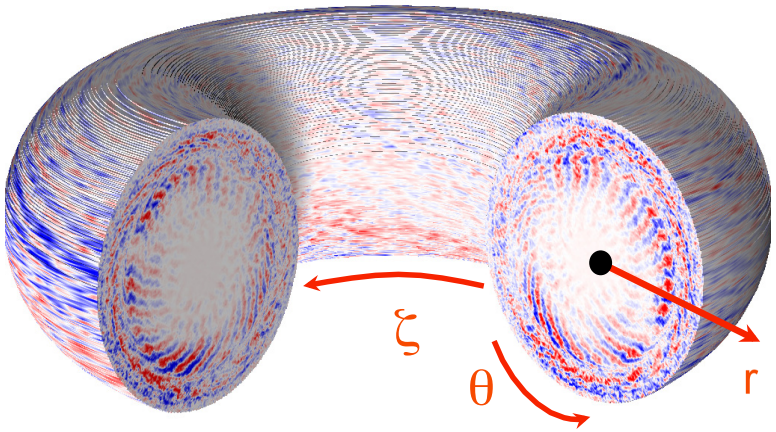
NATURE OF TRANSPORT ACROSS SHEARED ZONAL FLOWS



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FEST Seminar

ORNL, Oak Ridge, TN, Dec 17, 2009

Fusion Energy Division

Oak Ridge National Laboratory

Managed by UT-Battelle for the Department of Energy

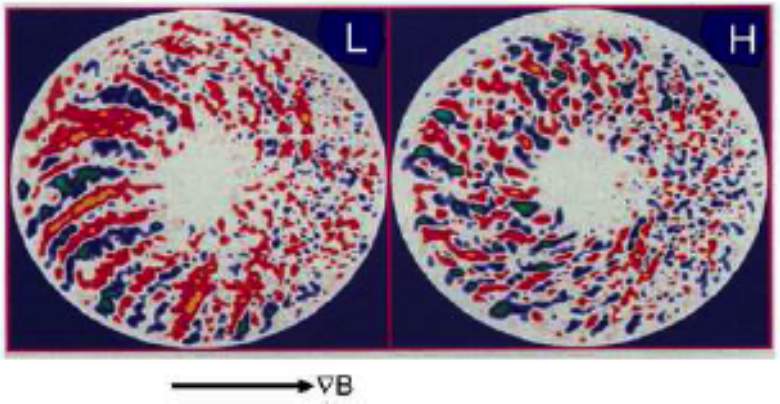
SHEARED ZONAL FLOWS AND SUPPRESSION OF RADIAL TRANSPORT

- Since the 90s, suppression of radial transport observed in enhanced confinement modes has been associated to radially-sheared poloidal flows. They may be driven by **turbulence** itself (via Reynold stresses) or **externally**.

[See: H. Biglary et al, *Phys. Fluids B* **4** (1990) 1385;
P.W. Terry, *Rev. Mod. Phys.* **72** (2000) 109;
P.H. Diamond et al, *Pl.Phys.Contr.Fus.* **47** (2005) R35]

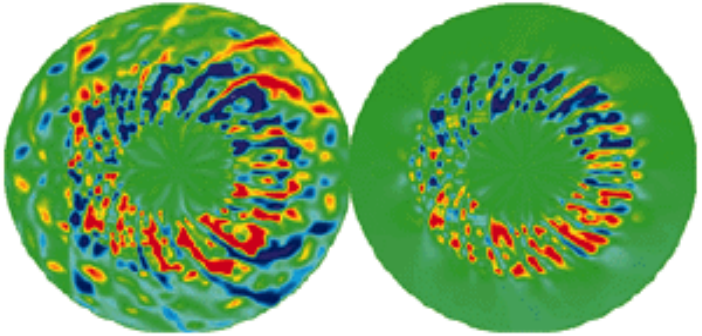
- Radial transport traditionally modeled using a *reduced effective diffusivity*, which presumably captures the "change in typical scales" the flow is assumed to cause.

Toroidal ITG gyro-kinetic simulations



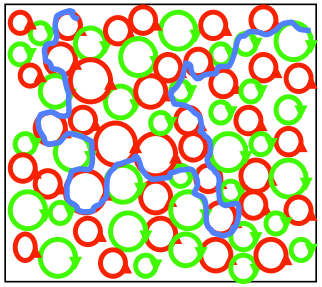
[See: Z. Lin et al, *Science* **281** (1998) 1835]

Fluid resistive interchange turbulence (cylinder)



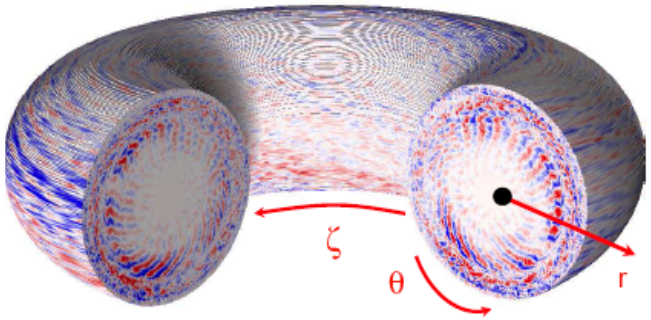
[See: B.A. Carreras et al, Phys. Fluids B 5 (1993) 1491]

OUTLINE*



1. Characterizing the nature of transport:

   ⇨ Lagrangian approach



2. Radial transport across poloidal sheared flows in (global, PIC, δf, electrostatic) gyrokinetic ion-temperature-gradient (ITG) turbulence with UCAN code:

   - CASE I: self-consistent Reynold-stress driven flow
   - CASE II: artificially suppressed zonal flow [$\cancel{\phi_{00}(r,t)}$]
   - CASE III: externally-driven zonal flow [$\cancel{\phi_{00}(r,t)}$ + $\phi_{ext}(r)$]

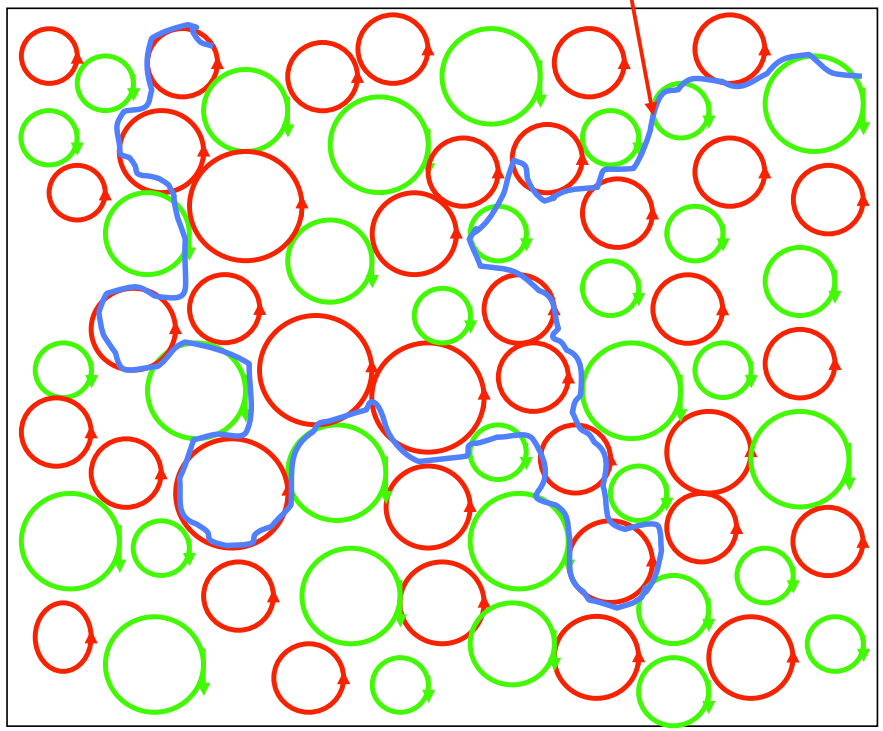
$$\phi_{\rm ext}(r) \simeq \langle \phi_{0,0}(r,t) \rangle_{\rm sat}$$

3. Mechanisms for self-similar (*a.k.a.* non-diffusive, scale-free, fractional,…) transport in the presence of sheared flows

LAGRANGIAN APPROACH

"Lagrangian" trajectories
(or characteristics)

$$\frac{d\mathbf{R}}{d\tau} = \tilde{\mathbf{V}}(\mathbf{R}, \tau), \quad \mathbf{R}(t) = \mathbf{r}$$



LAGRANGIAN APPROACH

$$\frac{\partial n}{\partial t} + (\mathbf{V}\cdot\nabla)\, n = 0$$

n, passive scalar
V, turbulent flow field

$n = n_0 + \tilde{n}$

$\vec{V}_0 = 0$

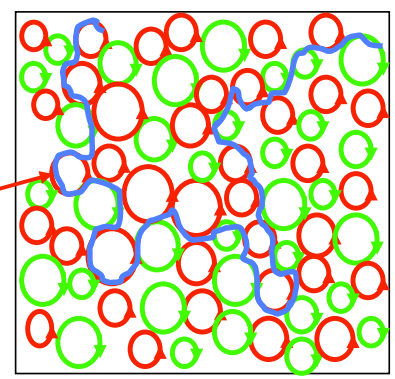
$$\frac{\partial n_0}{\partial t} = -\left\langle \tilde{\mathbf{V}}\cdot\nabla\tilde{n}\right\rangle$$

$$\frac{\partial \tilde{n}}{\partial t} + \tilde{\mathbf{V}}\cdot\nabla\tilde{n} = -\tilde{\mathbf{V}}\cdot\nabla n_0 + \left\langle \tilde{\mathbf{V}}\cdot\nabla\tilde{n}\right\rangle$$

$$\frac{\partial n_0}{\partial t} = \nabla\cdot\int_0^t dt' \left\{\left\langle \tilde{\mathbf{V}}(\mathbf{r},t)\tilde{\mathbf{V}}\left(\mathbf{R}(t'|\mathbf{r},t),t'\right)\nabla n_0\left(\mathbf{R}(t'|\mathbf{r},t),t'\right)\right\rangle\right\}$$

"Lagrangian" trajectories
(or characteristics)

$$\frac{d\mathbf{R}}{d\tau} = \tilde{\mathbf{V}}(\mathbf{R},\tau), \quad \mathbf{R}(t) = \mathbf{r}$$



[See: R. Balescu, *Aspects of turbulent transport in plasmas*, IOP (2006)]

LOCALIZATION HYPOTHESIS

$$\frac{\partial n_0}{\partial t} = \nabla \cdot \int_0^t dt' \left\{ \left\langle \tilde{\mathbf{V}}(\mathbf{r},t)\tilde{\mathbf{V}}\left(\mathbf{R}(t'|\mathbf{r},t),t'\right) \nabla n_0\left(\mathbf{R}(t'|\mathbf{r},t),t'\right) \right\rangle \right\}$$

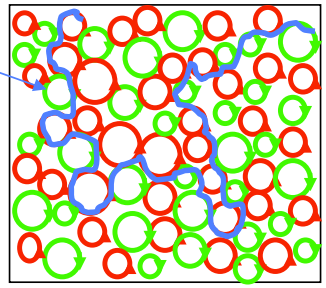
[See: R. Balescu, *Aspects of turbulent transport in plasmas*, IOP (2006)]

$$\frac{\partial n_0}{\partial t} = \nabla \cdot \left[\left(\int_0^t C_L(t,t')dt' \right) \nabla n_0(\mathbf{r},t) \right] := D_{\text{eff}} \nabla^2 n_0(\mathbf{r},t)$$

Lagrangian correlation

$$\mathbf{C}_L(t,t') \equiv \left\langle \tilde{\mathbf{V}}(\mathbf{r},t)\tilde{\mathbf{V}}(\mathbf{R}(t'|\mathbf{r},t),t') \right\rangle$$

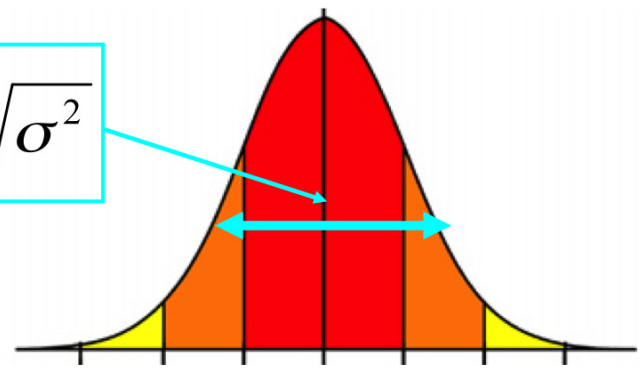
Lagrangian trajectory

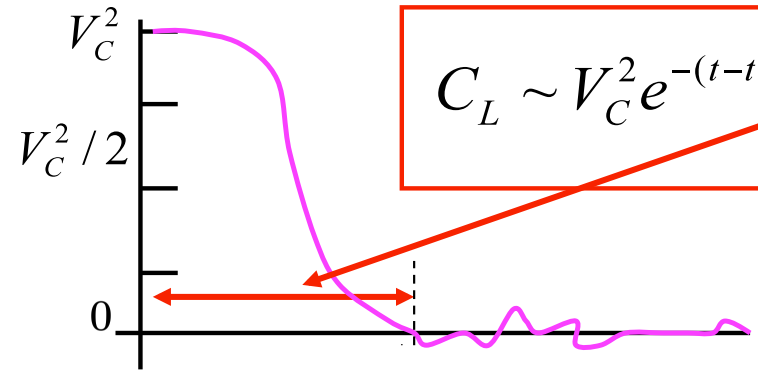


CHARACTERISTIC SCALES

GAUSSIAN + **MARKOVIAN**

$$V_c \sim \sqrt{\sigma^2}$$

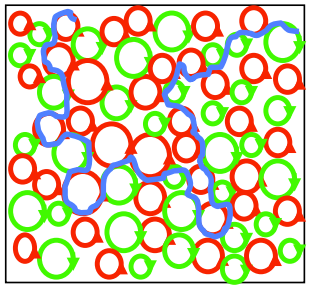


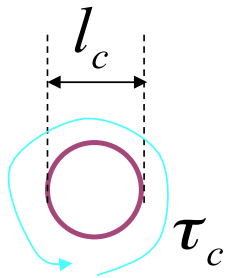


$$C_L \sim V_C^2 e^{-(t-t')/\tau_C}$$

$$\frac{\partial n_0}{\partial t} \simeq D\frac{\partial^2 n_0}{\partial x^2}, \quad D \sim (\tilde{V}_c^2 \tau_c)$$

Lagrangian trajectory





$$l_c \sim V_c \tau_c$$

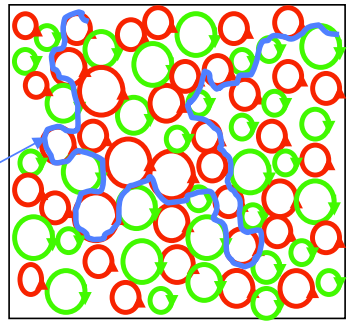
SCALE-FREE TURBULENT TRANSPORT?

