Healing of magnetic islands in stellarators with plasma flow

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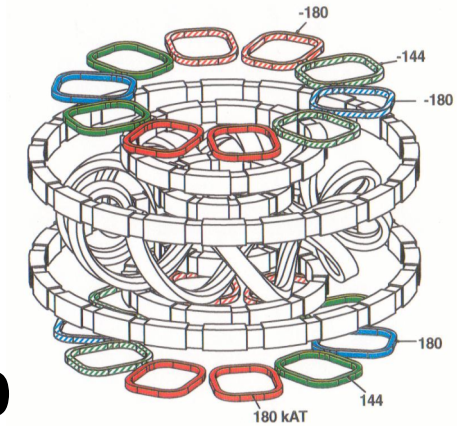
Theses

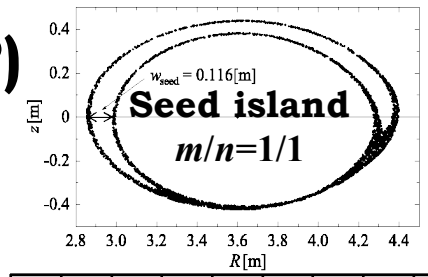
- An interpretation of the observed ‘healing’ of vacuum magnetic islands on LHD as being due to the properties of the plasma flow.
  - Reminiscent of ‘mode locking/unlocking’ physics of tokamak/RFP physics with two asymptotic states
    - Large nonrotating island locked to an external field error source
    - Rotational suppression of vacuum island --- eddy currents flowing at the rational surface shield the external field error from penetrating
  - Transitions between these two asymptotic states are described by coupled torque balance and island evolution equations.
    - Emphasis on the important role of neoclassical physics on flow profiles in conventional stellarators
  - Initial comparisons between theory and experiment are favorable

LHD experiment includes vacuum island

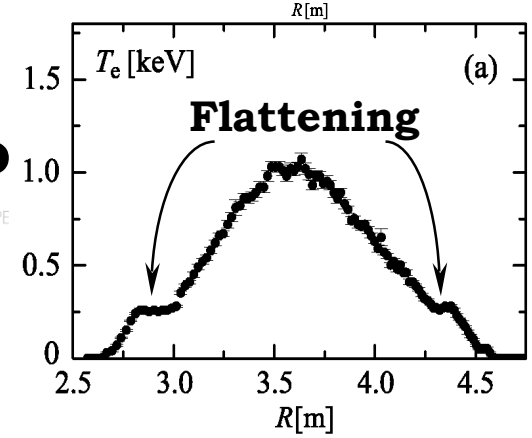
-Major radius R_0 = 3.5~4.1[m]
-Minor radius a ~ 0.6[m]
-Magnetic field $B_t \leq 3$[T]



-Super conductor coils
A pair of helical coils L/M = 2/10
3 pairs of poloidal coils
-Perturbation coil system (RMP)
Static magnetic island m/n=1/1



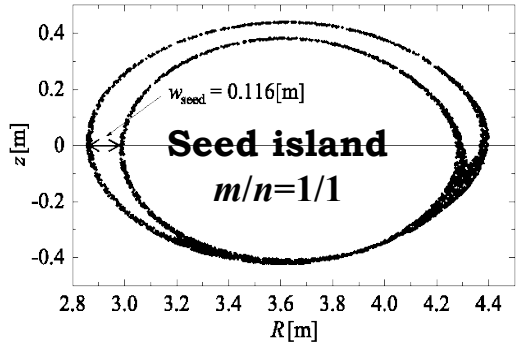
-Diagnostics
Thomson scattering system
Charge-exchange spectrosco
Magnetics



- Intended magnetic island
m/n = 1/1
Non rotating

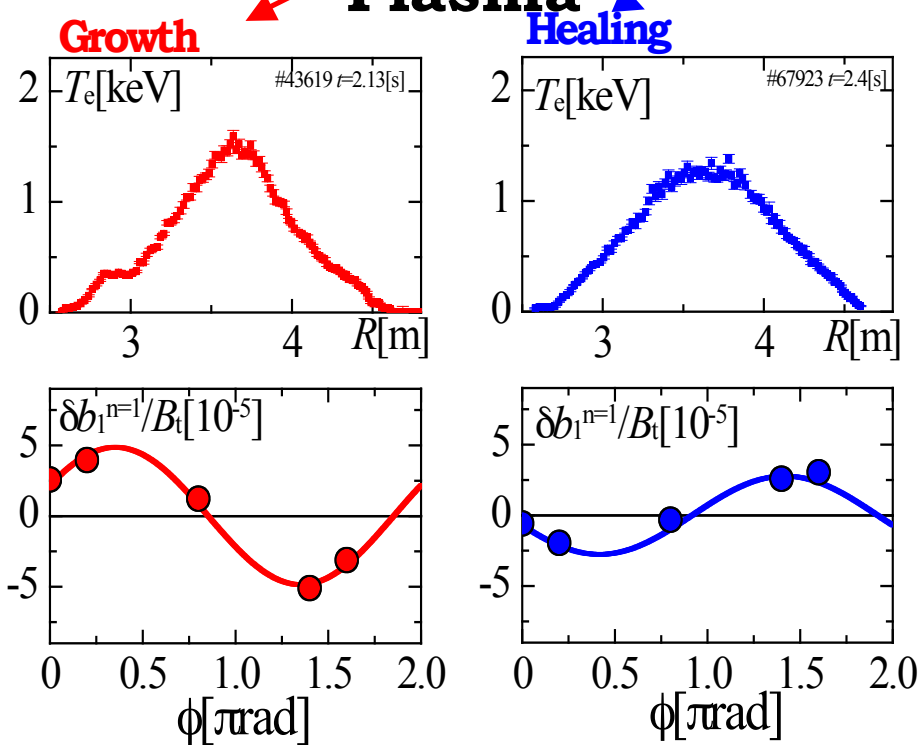
[NARUSHIMA, Y., *et al.*, PFR. 2 (2007) S1094]

Depending on parameters, the island may grow or be suppressed with $\beta \neq 0$



