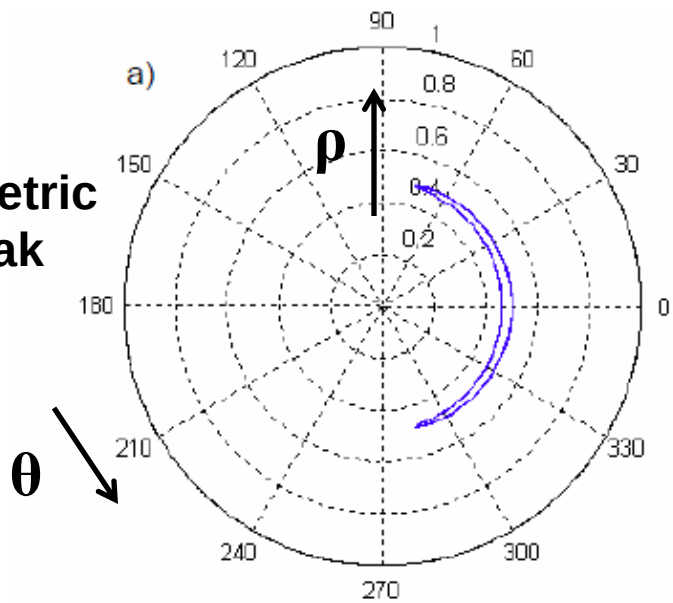
[A. Briesemeister et. al., PPCF 2013]

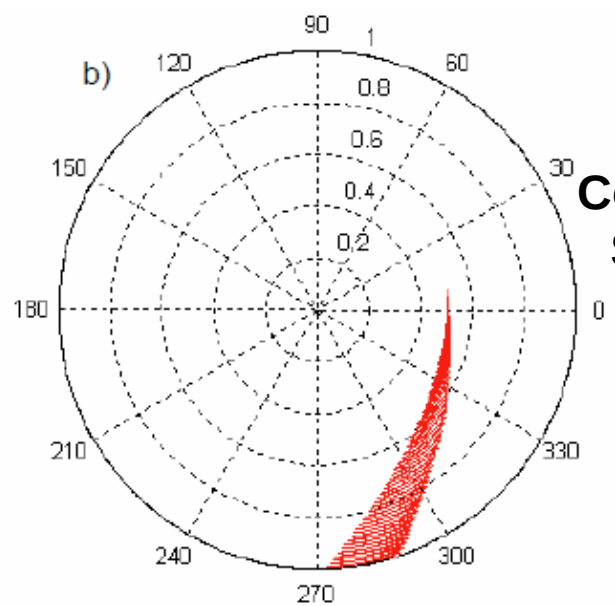
Conventional stellarators have direct loss orbits that lead to unacceptable particle losses in the low collisionality regime

- **Particles trapped in the helical ripple do not generally have closed orbits**
  - **Bounce points of different |B| ripples are not on the same surface**
  - **Collisionless radial drift**



Axisymmetric Tokamak





Conventional Stellarator

Figure 1.4: Toroidal projections of trapped particle trajectories in a) tokamak and b) stellarator magnetic fields

[J. Canik dissertation, 2007]