GAUSSIAN + MARKOVIAN

NO 'scale-free' transport, since global transport dynamics set by length/time scales of the elements (i.e., "eddies")

↓

Lagrangian trajectories



Scale-free turbulent transport requires:

NON-GAUSSIAN (i.e., *Lagrangian velocity dist. ~ Levy pdfs*)

NON-MARKOVIAN (i.e., *long-term correlations along characteristics*)

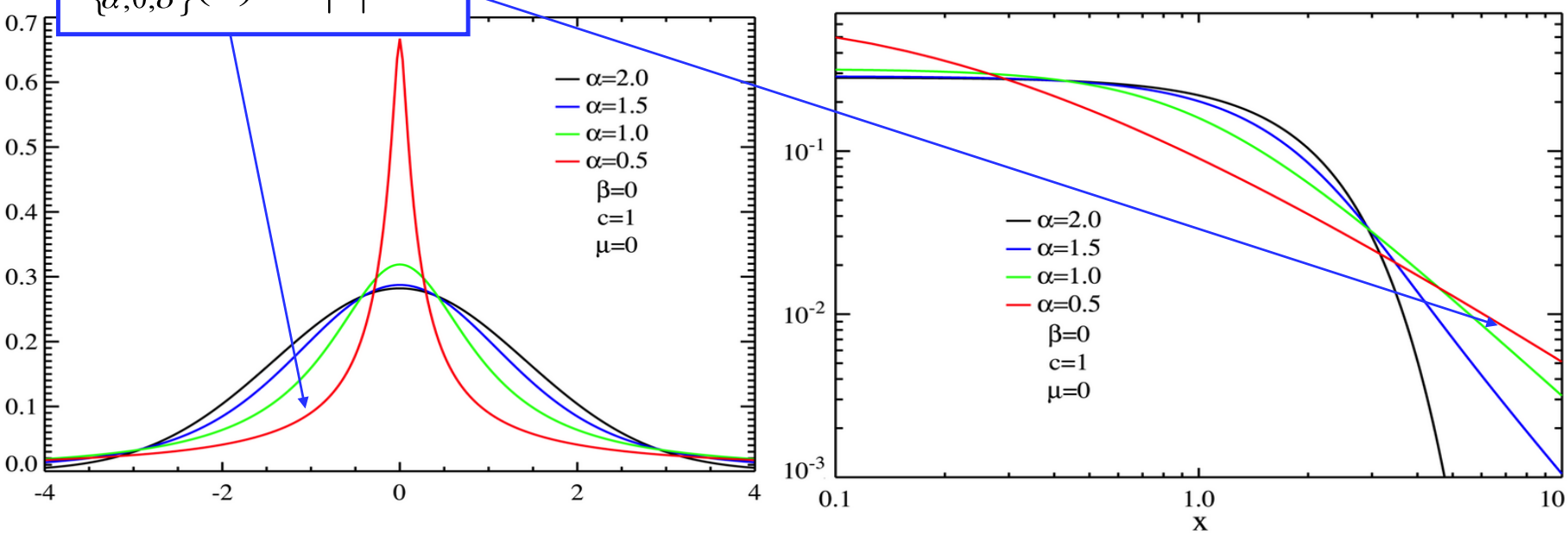
NON-GAUSSIANITY: SYMMETRIC LEVY PDFs

Levy distributions: complete central limit theorem if relaxing condition on exponential decay.

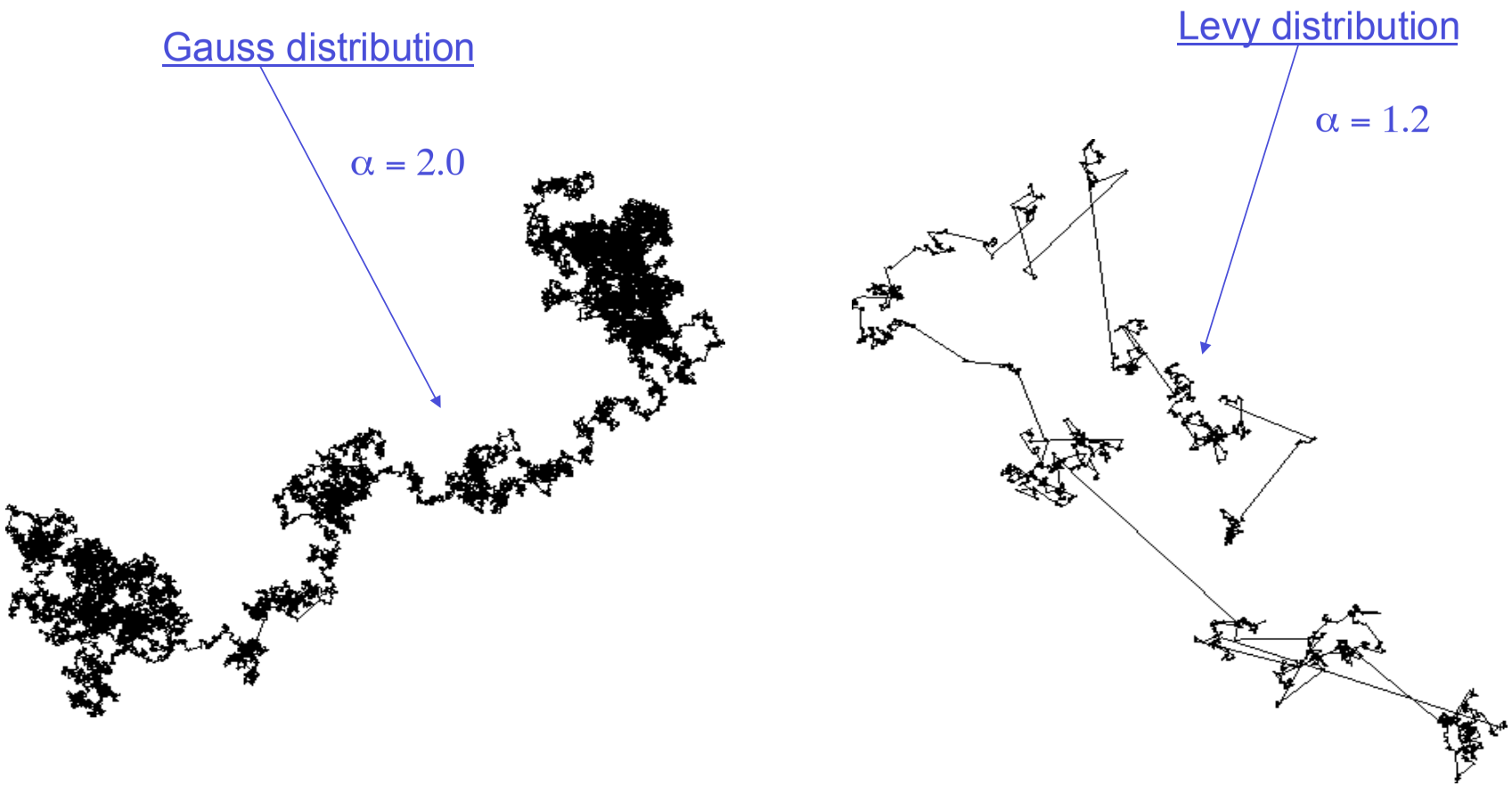
$$P_{\{\alpha,0,\sigma\}}(k) = \exp\left(-\sigma^{\alpha}|k|^{\alpha}\right) \quad \alpha \le 2$$

[See: Samorodnitsky & Taqqu, *Stable non-Gaussian Processes*, Chap.& Hall (1994)]

$$P_{\{\alpha,0,\sigma\}}(x) \rightarrow |x|^{-(1+\alpha)}$$

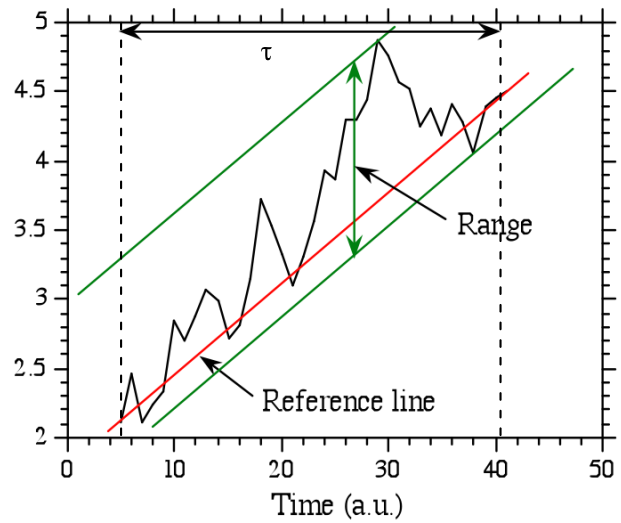


GAUSS vs. LEVY RANDOM WALK



LAGRANGIAN MEMORY

Quantify correlation along the Lagrangian trajectory using the Hurst (or self-similarity) exponent of the Lagrangian radial velocity time series:



$$[R/S](\tau) \equiv \frac{\max_{1\le k\le\tau} W(k,\tau) - \min_{1\le k\le\tau} W(k,\tau)}{\sqrt{\langle V^2\rangle_\tau - \langle V\rangle_\tau^2}} \sim \tau^{H}$$

$$W(k,\tau) \equiv \sum_{i=1}^{k} V_i - k\,\langle V\rangle_\tau$$

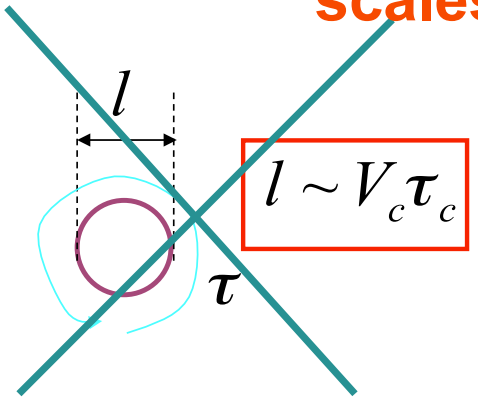
NOTE: Use $(\sigma^s)^{1/s}$, $s<\alpha$ if α-Levy

$H=1/\alpha$, random
$H> 1/\alpha$, correlated positively
H $H < 1/\alpha$, correlated negatively

$H=1/2$, diffusion
$H > 1/2$, superdiffusion
$H < 1/2$, subdiffusion

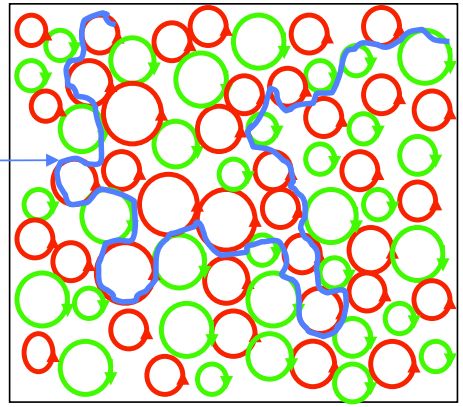
‘SCALE-FREE’ TURBULENT TRANSPORT

No typical scales!



$$l \sim V_c \tau_c$$

Lagrangian trajectories



“Fractional Renormalization”

[See: R. Sanchez et al, Phys. Rev. E **74**, 016305 (2006)]