Vacuum

Plasma



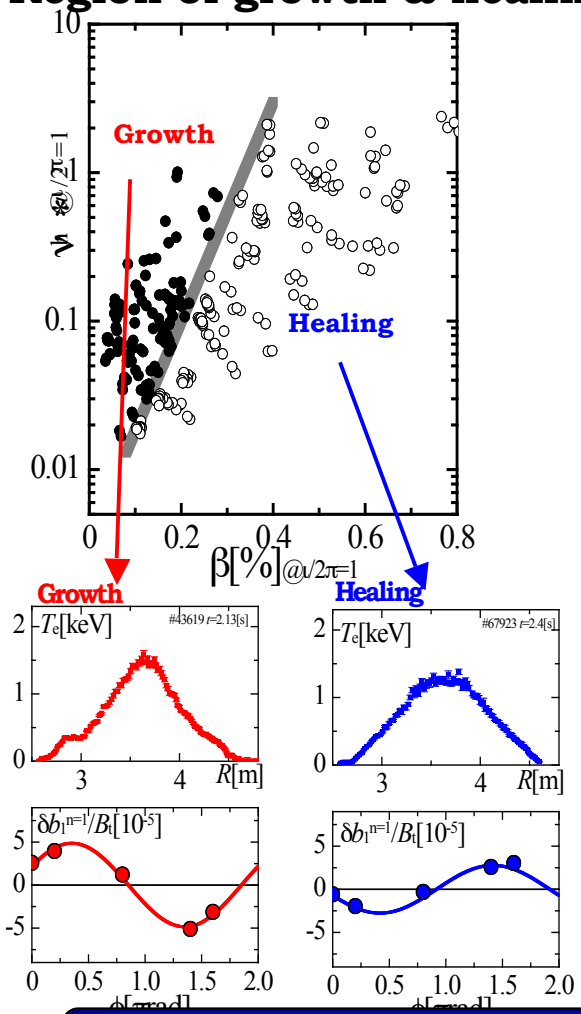
Local flattening of T_e indicates island

Reversal phase of plasma response in each state

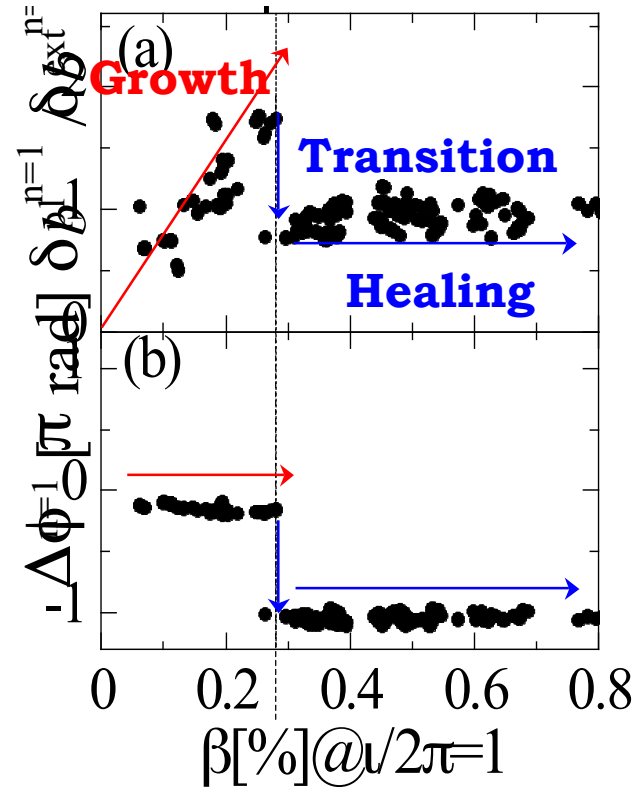
Island healing occurs at high β, low ν

[NARUSHIMA, Y. *et al* NF 48 (2008) 075010]

Region of growth & healing

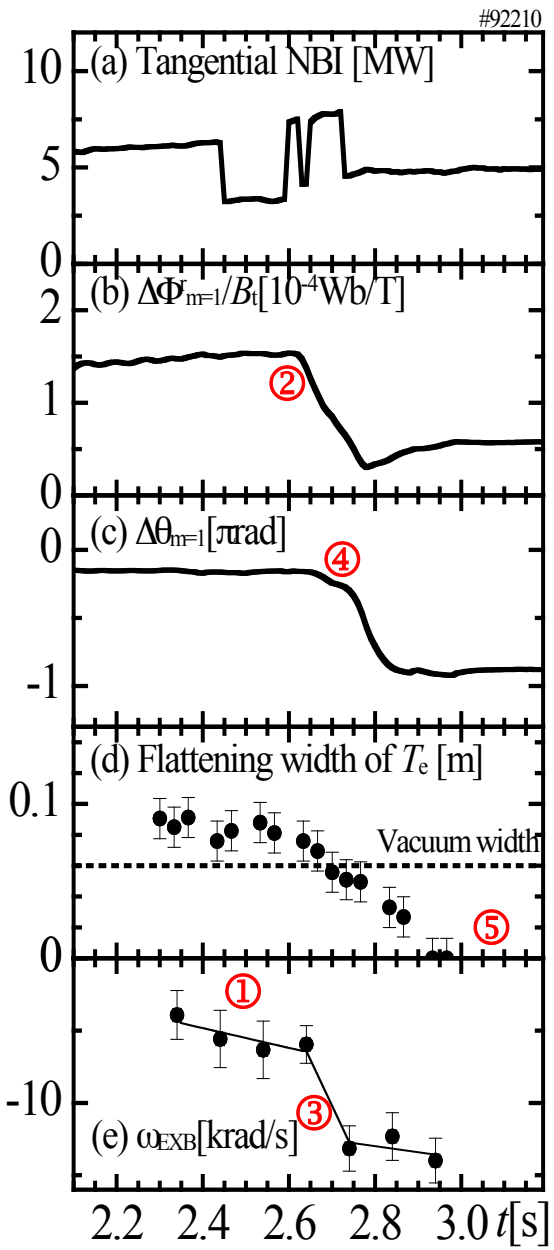


Structure of plasma response field



☝ **Growth and healing depends on β and ν**

Abrupt flow profile change at healing



From Narushima et al, IAEA '10

① $-\omega_{E\times B}$ **increases from t = 2.34s to 2.54s prior to island being healed**

② $\Delta\Phi^r_{m=1}/B_t$ **starts decreasing at t = 2.62s (Island width decreases)**