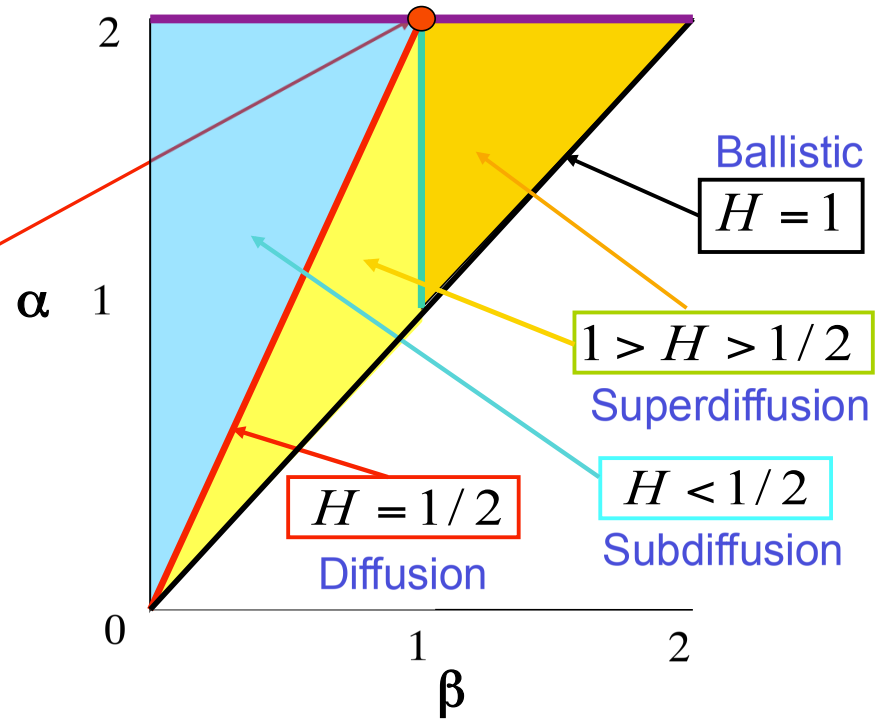
β

$$\frac{\partial n_0}{\partial t} = {}_0\mathrm{D}_t^{1-\alpha H}\left[D'' \frac{\partial^\alpha n_0}{\partial |x|^\alpha}\right]$$

$$\frac{\partial n_0}{\partial t} = D t^{\alpha H - 1} \frac{\partial^\alpha n_0}{\partial |x|^\alpha}$$

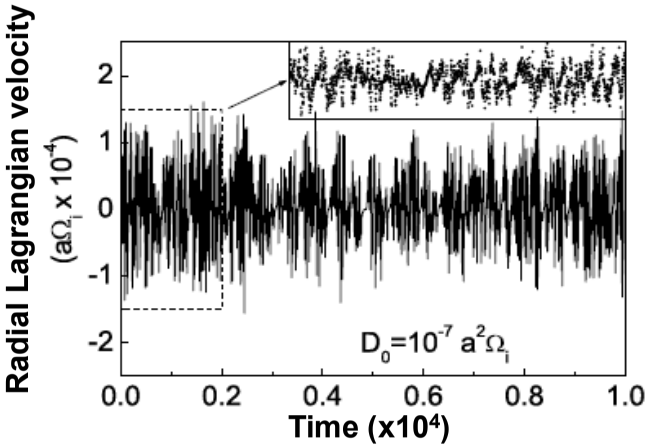
SUB-, SUPER- AND DIFFUSIVE TRANSPORT

$$\frac{\partial n(x,t)}{\partial t} \approx D \frac{\partial^2 n(x,t)}{\partial x^2}$$



[See: R. Metzler and J. Klafter, Phys. Rep. **339**, 1 (2000); G. Zaslavsky, Phys.Rep. **371**, 461 (2002)]

DETECTING "SCALE-FREE" TRANSPORT



METHOD 1

Estimate α (statistics): [See: J.A. Mier et al, Phys. Rev. Lett.**101**, 112301 (2008)]

construct probability density function of L. velocities

Estimate H (correlation): $[R/S]_k(\tau) \sim \tau^H$

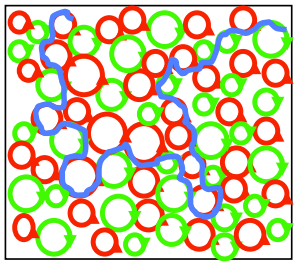
compute Hurst exponent of L. velocity series.

Self-similarity requires that, for α-Levy, H-correlated Lagrangian velocities, propagator *P(x,t)* is α-Levy pdf and scaling exponent H.

"Lagrangian" trajectories (or characteristics)

$$\frac{d\mathbf{R}}{d\tau} = \tilde{\mathbf{V}}(\mathbf{R},\tau)$$

$$\mathbf{R}(t) = \mathbf{r}$$



METHOD 2

Estimate α: $P(x,t) = t^{-H} L_\alpha\left(x/t^H\right)$

form of the propagator

Estimate H : $\sigma_k(\tau) \sim \tau^{kH}$ $\quad k < \alpha$

scaling of the moments

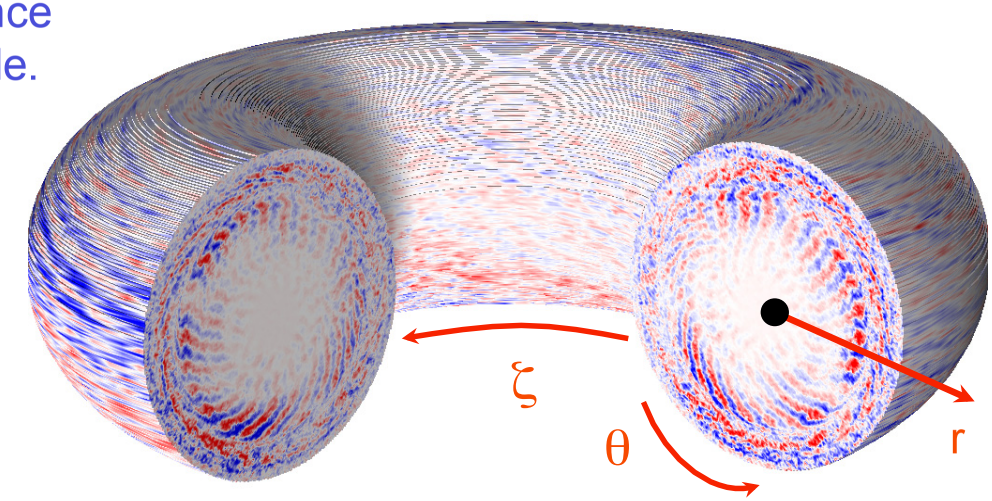
NATURE OF TURBULENT TRANSPORT ACROSS RADIALLY-SHEARED POLOIDAL FLOWS

Ion-temperature-gradient (ITG) turbulence simulations realized with the UCAN code.

UCAN characteristics:

- **δf**-method
- Particle-in-Cell (PIC)
- kinetic ions, adiabatic electrons
- global, toroidal geometry
- collisionless
- electrostatic
- low-β

[See: R. Sydora et al, Pl. Phys. Contr. Fusion 38, A281 (1996)]



RUN parameters:

- **512x512x256 ions**
- 256x256x128 spatial grid points
- 8 particles/cell

PHYSICAL PARAMETERS (DIIID–like):

- Central ion/electron temp [Kev]: 0.68/0.68
- Magnetic field [T]: 1.8
- Axis density [$x10^{20}m^{-3}$]: 0.2
- Major/minor radius [m]: 2.5/0.56
- Ion Larmor radius [mm]: 2.85
- Temporal length of simulation [ms]: 5.56

PIC gyrokinetic simulations of ITG turbulence

ITG are drift waves driven by a gradient in ion temperature.

Collisionless, gyro-averaged kinetic equation:

$$\frac{\delta f}{\delta t} + \dot{\mathbf{R}} \cdot \nabla f + v_{\parallel} \nabla_{v_{\parallel}} f = 0$$

Low noise, delta-f method:

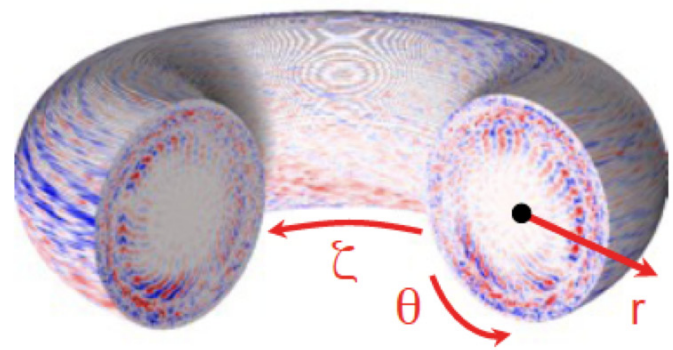
$$f = \delta f + f_0$$

$$f_0(r, v_{\parallel}) = n_o \left(2\pi T_0^i / m_i\right)^{-3/2} \exp\left(-\frac{(m_i/2)v_{\parallel}^2 + \mu B}{T_0^i}\right)$$

Characteristics:

$$\dot{v}_{\parallel} = -\left[\mathbf{b} + \frac{v_{\parallel}}{\Omega_i}\mathbf{b} \times (\mathbf{b} \cdot \nabla \mathbf{b})\right] \cdot \left(\frac{Ze}{m_i}\frac{\partial \bar{\phi}}{\partial \mathbf{R}} + \mu \nabla B\right)$$

$$\dot{\mathbf{R}} = v_{\parallel}\mathbf{b} - \frac{1}{B^2}\left(\nabla_{\mathbf{R}}\bar{\phi} \times \mathbf{B}\right) + \frac{1}{\Omega_i}\left(\mu \nabla B + v_{\parallel}^2 \frac{\mathbf{b} \cdot \nabla B}{B}\right)$$



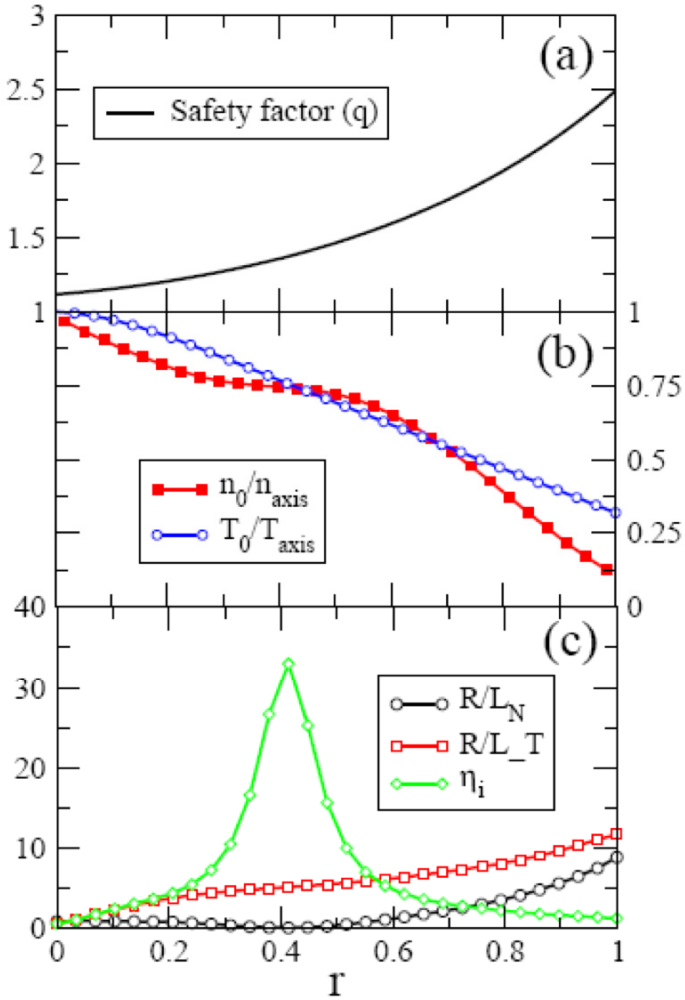
Poisson equation:

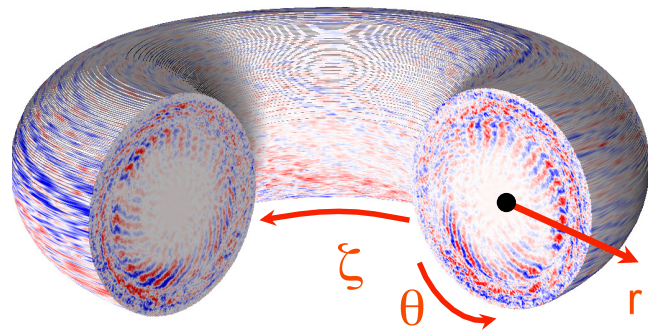
$$\frac{T_e}{T_i}\frac{\bar{\phi} - \phi}{\lambda_D^2} = e(Z\bar{n}_i - n_e)$$

Weights: $w = \dfrac{\delta f}{f_0}$

$$\frac{dw_i}{dt} = -(1 - w_i)\left[\frac{v_{\parallel}}{v_{th}}\frac{Ze}{m_i}\frac{\partial \bar{\phi}}{\partial \mathbf{R}} - \frac{1}{B^2}\left(\frac{\partial \bar{\phi}}{\partial \mathbf{R}} \times \mathbf{B}\right)\right] \cdot G(n_0, T_o^i)$$

PROFILES USED FOR SIMULATIONS





Most unstable location circa r/a ~ 0.4

$$R/L_T = 5.25,\ R/L_n = 0.16,\ q = 1.36$$

$$\hat{s} \equiv (r/q)(dq/dr) = 0.28$$

Ion trajectories

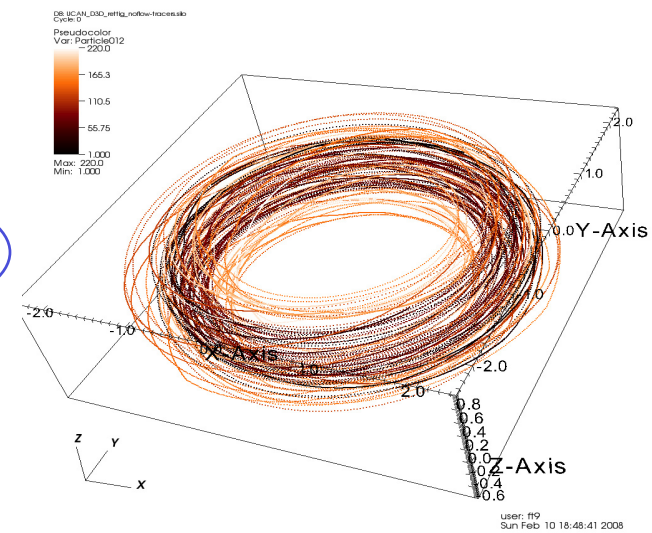
The spatial part of the characteristic along which markers are pushed gives the gyro-averaged ion orbit in real space:

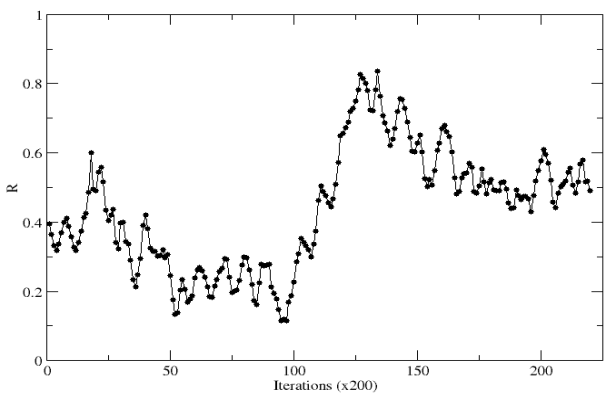
$$\dot{\mathbf{R}} = v_{\parallel}\vec{b} - \frac{c}{B}\nabla_R\varphi\times\vec{b} + \frac{1}{\Omega}\vec{b}\times\left[\mu\nabla B + v_{\parallel}^2\frac{\vec{b}\cdot\nabla B}{B}\right]$$

Parallel motion

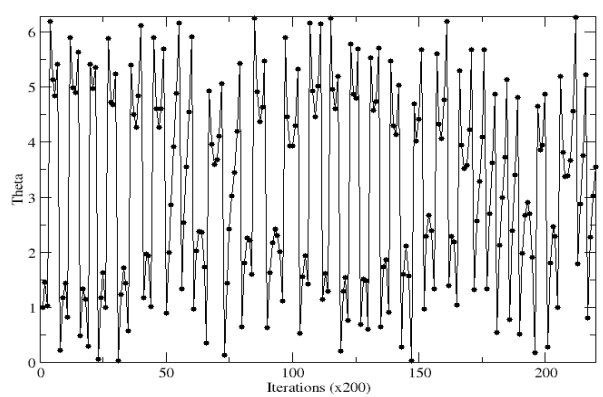
ExB drifts

magnetic drifts

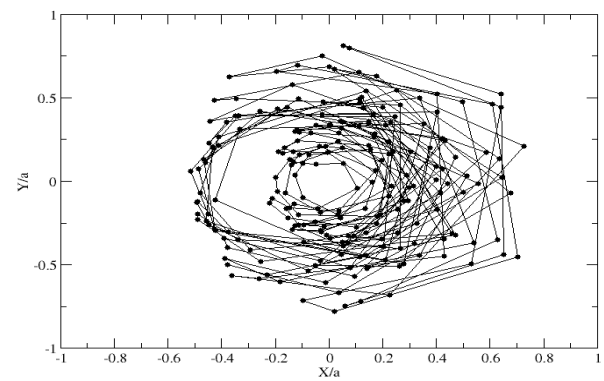




r vs time

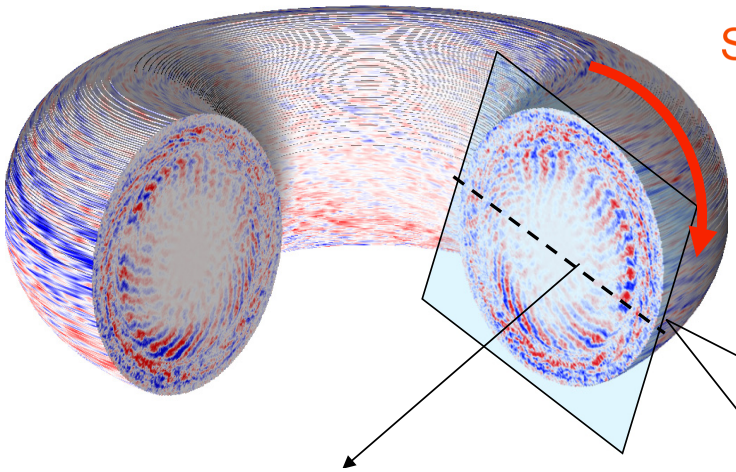


θ vs time

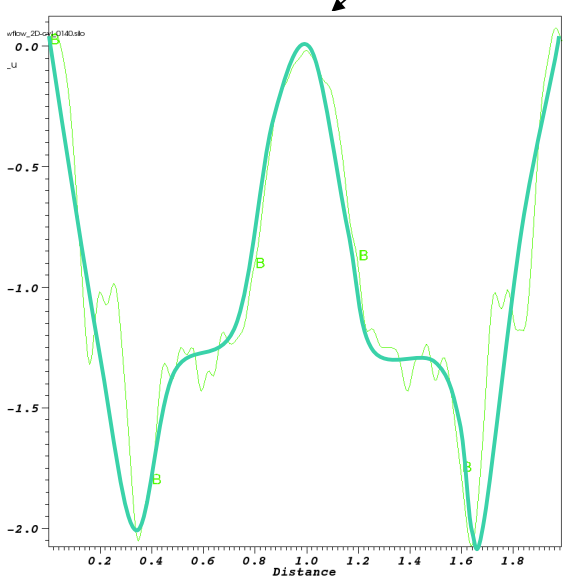


r-θ Poincare plot

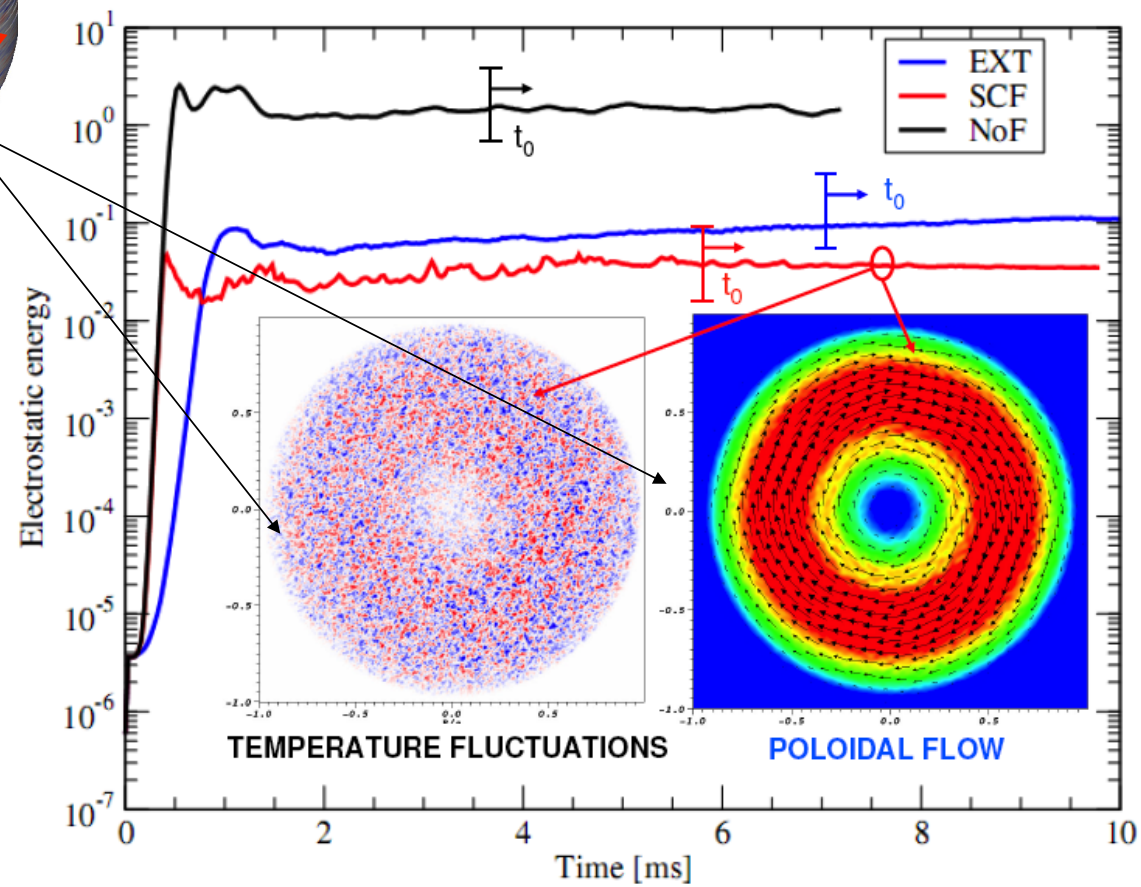
SATURATED PHASE



Sheared poloidal (zonal) flow

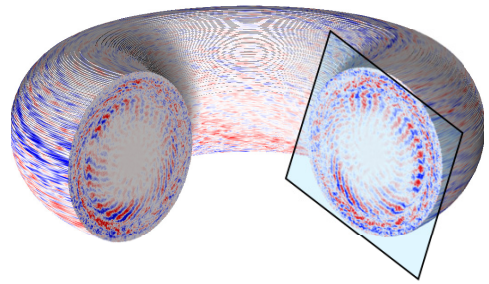


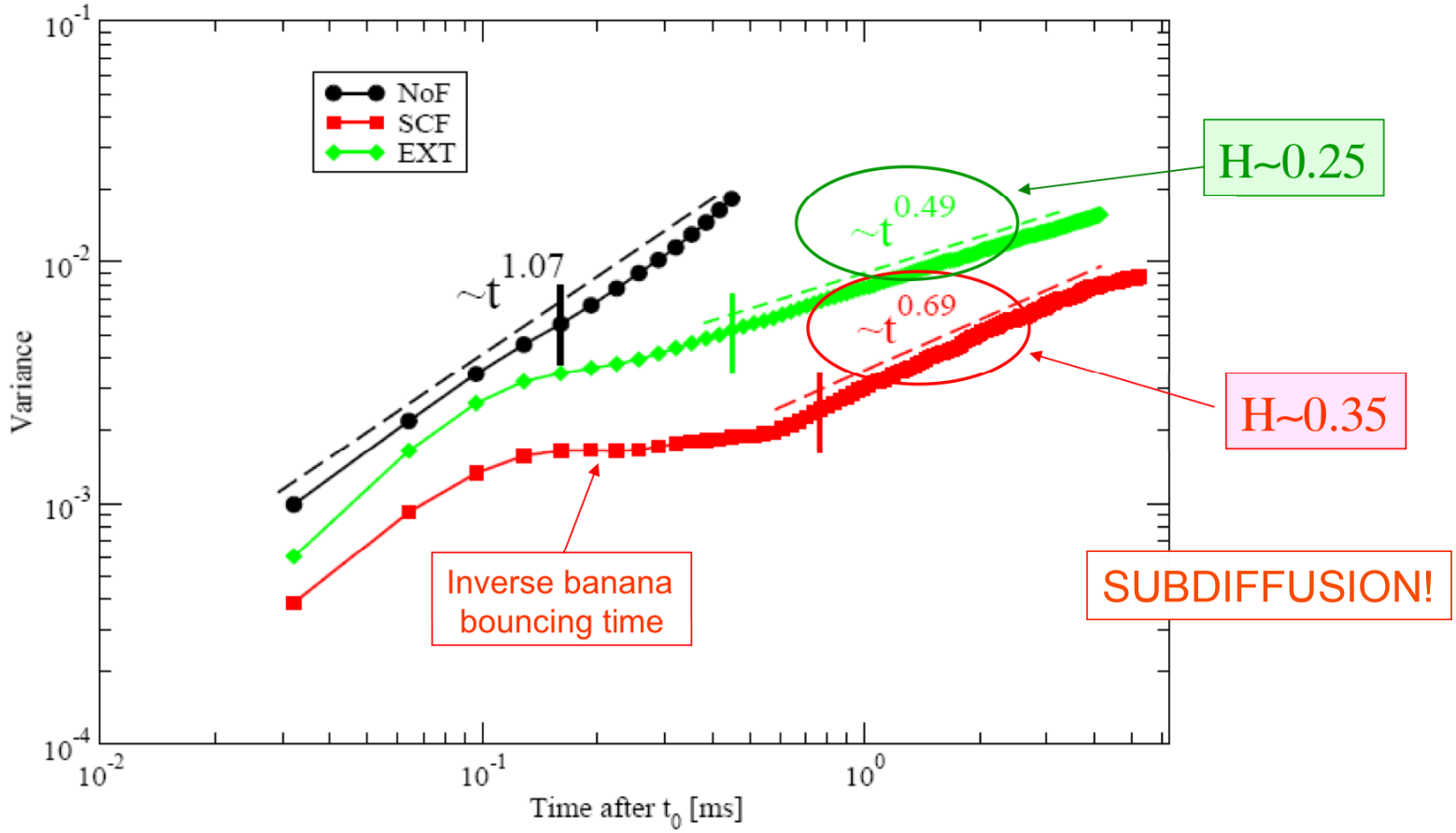
Electrostatic energy



ESTIMATING H

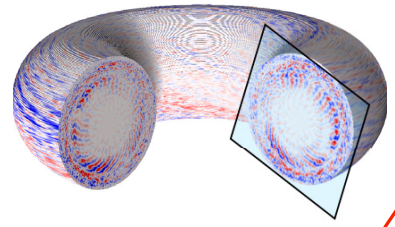
[See: R. Sanchez et al, Phys. Rev. Lett.**101**, 200302 (2008)]