③ $-\omega_{E\times B}$ **further increases from 6 to 13krad/s**

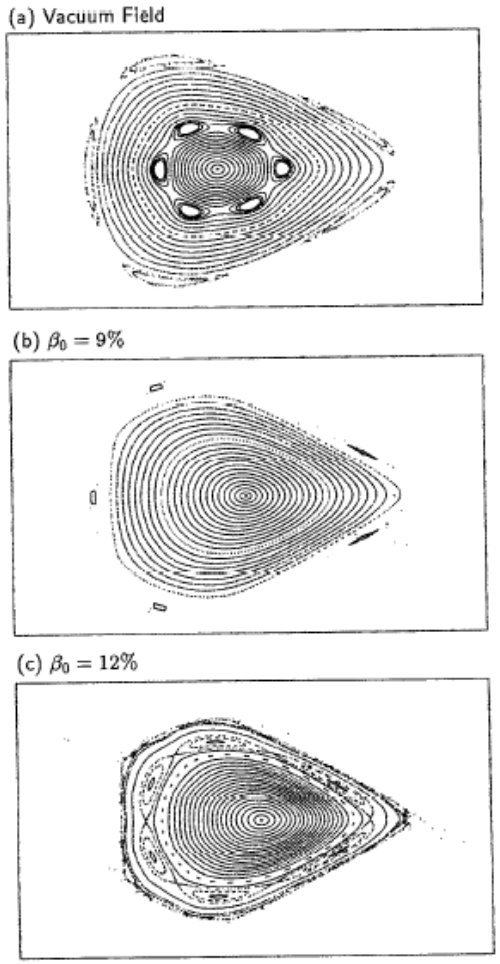
④ $\Delta\theta_{m=1}$ **starts shifting (rotating) to e-dia. direction at t = 2.67s**

⑤ **Healing**

Outline

- Background
- Magnetic island asymptotic theory
- Torque balance relation
- Healing a locked island
- Mode penetration into a rotating plasma
- Summary

Finite β healing of magnetic islands in stellarator configurations has been studied



Plasma pressure effects alter nonlinear island widths through Pfirsch-Schluter currents (among other effects)

- β healing if the phase of the resonant Pfirsch-Schluter current is π radians out of phase with respect to the vacuum island
- Analytic understanding (Bhattacharjee et al '95)

---> However, efforts to model LHD island physics with these models failed --- Narushima NF '08

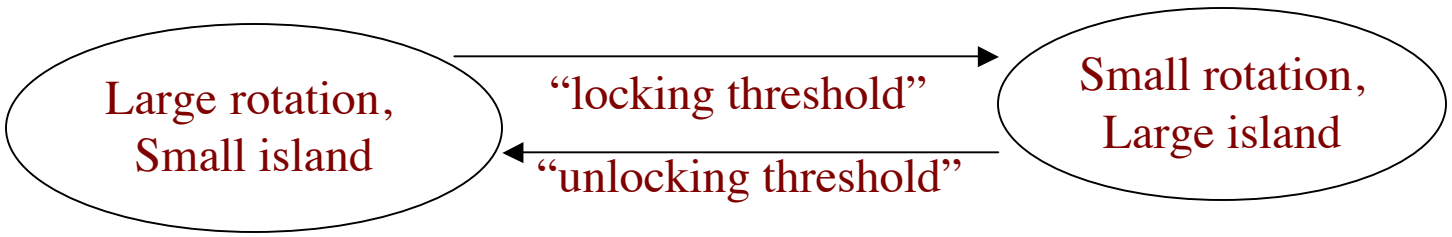
From Hayashi et al PoP '94 using HINT

LHD observations are reminiscent of mode locking/unlocking physics of tokamaks

- In the presence of externally produced 3-D external magnetic perturbations, two classes of problems are generally of interest in tokamaks/RFPs
  - For tearing stable, plasmas, resonant external source provides a source for island formation
    - Mode penetration inhibited by plasma rotation
    - At high enough resonant field amplitude, forced reconnection
  - Interaction of a rotating magnetic island with a resonant field error
    - After a sufficient field amplitude is breached, island ceases to rotate and becomes 'locked' to the external 3-D field
    - Mode locking is to be avoided ---> Often leads to disruptions

A theoretical construct for understanding tearing mode/field error interaction has developed

- Interaction of tearing modes (both stable and unstable) with resonant external 3-D magnetic perturbations is governed by coupled electromagnetic and fluid flow information (Fitzpatrick, NF '93)
  - Island amplitude and phase determined a coupled modified Rutherford theory and torque balance
  - For tearing stable plasmas, two asymptotic states



- Predictions for transitions between asymptotic states

$$\tilde{B}^{3-D}_{locking} \neq \tilde{B}^{3-D}_{unlocking}$$

Magnetic fields are described as the sum of the two fields

- Magnetic field $\vec{B} = \vec{B}_0 + \vec{B}_1$

$$\vec{B}_0 \cdot \nabla\psi = 0 \quad \vec{B}_1 \cdot \nabla\psi \sim m_o A_o \sin(n_o\phi - m_o\theta)$$

  - Island formation at the rational surface $\iota/2\pi = n_o/m_o$
  - Helical flux function describes island of width w

$$\vec{B} = \nabla\psi \times \nabla\alpha + \nabla\zeta \times \nabla\Psi^*$$

$$\Psi^* = \frac{1}{2}\iota' x^2 - A\cos(m_o\alpha) \quad \Rightarrow \quad w = 4\sqrt{\frac{A}{\iota'}}$$

$$\alpha = \theta - \frac{n_o}{m_o}\zeta$$

    - In vacuum, resonant component of 3-D field described by $A^V = A_o^V \cos(m_o\alpha - \Delta\phi)$
      - Without plasma response --> locked island

$$A = A^V \quad \Delta\phi = 0$$

Analytic description of island formation in 3-D equilibria use tearing mode formalism

- Matched asymptotic solutions
  - Outer region --- linear marginal ideal MHD --- singular at rational surfaces ---> asymptotic matching data Δ'
  - Toroidal and helical coupling of harmonics. To simplify the analysis, we concentrate on the dynamics of a single island chain and assume that that response at the couples surfaces are small
  - Outer region affected by boundary condition

$$\Delta'_{m_o n_o} = \Delta'_o + \Delta'_{BC}$$

- $\Delta_o' < 0$ indicating tearing stable
- $\Delta_{BC}' \sim A^V$

Matched asymptotic solutions include the effects of the external 3-D source

- Neglecting variations in the geometric coupling to other harmonics in the exterior region solution

$$\Delta_c' = \Delta_o'[1 - k_v \frac{A^V}{A_o}\cos(\Delta\phi)] \qquad \Delta_s' = -k_v \Delta_o' \frac{A^V}{A_o}\sin(\Delta\phi)$$

$$\mu_o \frac{\vec{J}_1 \cdot \vec{B}_0}{\vec{B}_0 \cdot \nabla\zeta} \approx \delta(\rho - \rho_s) g^{\rho\rho} A_o [\Delta_c' \cos(m_o\alpha) + \Delta_s' \sin(m_o\alpha)]$$