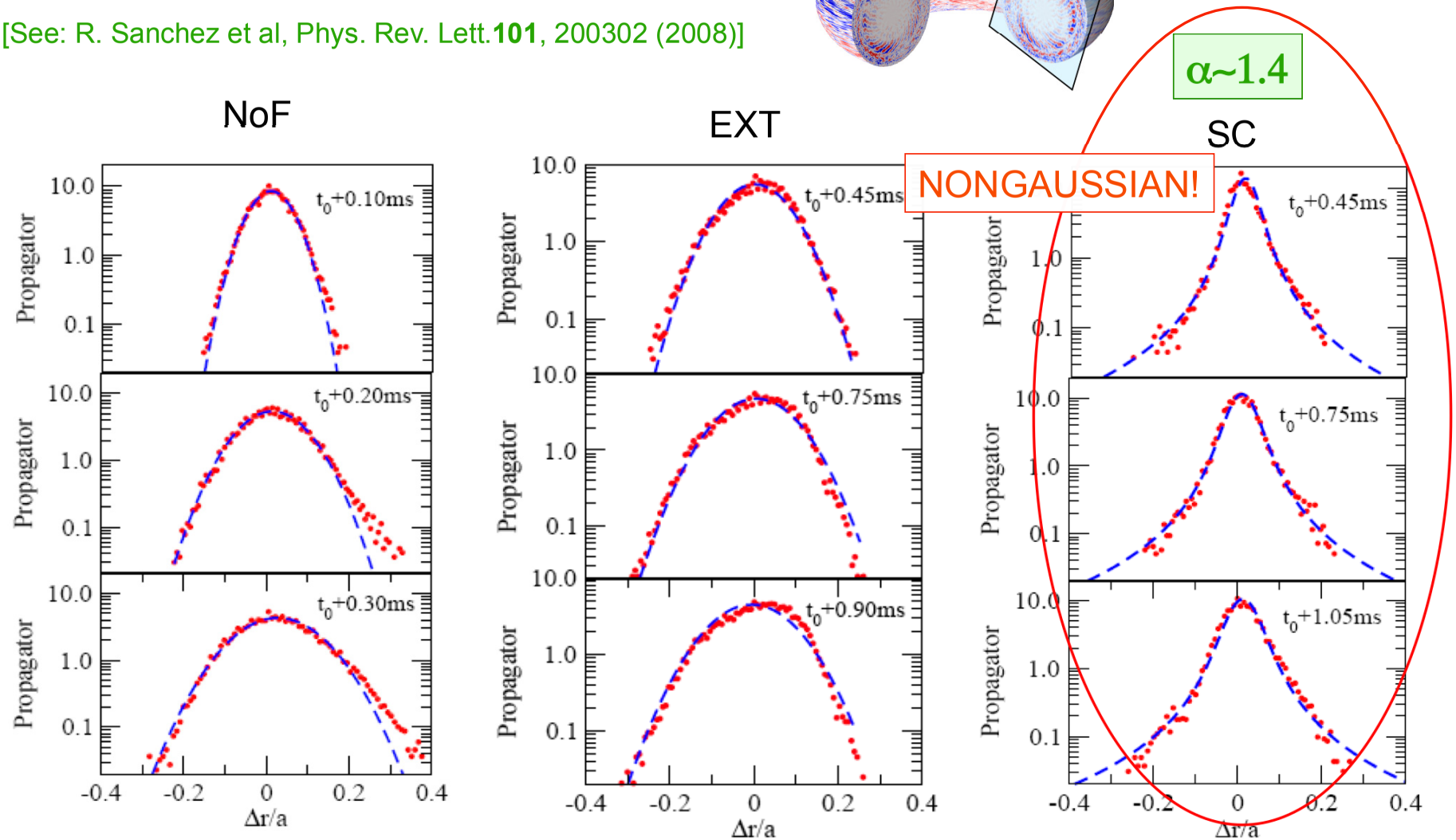




ESTIMATING α

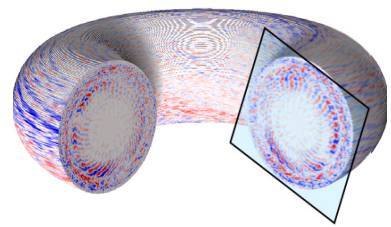
[See: R. Sanchez et al, Phys. Rev. Lett.**101**, 200302 (2008)]



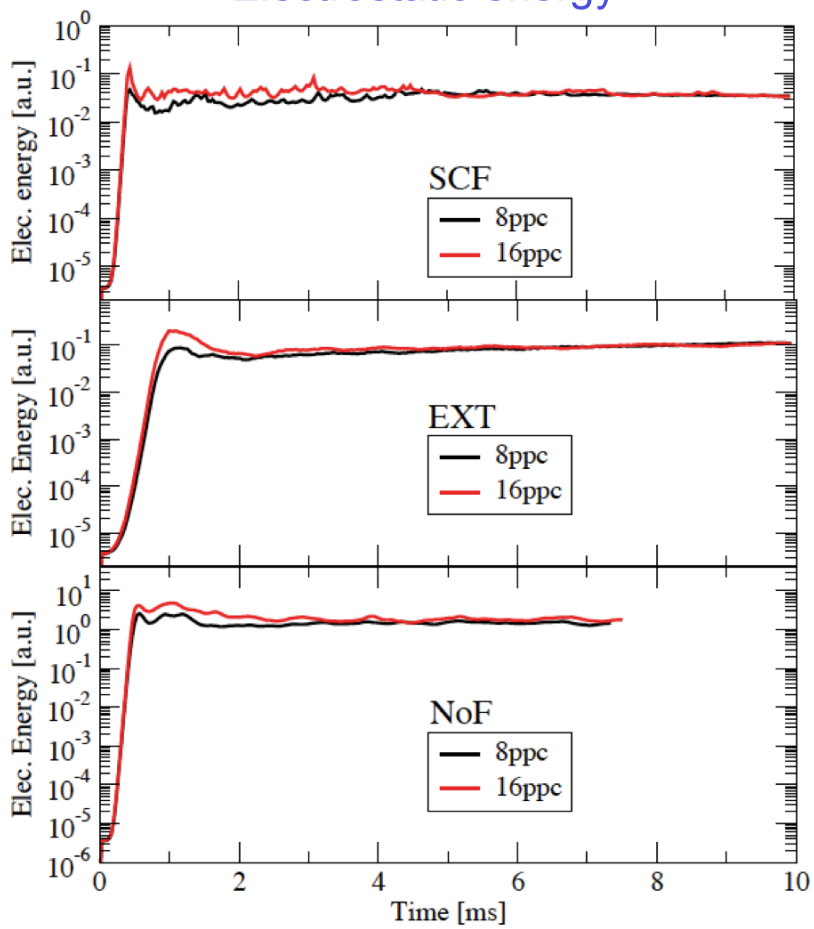


Convergence with part-per-cell

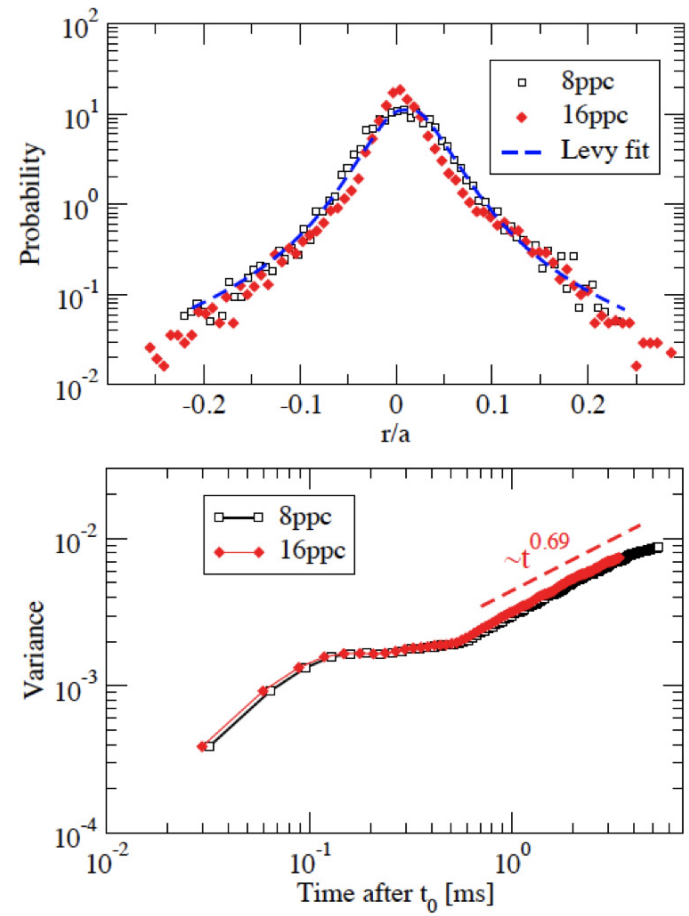
[See: R. Sanchez et al, Phys. Plasmas **12**, 055905 (2009)]



Electrostatic energy

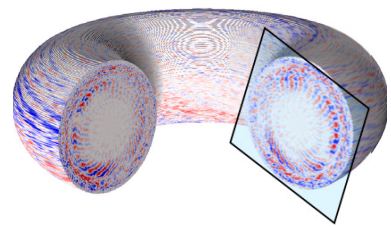


Self-consistent case

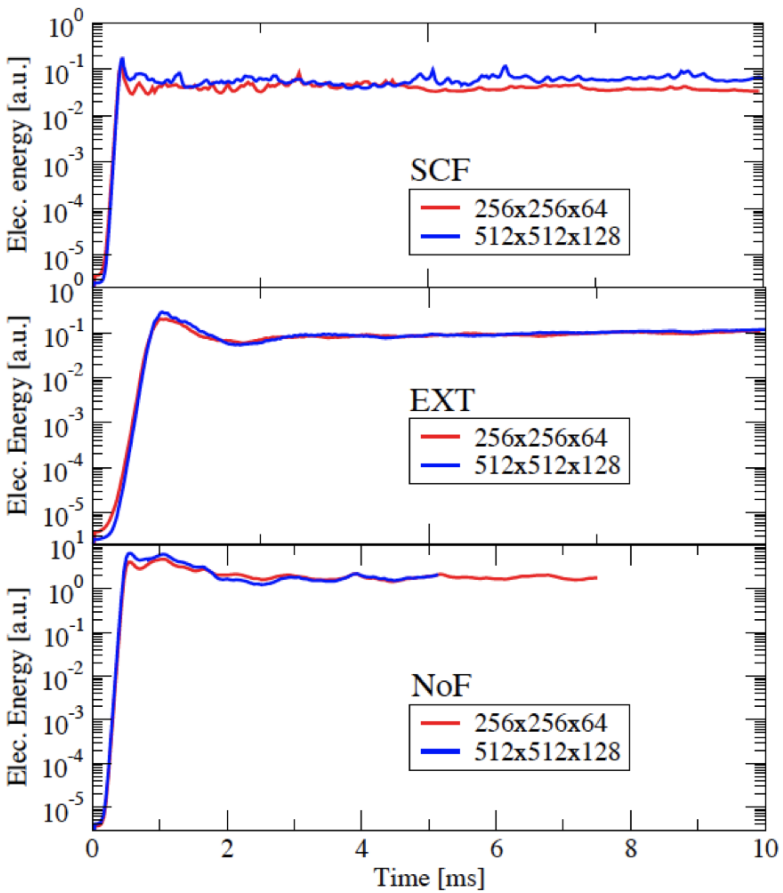


Convergence with grid

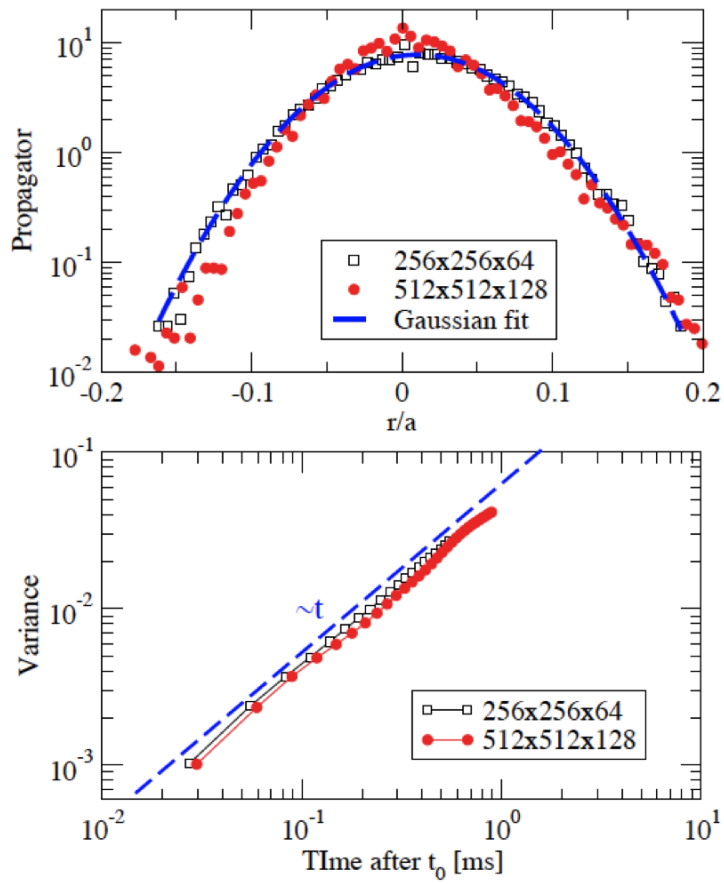
[See: R. Sanchez et al, Phys. Plasmas **12**, 055905 (2009)]



Electrostatic energy

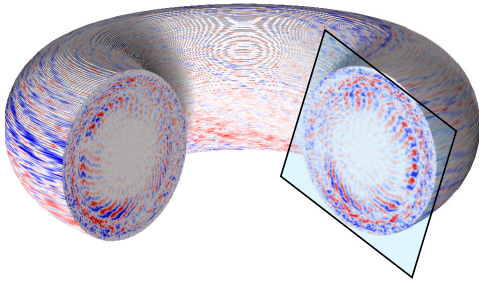


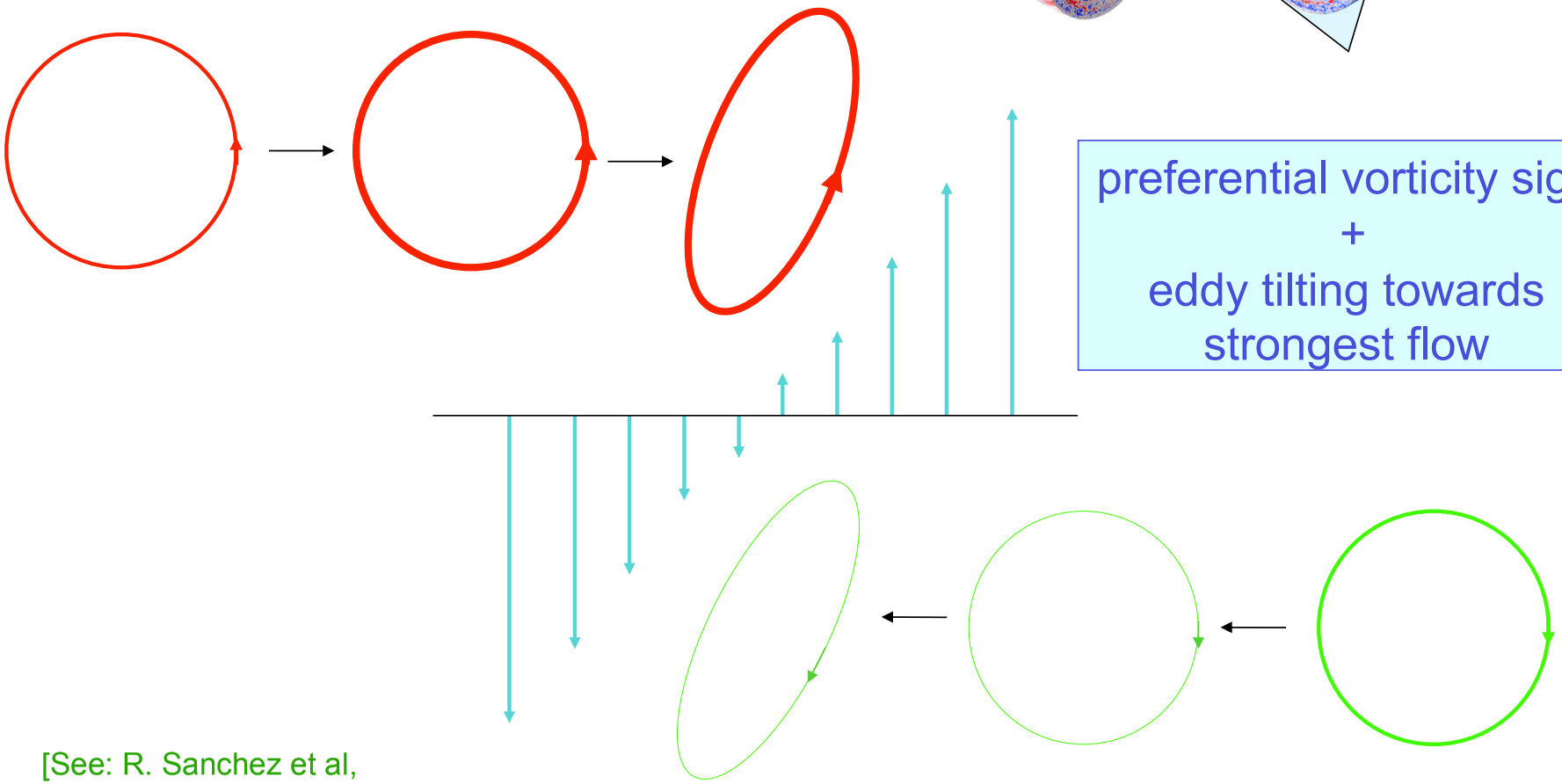
No flow case



Which are the physical mechanisms causing self-similar transport across a sheared poloidal flow?

Mechanism for subdiffusion



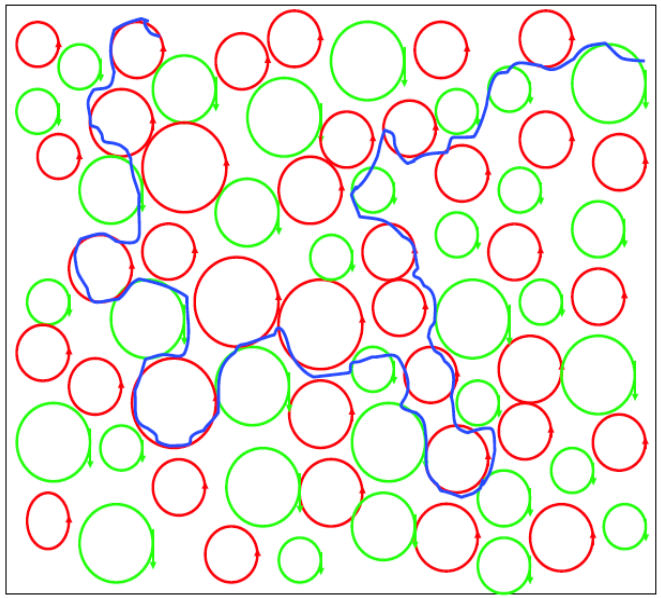


preferential vorticity sign
+
eddy tilting towards
strongest flow

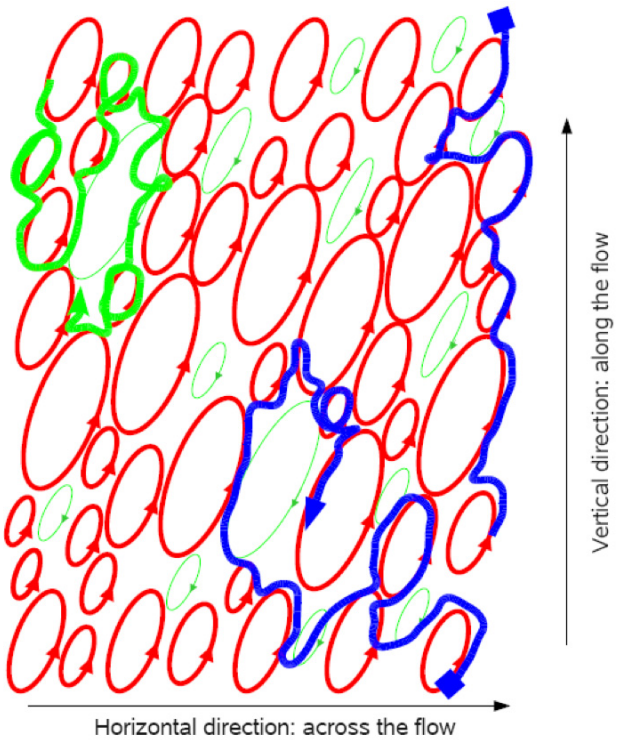
[See: R. Sanchez et al,
Physics of Plasmas 16, 055905 (2009)]

Mechanism for subdiffusion

NO FLOW



WITH FLOW



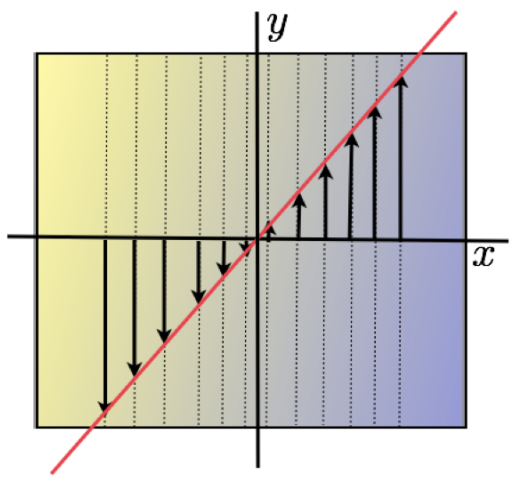
Presence of a sheared flow makes successive radial Lagrangian velocites anti-correlated in time!

Mechanism for subdiffusion

In a toroidal/cylindrical geometry with quasi-2D turbulence, the preferential vorticity sign argument cannot be carried over. The reason is that a constant poloidal rotation yields a constant vorticity that does not affect the turbulence.

The equivalent case to the Cartesian linear sheared flow is a linear angular velocity profile, or a quadratic poloidal velocity profile. The quantity of merit is now a preferential sign of the radial derivative of the axial vorticity.

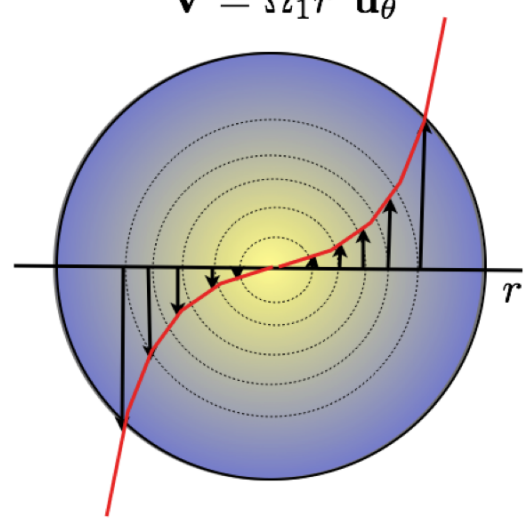
$$\mathbf{V} = Ax\ \mathbf{u_y}$$



$$w_z = A$$

$$\mathbf{\Omega} = \Omega_1 r\ \mathbf{u_z}$$

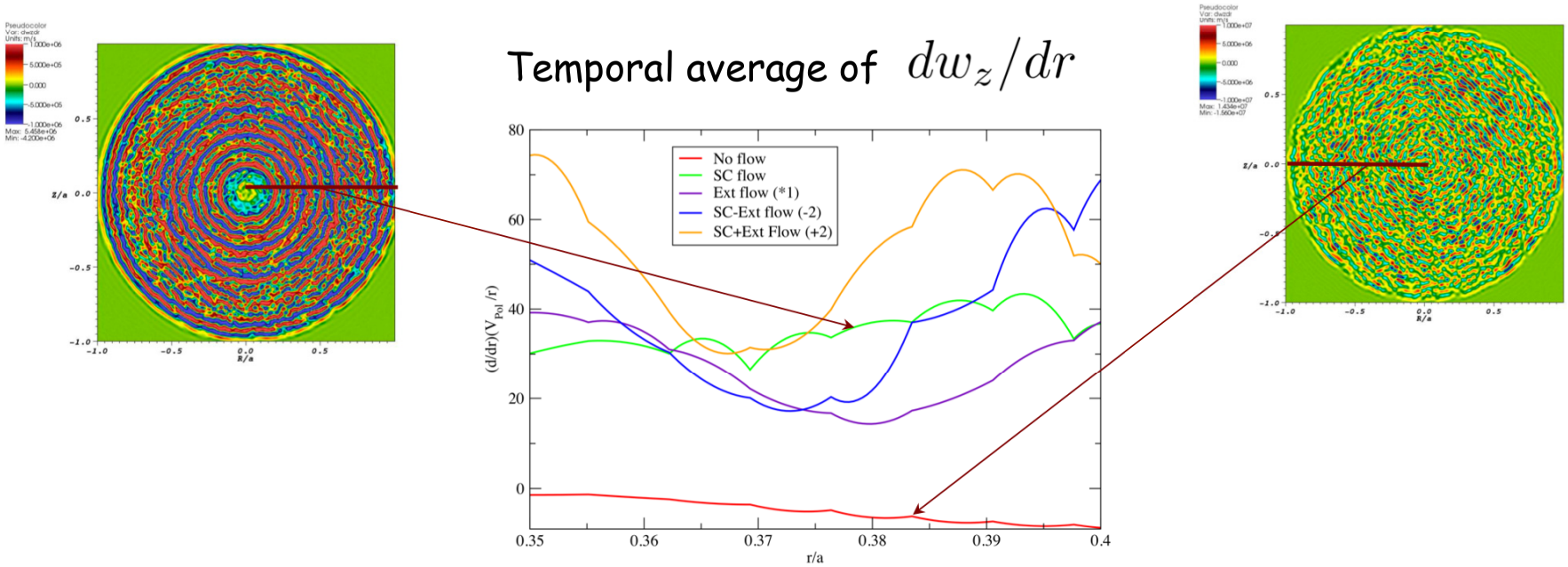
$$\mathbf{V} = \Omega_1 r^2 \mathbf{u}_\theta$$



$$\frac{dw_z}{dr} = 3\Omega_1$$

Mechanism for subdiffusion

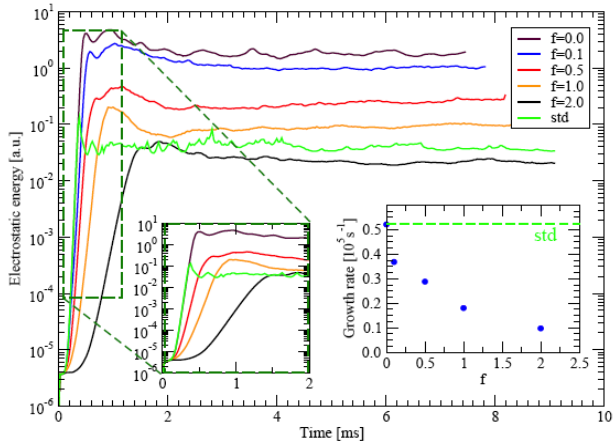
If this is the mechanism responsible for subdiffusion, it would not care about the nature of the shear. Only its magnitude its important. Below, we compare the transport exponents for ions initially contained within in the radial interval (0.35, 0.40), for cases with flows of different (average) shear strength and different nature (self consistent, externally driven, or a combination of the two with opposing or additive external flows).