  - Asymptotic matching

$$-\int dx \int_0^{2\pi} \frac{d\alpha}{\pi} \int_0^{2\pi} d\zeta \cos(m_o\alpha) \frac{\mu_o \gamma}{g^{\rho\rho}} \frac{\vec{J}_1 \cdot \vec{B}_0}{B^2} = \Delta_c A_o$$

$$-\int dx \int_0^{2\pi} \frac{d\alpha}{\pi} \int_0^{2\pi} d\zeta \sin(m_o\alpha) \frac{\mu_o \gamma}{g^{\rho\rho}} \frac{\vec{J}_1 \cdot \vec{B}_0}{B^2} = \Delta_s A_o$$

  - In the vacuum limit

$$\Delta_c = \Delta_s = 0 \quad \Rightarrow \quad A = A^V \quad \Delta\phi = 0$$

Two cases to consider

- Locked island ---> No Island phase velocity
  - Island rotates with plasma -> No plasma rotation at the rational surface
  - Healing criteria = conditions under which plasma response causes the island to unlock from the wall.
    - Plasma rotates
    - No island
- Mode penetration ---> Plasma flowing at rational surface
  - Plasma rotation inhibits the penetrationof the vacuum magnetic island source
  - Mode penetration criteria = conditions under which external 3-D source produces a magnetic island
    - Island formation
    - Plasma rotation at rational surface ceases

Inner layer solution describes the plasma response

- Layer current responses segregated into ‘cosine’ and ‘sine’ components
  - ‘cos’ components contribute to island growth/decay --- contributes to generalized nonlinear Rutherford equation
  - ‘sin’ components describe a localized electromagnetic torque $\int_0^{2\pi} d\alpha \int_0^{2\pi} d\zeta \sqrt{g}\, \vec{e}_\alpha \cdot \tilde{\vec{J}} \times \tilde{\vec{B}} \cong T_{EM\,0}\delta(\rho - \rho_o)$

$$T_{EM\,0} = \frac{2\pi^2 \psi'}{\mu_o \gamma} m_o A_o^2 \Delta_s$$

- For simplicity, we’ll neglect pressure driven contributions to ‘cos’

$$k_1 \frac{\mu_o}{\eta}\frac{dw}{dt} = \Delta_c \quad \Rightarrow w = w^V \sqrt{k_v \cos(\Delta\phi)}$$

$$T_{EM\,0} = \frac{\pi^2 \psi'^3}{\mu_o \gamma}\frac{m_o k_v^2(-\Delta_o')}{256}\left(\frac{\iota_o'}{2\pi}\right)^2 (w^V)^4 \sin(2\Delta\phi)$$

In conventional mode locking theory, viscous torque balances electromagnetic torque

- Momentum balance equation with phenomenological cross-field viscosity (Fitzpatrick NF '93)

$$\rho \frac{d\vec{v}}{dt} = \vec{J} \times \vec{B} + \nabla \cdot (\rho \nu_{\perp} \nabla \vec{v}) + \ldots$$

  – Localized $T_{EM} \sim T_{EM0}\delta(\rho - \rho_o)$**b** X **n** balance by viscous toruqe in steady-state

$$0 \cong T_{EM\,0} + \rho_M \nu_{\perp} \vec{e}_{\alpha} \cdot \frac{\partial \vec{v}}{\partial \rho} \Big|_{\rho_o -}^{\rho_o +}$$

    - Viscous torque determined by properties of flow profile
      – In tokamak, strong neoclassical poloidal flow damping, undamped (weakly damped) toroidal flows

$$T_{V0} = \rho_M \nu_{\perp} \vec{e}_{\alpha} \cdot \frac{\partial \vec{v}}{\partial \rho} \Big|_{\rho_o -}^{\rho_o +} \sim \rho_M \nu_{\perp} g_{\alpha\alpha} \frac{\Omega_{\zeta}}{a}$$

Momentum balance equation includes the effects of cross field viscosity

- Momentum balance equation.

$$\rho \frac{d\vec{v}}{dt} = \vec{J} \times \vec{B} - \nabla p - \sum_s \nabla \cdot \overleftrightarrow{\pi}_s + \nabla \cdot (\rho_M \nu_\perp \nabla \vec{v}) + S$$

  - In the absence of islands

$$\vec{B} \cdot \nabla \psi = 0 \quad \vec{v} = \left(\frac{d\Phi}{d\psi} + \frac{1}{n_i q_i}\frac{dp_i}{d\psi}\right)\frac{\vec{B} \times \nabla \psi}{B^2} + \frac{v_\parallel}{B}\vec{B}$$

$$\vec{v} = \Omega^\alpha \vec{e}_\alpha + \Omega^\xi \vec{e}_\xi \quad \Omega^\alpha \cong \frac{d\Phi}{d\psi} + \frac{1}{n_i q_i}\frac{dp_i}{d\psi} = \Omega^\alpha(\psi) \quad \Omega^\xi = -\frac{I}{\gamma}\Omega^\alpha + \frac{v_\parallel B}{\gamma}$$

  - From momentum balance

$$< \vec{e}_\alpha \cdot Momentum\,Balance >= 0$$

$$0 = - < \vec{J} \cdot \nabla \psi > - \sum_s < \vec{e}_\alpha \cdot \nabla \cdot \overleftrightarrow{\pi}_s > + < \vec{e}_\alpha \cdot \nabla \cdot (\rho_M \nu_\perp \nabla \vec{v}) > + < \vec{e}_\alpha \cdot \vec{S} >$$

For a locked island, 1/ν neoclassical transport operative at high temperature

- Viscous stress component denotes neoclassical transport

$$\sum_s < \vec{e}_\alpha \cdot \nabla \cdot \overleftrightarrow{\pi}_s > = -\sum_s q_s < \vec{\Gamma}_s^{neo} \cdot \nabla \psi >$$

  - For high T, low E_r plasmas

$$e < \vec{\Gamma}_s^{neo} \cdot \nabla \psi > = -C_{1/\nu} \rho_M \frac{\omega_{ts}^2}{\nu_s} [\Phi' \frac{q_s}{e} + \frac{p_s'}{n_s q_s} + k_s \frac{T_s'}{q_s}]$$

$$e < \vec{\Gamma}_s^{neo} \cdot \nabla \psi > = -C_{1/\nu} \rho_M \frac{\omega_{ts}^2}{\nu_s} [\Omega^\alpha - \Omega_{amb,s}^\alpha]$$

  - $C_{1/\nu} \sim \varepsilon_{eff}^{3/2}$ for conventional stellarator
- With a neoclassical transport model, the momentum balance equation becomes a transport equation for Ω^α
  - More generally, nonlinear dependence on Ω^α --- multiple roots

In the presence of a locked island, a similar procedure can be used to determine rotation

- In the presence of a locked island

$$\Phi = \Phi(\Psi^*), n = n(\Psi^*), T = T(\Psi^*)$$

$$\Omega^\alpha = [\frac{d\Phi}{d\Psi^*} + \frac{1}{n_s q_s}\frac{dp_s}{d\Psi^*}]\frac{\iota' x}{2\pi} = <\Omega^\alpha>_* \frac{x}{<x>_*}$$

  - Transport equation for $<\Omega^\alpha>$ obtained from momentum balance

$$0 = -<\vec{J}\cdot\nabla\Psi^*>_* - \sum_s <x\vec{e}_\alpha\cdot\nabla\cdot\vec{\pi}_s>_* + <x\vec{e}_\alpha\cdot\nabla\cdot(\rho_M\nu_\perp\nabla\vec{v})>_* + <x\vec{e}_\alpha\cdot\vec{S}>_*$$

$$<\Omega^\alpha> = \Omega_o^\alpha + \delta_r^2 \frac{\iota'^2 <x>^2}{c_r <x^2 g^{\rho\rho}/B^2>}\frac{d}{d\Psi^*}[\frac{<x^4 g^{\rho\rho}/B^2>}{<x>}\frac{d}{d\Omega^*}\frac{<\Omega^\alpha>}{<x>}]$$

$$\Omega_o^\alpha = <\Omega_{ambi,i}^\alpha> + O(S)$$

$$\delta_r^2 = \frac{\nu_\perp \nu_i}{C_{1/\upsilon}\omega_{ti}^2}$$

Measures the relative importance of Cross-field viscosity to neoclassical transport

With large neoclassical transport, a flow boundary layer is present near the island

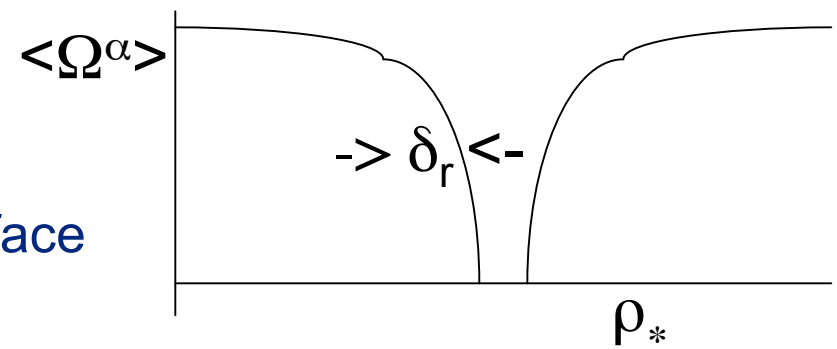
- Momentum balance provides profile near island

$$< \Omega^{\alpha} >= \Omega_o^{\alpha} + \delta_r^2 \frac{\iota'^2 < x >^2}{c_r < x^2 g^{\rho\rho} / B^2 >} \frac{d}{d\Psi *} [\frac{< x^4 g^{\rho\rho} / B^2 >}{< x >} \frac{d}{d\Omega *} \frac{< \Omega^{\alpha} >}{< x >}]$$

$$\delta_r^2 = \frac{\nu_{\perp} \nu_i}{C_{1/\upsilon} \omega_{ti}^2}$$ At high T, δ_r << a in conventional stellarators

  - Locked island condition provides internal boundary condition at $\Psi^* = \Psi^*_{sx}$. Produces boundary of width δ_r near island

$$< \Omega^{\alpha} > \cong \Omega_o^{\alpha} - \Delta\Omega^{\alpha} e^{-|\rho_*|/\delta_r}$$



  - Viscous torque at rational surface

$$T_{v0} = -2V' \rho_M \nu_{\perp} < \frac{\psi'^2 g^{\rho\rho}}{B^2} > \frac{\Delta\Omega_o^{\alpha}}{\delta_r}$$

Torque balance equation determines self-consistent $\Delta\phi$

- From torque balance $T_{VS\theta s} + T_{EM\theta s} = 0$.

$$D_\Omega = D_w \sin(2\Delta\phi)$$

$$D_\Omega \equiv \rho_s \frac{\partial \Delta\Omega^\alpha}{\partial \rho} \Big|_{r_s -}^{r_s +} = -2\frac{\rho_s}{\delta_r}\Delta\Omega^\alpha$$

$$D_w \equiv \frac{\omega_A^2 \tau_V n_0^2 \hat{s}^2}{512}\left(\frac{w^V}{r_s}\right)^4 \qquad \tau_V \cong \frac{r_s^2}{\nu_\perp} \qquad \omega_A^2 \cong \frac{B_o^2}{\mu_o \rho_o R_o^2}$$