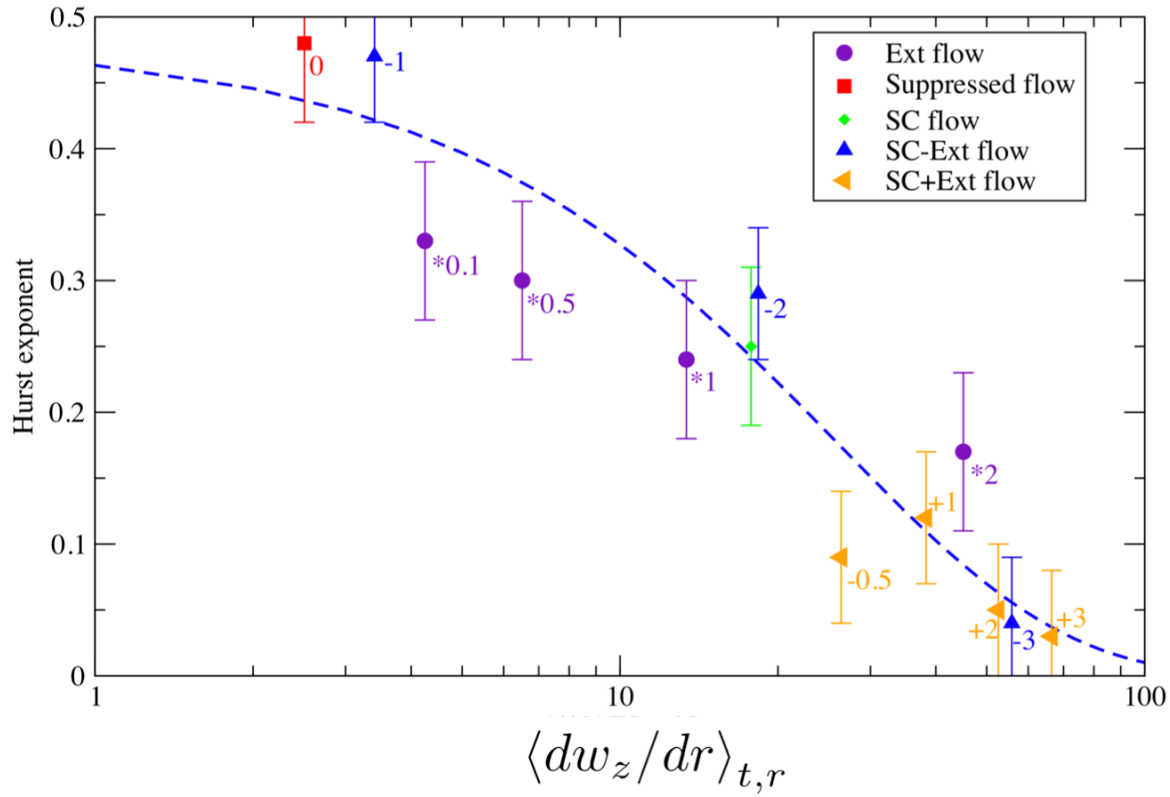
Mechanism for subdiffusion

Subdiffusion only cares about
shear in mean flow component!

Electrostatic Energy Time series of some cases

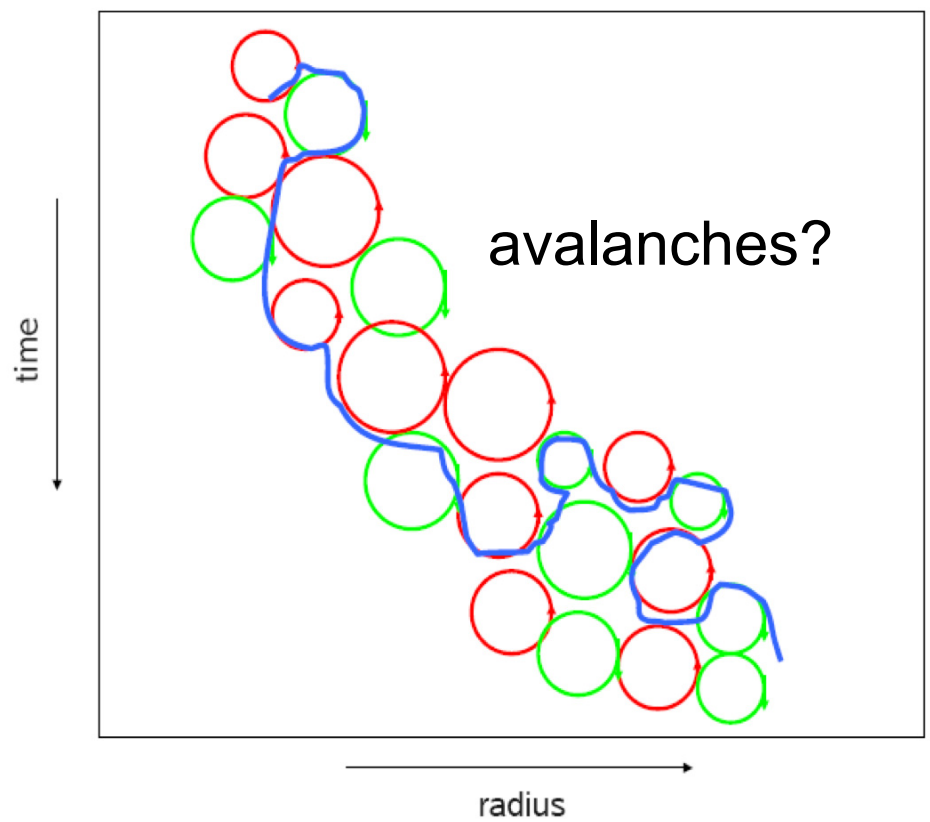


Hurst exponent for varying shears and origins



Mechanism for non-Gaussianity

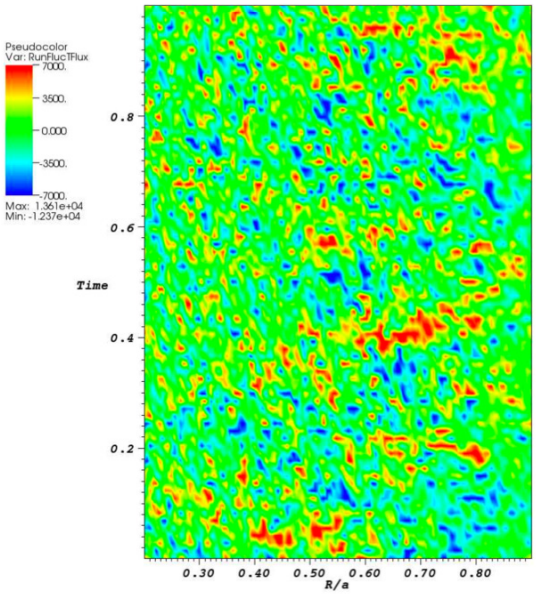
But why do statistics become non-Gaussian and exhibit heavy tails when the flows are self-consistently generated by turbulence? Traditionally, heavy tails are associated with avalanche-like transport. Are there radial avalanches here, which are no longer present if the interaction of ZFs and fluctuations is suppressed? And avalanches of what?



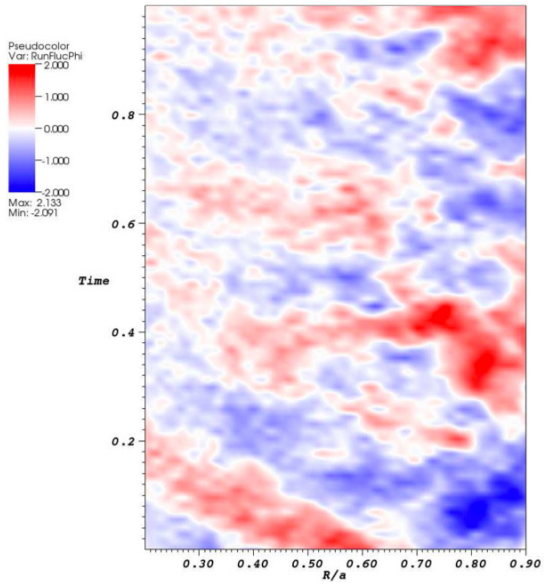
Mechanism for non-Gaussianity

CASE A: coupling ϕ_{00} and fluctuations <u>suppressed</u>.

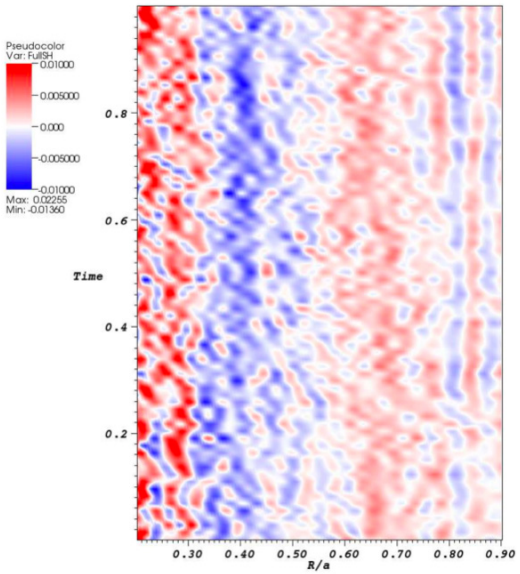
Ion Heat flux



$\phi - \phi_{00}$

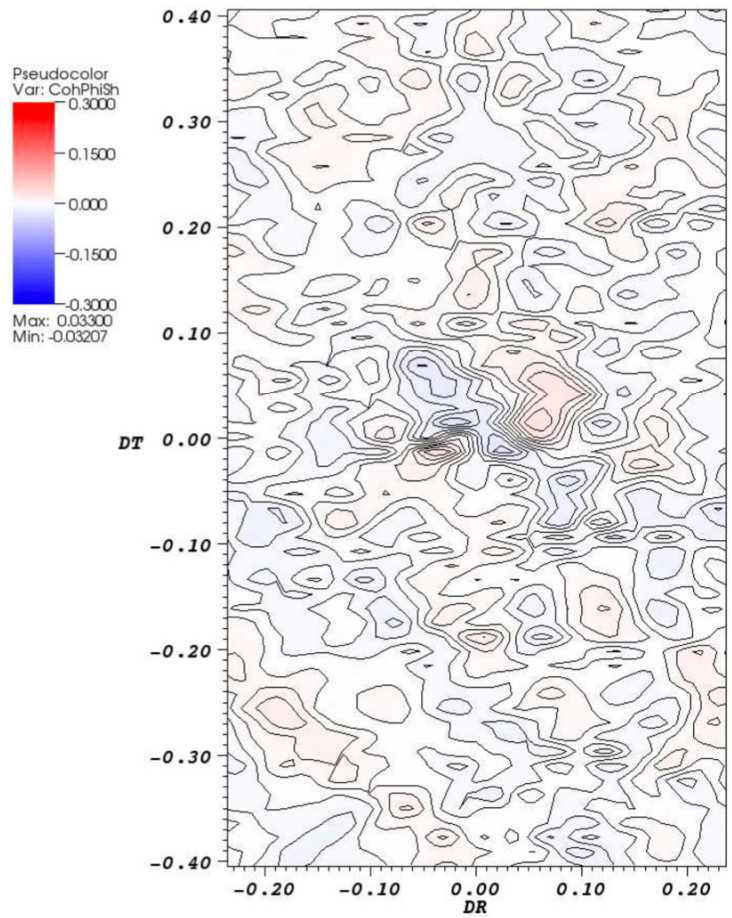


dw_z/dr



Mechanism for non-Gaussianity

CASE A: coupling ϕ_{00} and fluctuations <u>suppressed</u>.