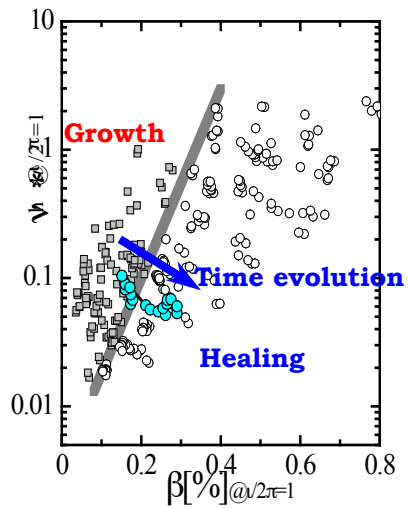
  - For $D_\Omega < D_A$

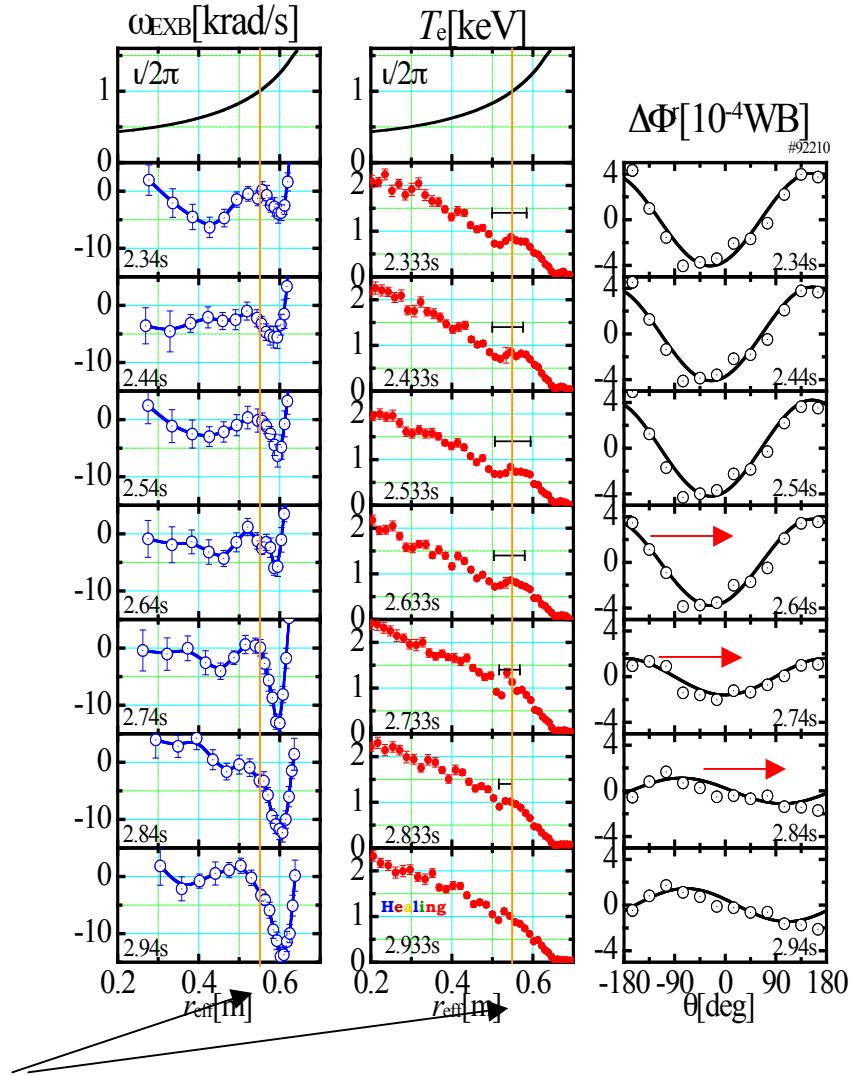
$$\sin(2\Delta\phi) = \frac{-D_\Omega}{D_A}$$

  - For $D_\Omega > D_A$ steady state torque balance can't be maintained --> Viscous torque overwhelms electromagnetic torque and the plasma starts to rotate and the island is healed.
  - 'Healing criteria'

$$|D_\Omega| = D_A$$

Healing associated with change in rotation





- Transition to healing phase associated with $\Delta\Omega(\rho_s)$

Rational surface

The torque balance equation predicts a critical condition for island healing

- The healing threshold is given by

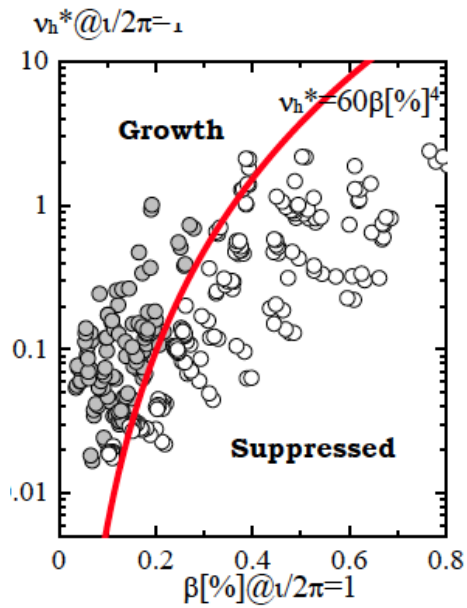
$$D_{\Omega} = | D_{w} |$$

  – Assuming rotation scales with diamagnetic flows, and cross-field viscosity is gyro-Bohm, a critical-β as a function of ν^* can be derived

$$\Omega^{\alpha} \sim \frac{T}{eBL_{eq}^2} \qquad \nu_{\perp} \sim \rho * \frac{T}{eB}$$

$$\rightarrow \beta_{crit} \sim \nu *^{1/4} \left(\frac{w^{v}}{\rho_s}\right)^2 \frac{c}{\omega_{pi}\rho_s}$$

$\beta_{crit} \sim (\nu^*)^{1/4}$ consistent with LHD data



From Narushima private comm.

The island width is dramatically reduced in a rotating plasma

- Linear theory used to describe plasma response

$$A_o = \frac{A^V}{1 + \Delta_L / (-\Delta_o^{'})}$$

  - Δ_L denotes the layer response with layer time τ_L

$$\frac{\Delta_L}{-\Delta_o^{'}} \cong i\omega\tau_L = i m_o \Omega^{\alpha} \tau_L$$

  - In many practical applications $\omega\tau_L$ >>1
  - Island width and phase

$$A_o = \frac{A^V}{\sqrt{1 + \omega^2 \tau_L^2}} \qquad \sin(\Delta\phi) = -\frac{\omega\tau_L}{\sqrt{1 + \omega^2 \tau_L^2}}$$

$$T_{EM\,0} \sim (w^V)^4 \frac{2\omega\tau_L}{1 + \omega^2 \tau_L^2}$$

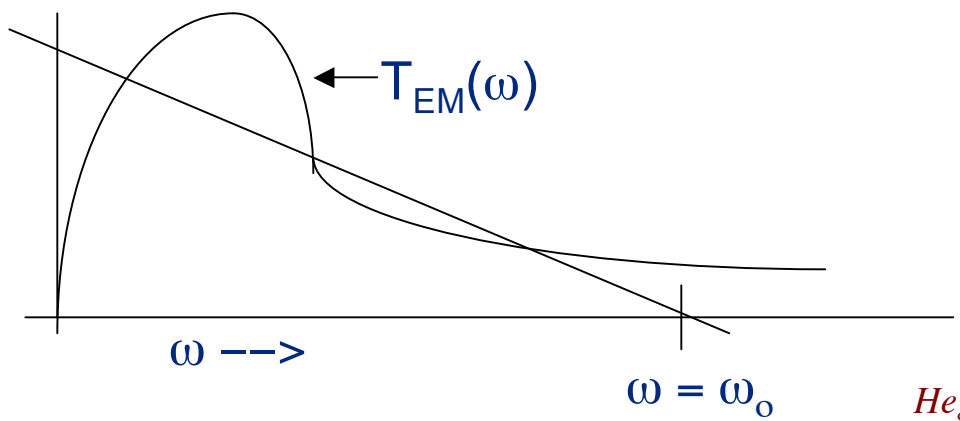
Torque balance predicts bifurcation process between asymptotic states