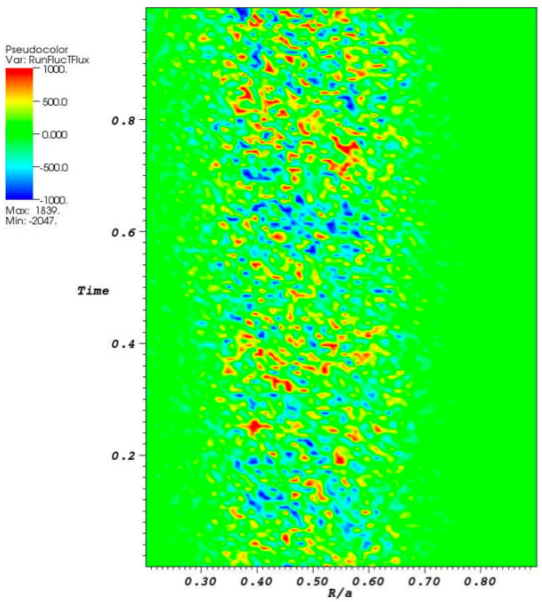


$$\mathrm{Coh}(\mathrm{dw_z}/\mathrm{dr}, \phi - \phi_{00})$$

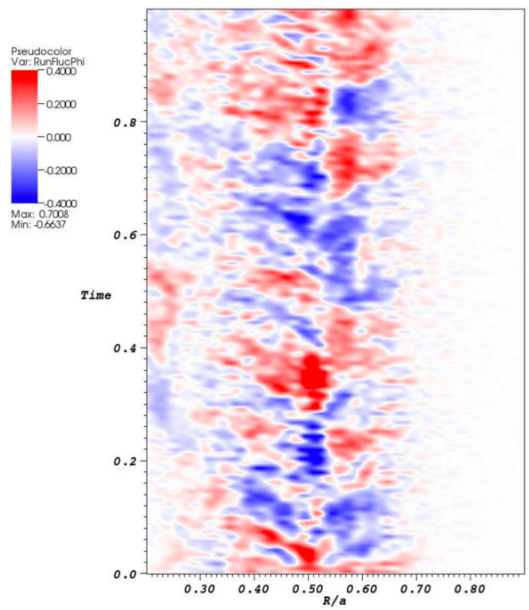
Mechanism for non-Gaussianity

CASE B: coupling ϕ_{00} and fluctuations <u>suppressed</u>, substituted by external flow.

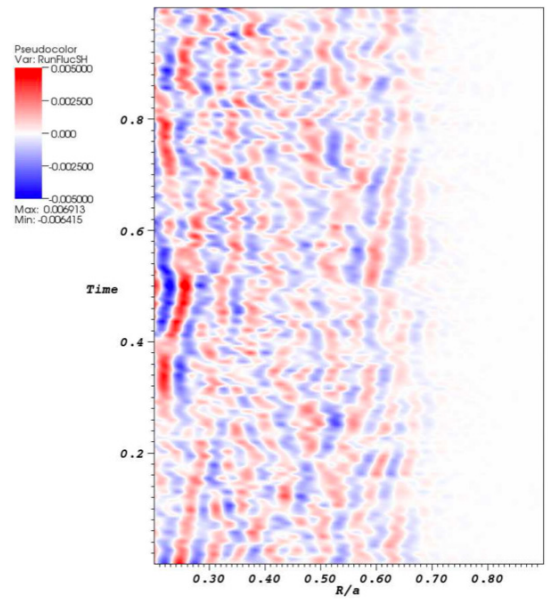
Ion Heat flux



$\phi - \phi_{00}$

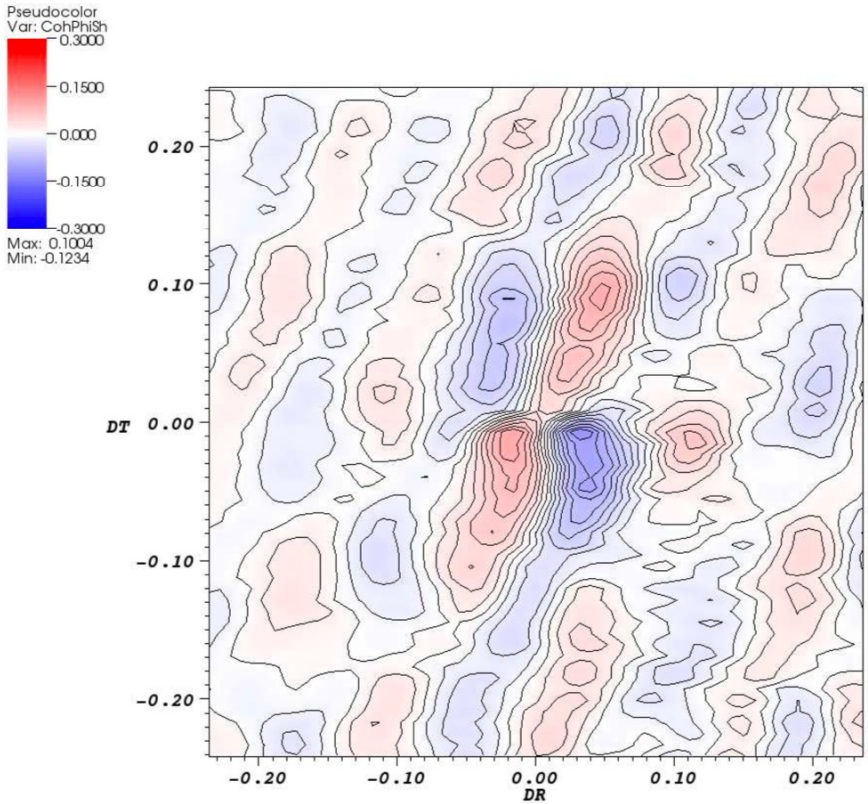


dw_z/dr



Mechanism for non-Gaussianity

CASE B: coupling ϕ_{00} and fluctuations <u>suppressed</u>, substituted by external flow.

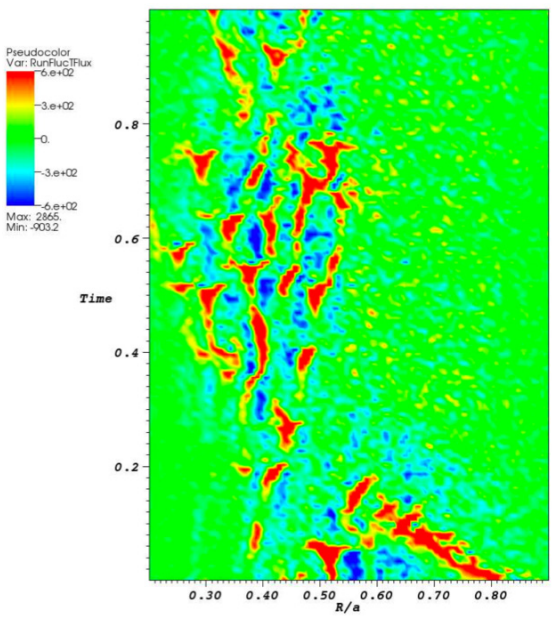


$$\mathrm{Coh}(\mathrm{dw_z/dr}, \phi - \phi_{00})$$

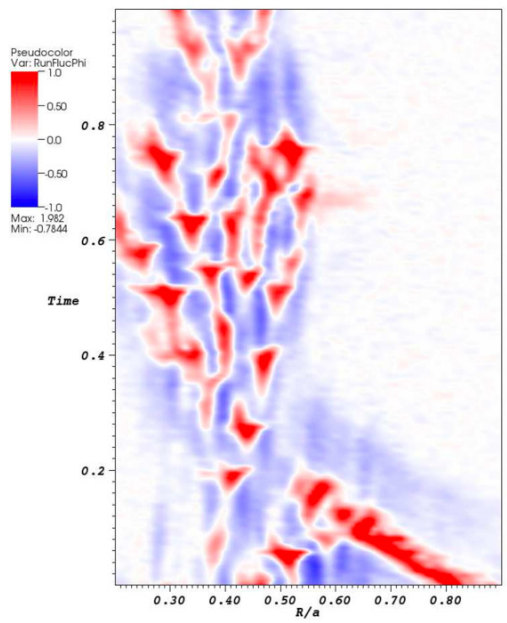
Mechanism for non-Gaussianity

CASE C: Standard GK simulation ➔ ϕ_{00} and fluctuations <u>coupled</u>.

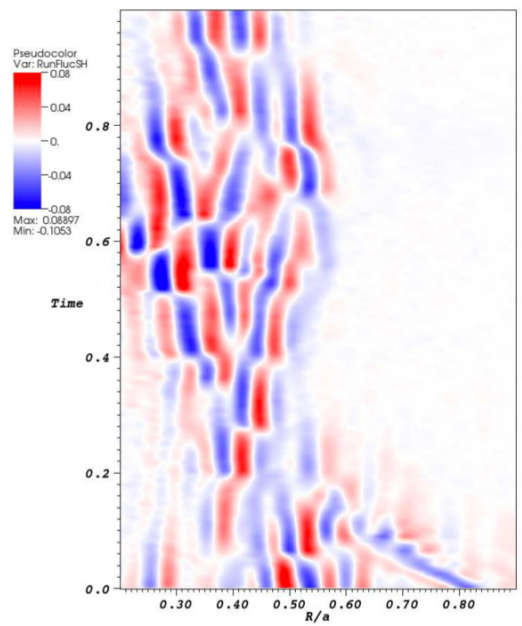
Ion Heat flux



$\phi - \phi_{00}$

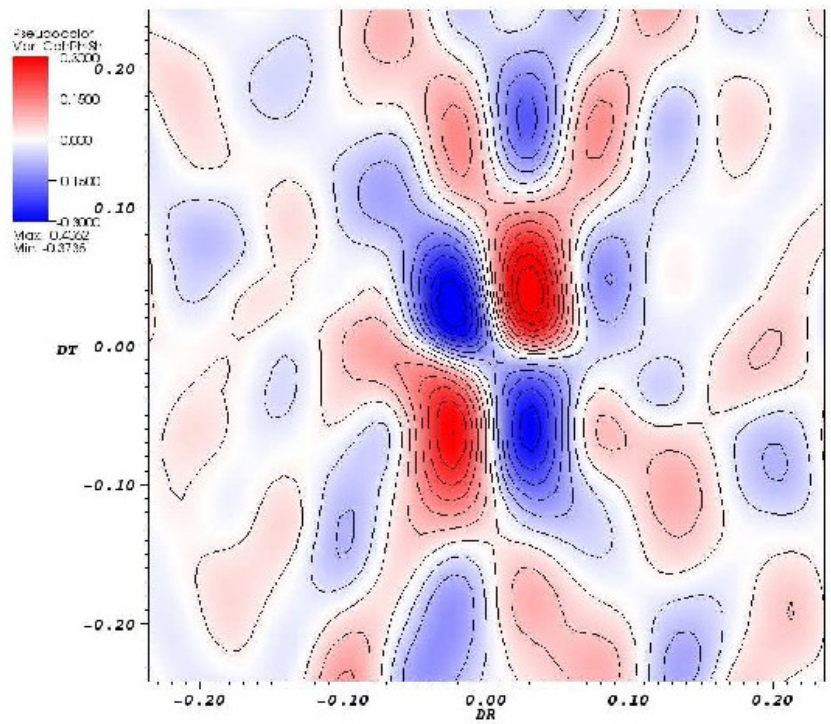


dw_z/dr



Mechanism for non-Gaussianity

CASE C: Standard GK simulation ➔ ϕ_{00} and fluctuations <u>coupled</u>.



$$\mathrm{Coh}(\mathrm{dw_z/dr}, \phi - \phi_{00})$$

CONCLUSIONS

- Transport across a radially-sheared poloidal flow is subdiffusive for timescales much longer that turbulent timescales.

- If full dynamics kept, it appears to have also a non-Gaussian character.

- Values of α and H will depend on shear strength, extension and lengthscale.

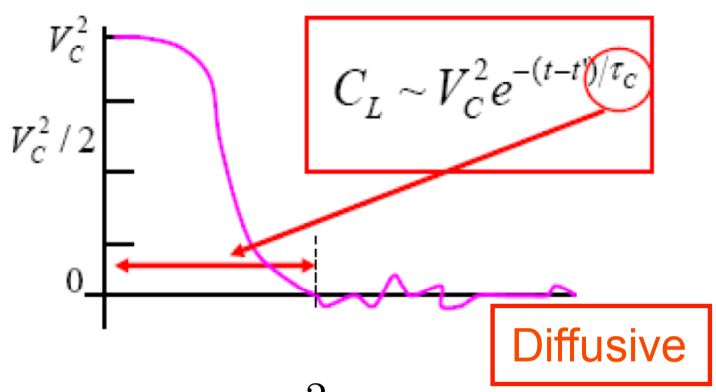
- Mechanism(s) are very general: probably applicable to ANY stable sheared flow: atmosphere, ocean,….

- Quasilinear estimates of diffusion may greatly overestimate transport in this case, even if *diffusive transport is eventually (maybe superdiffusive?) MUST be finally established*.

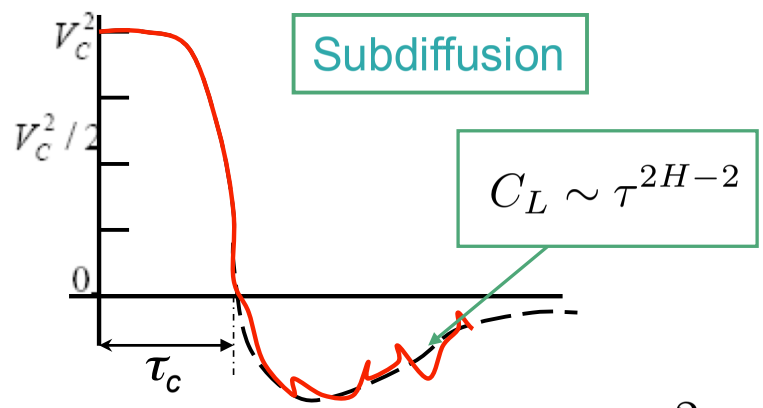
SUBDIFFUSION AND CORRELATION FUNCTION

Quasilinear estimates of diffusion may greatly overestimate transport in this case, even if *diffusive transport is eventually (?) established*.

$$D = \int_0^\infty d\tau C_L(\tau)$$



$$D \sim V_c^2 \tau_c$$



$$D \ll V_c^2 \tau_c$$

$$K \equiv V_c \tau_c / l_c \sim 1$$