- Torque balance has 3 solutions for ω

$$d_\Omega(\omega_o - \omega) = D_w \frac{2\omega\tau_L}{1+\omega^2\tau_L^2}$$

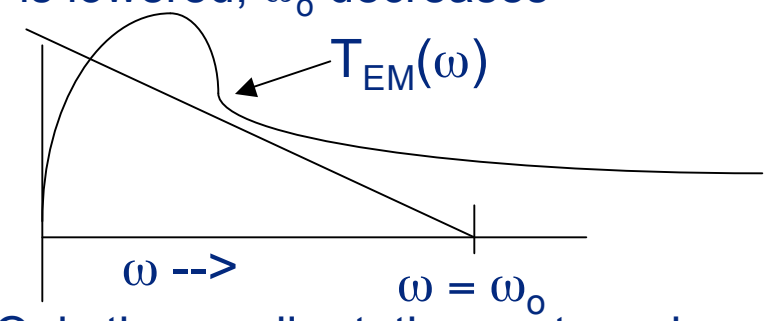
Asymptotic solutions:

- D_W small, only solution $\omega = \omega_o$ (large rotation, small island)
- D_W large, only solution $\omega = 0$ (small rotation, large island)
- Third root is dynamically unstable (not realizable)



At lower ω_o, only one solution is allowed

- As β is lowered, ω_o decreases



  - Only the small rotation root survives ---> Abrupt transition from the high rotation/no island state to the low rotation/large island state
    - "Locking" threshold when

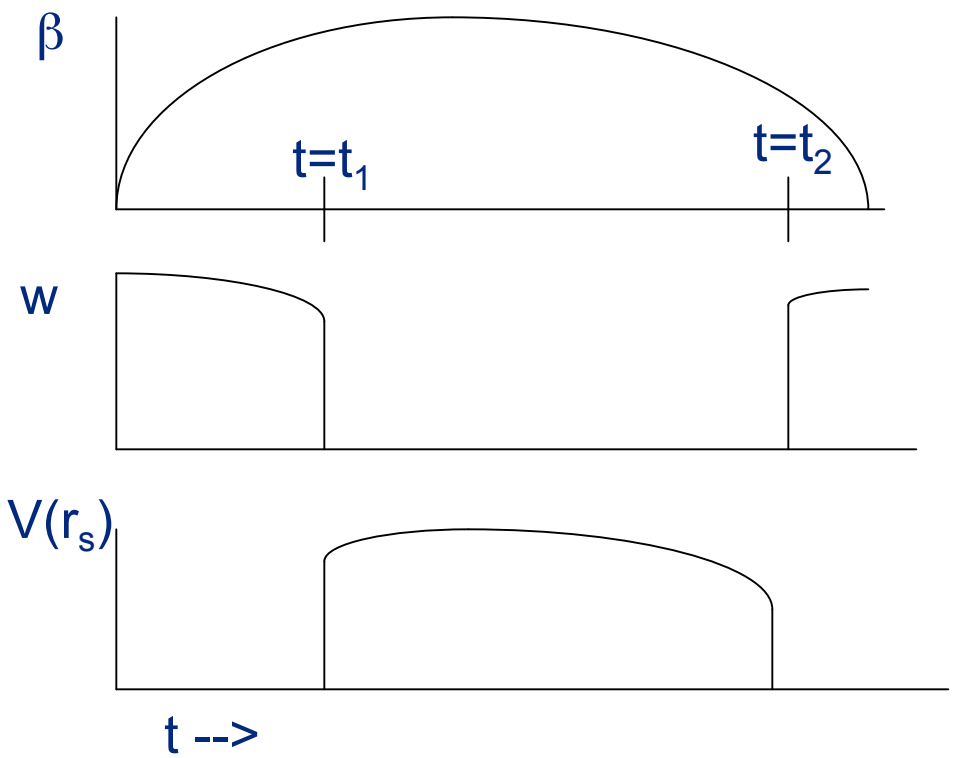
$$\omega_o = \omega_{oC} = \sqrt{\frac{8D_W}{d_\Omega \tau_L}}[1 + O(\frac{1}{\omega_o^2 \tau_L^2})]$$

    - At locking threshold, $D_\Omega < D_W$ ---> Hysteresis

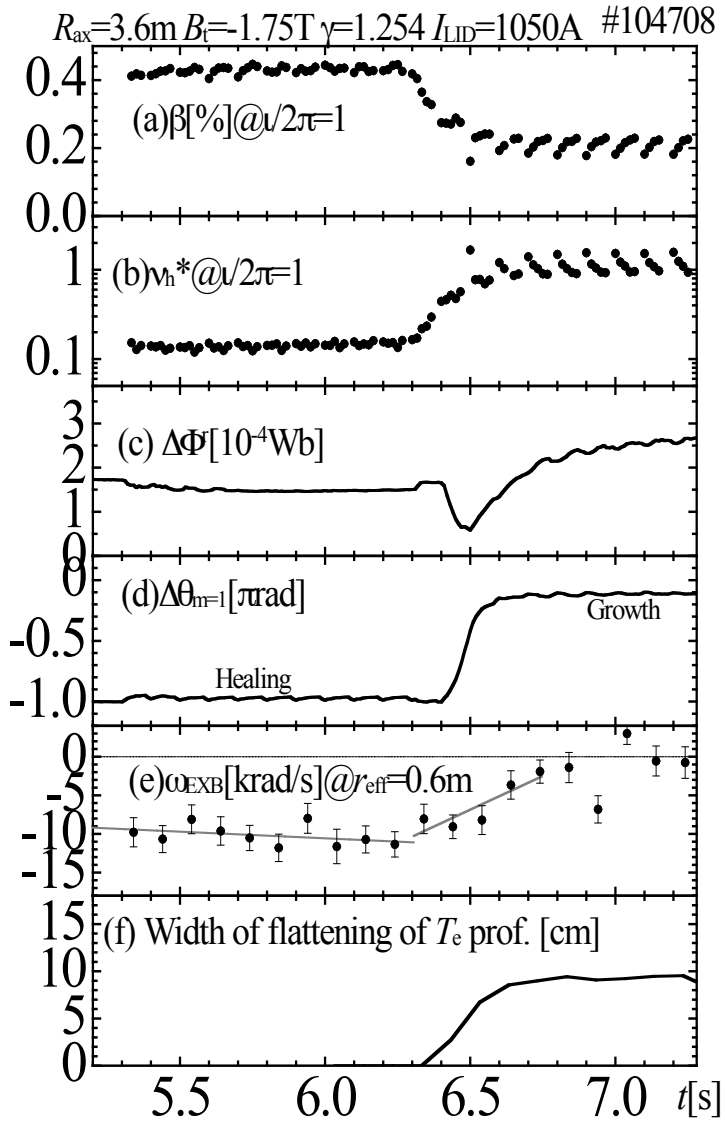
$$D_\Omega = d_\Omega(\omega_o - \omega) \cong \frac{D_W}{\omega \tau_L} << D_W$$

An interpretation of observations on LHD can be made using theory

- Evolution of the stored energy, island width and rotation at r_s
- At $t = 0$, large vacuum Island
- For $0 < t < t_1$ viscous Torque raises with β
- At $t = t_1$, $D_w = D_\Omega$ Plasma abruptly jumps To rotating state at r_s
- For $t_1 < t < t_2$ flow shields Island formation
- At $t = t_2$ plasma abruptly Jumps to the low rotation/ Large island state at r_s

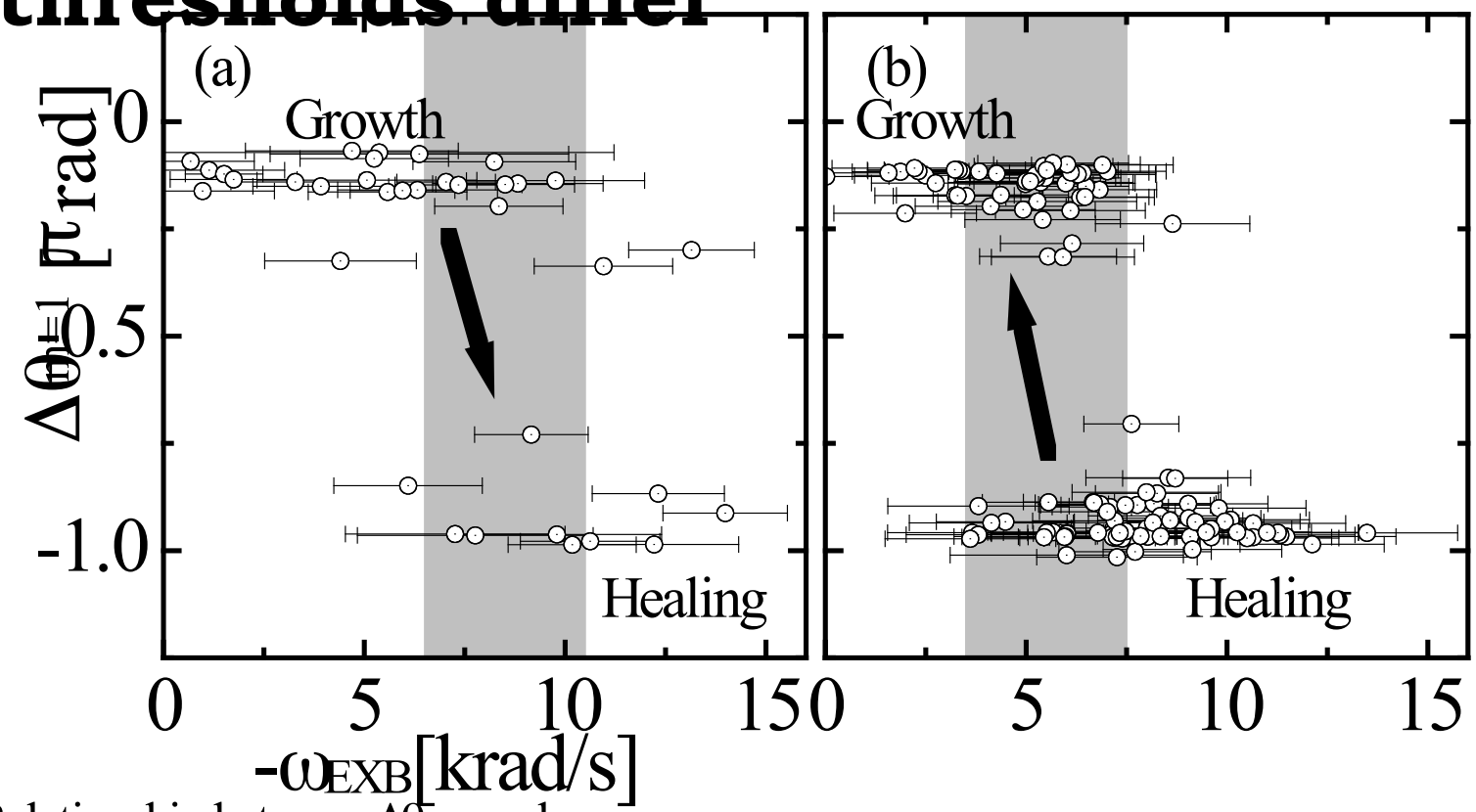


Relocking of vacuum island is seen



LHD results reported by Narushima et al at EPS '11

Observed healing and penetration thresholds differ



Relationship between $\Delta\theta_{m=1}$ and ω_{EXB}.
Transition from growth to healing. (b) from healing to growth.
Arrowed lines indicate time evolution.
Thresholds of transition are (a)ω_{EXB} ~ -8.5krad/s and (b)ω_{EXB} ~ -5.5krad/s, respectively.

☝ **These results suggest existence of hysteresis in magnetic island transition dynamics.**

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Conclusions

- A theory accounting for the evolution of the magnetic island width and phase is obtained from coupled electromagnetic and fluid flow information.
  - Theory generalized previous analytic calculations in cylindrical geometry for tokamak/RFP applications.
  - Unlike axisymmetric equilibria, neoclassical damping physics is crucial for determining flow profile properties
  - Generally, the healing criteria differs from penetration criteria. ---> Hysteresis in the flow/island width evolution.
  - The appearance of large shear flow near the island is a property of locked islands in conventional stellarators ---> Transport barrier.
- Implication: Plasma rotation physics can play a crucial role in healing magnetic islands. This physics is not incorporated in 3-D MHD equilibrium codes (PIES, SIESTA, HNT,etc.). However, extended MHD codes should be able to include this physics.

References of interest

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