New directions in stellarator optimization/design

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Recent interest in stellarator design/optimization

- Fusion ReNeW document recommends:
  - Prepare to build 2 U.S. POP stellarator facilities
  - Longer-term U.S. PE class stellarator
- NCSX restart is off-the-table for now
- Potential physics targets have evolved since NCSX/QPS
  - Flux surface quality (SIESTA, CAS3D, etc.)
  - Plasma microturbulence levels (geodesic curvature)
  - Fast ion driven Alfvén instabilities
  - Plasma flow optimization
  - Coupling to divertors
- Incorporate various engineering lessons learned

Compactness: vice or virtue? (near term devices)

- Lower surface to volume ratio desirable: lower recycling, better neutral/impurity shielding, etc.
  - However, new wall conditioning methods (lithium) can lower recycling
- Compact geometries ➡ lower development costs
  - However, compactness -> awkward geometries (NCSX inner coil welding, QPS split center coil)

QPS coil fixture

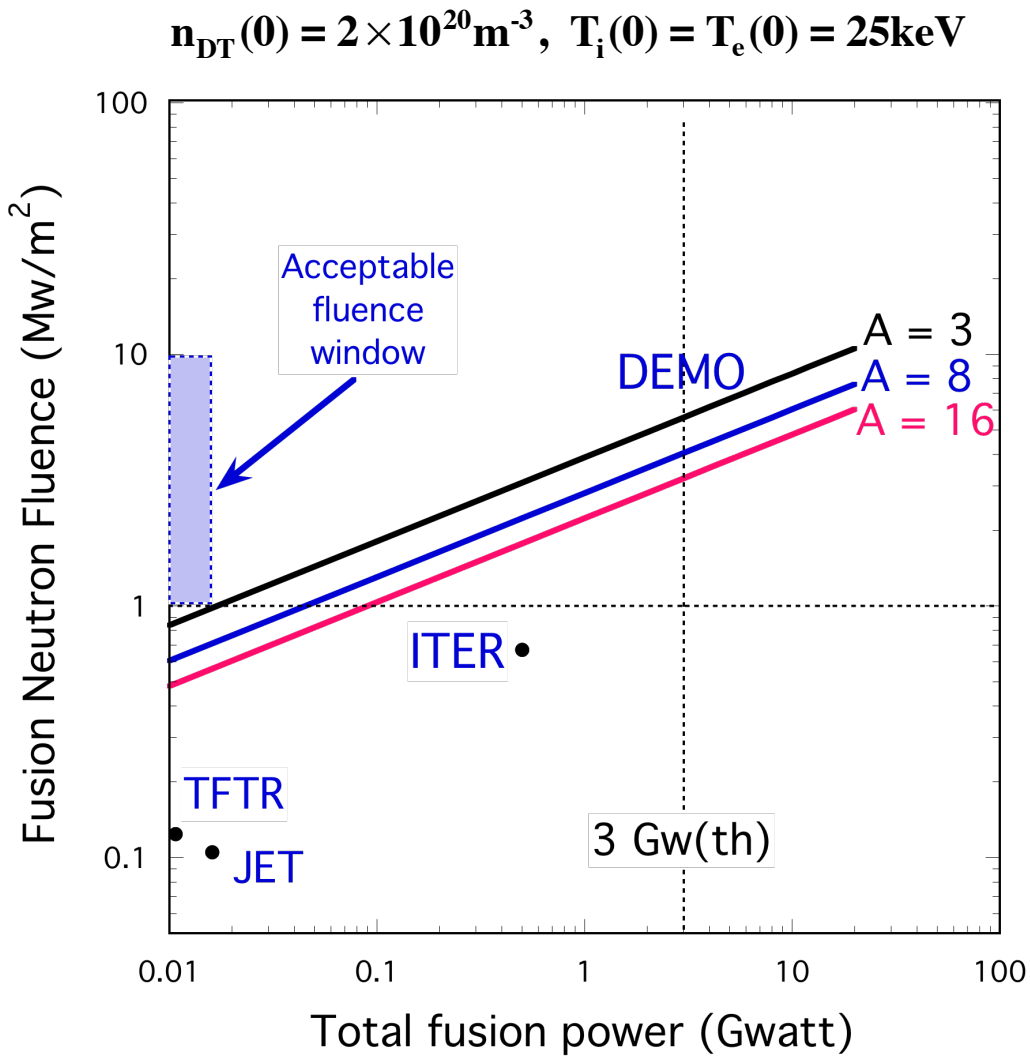
Degree of compactness in a fusion reactor will limit total power output (wall loading constraint)

- Geometry, plasma parameters, nuclear cross-sections determine heat flux
  - Surface to volume ratio of simple torus: S/V = 2/<a>
  - Power per unit area and per unit circumference

$$\frac{P}{A} = \frac{n^2 \langle \sigma \mathrm{v} \rangle V}{A} = \frac{n^2 \langle \sigma \mathrm{v} \rangle a}{2} \propto P^{1/3} / (R/a)^{1/3}$$

$$\frac{P}{C} = \frac{n^2 \langle \sigma \mathrm{v} \rangle V}{2\pi R} = n^2 \langle \sigma \mathrm{v} \rangle \pi a^2 \propto P^{2/3} / (R/a)^{2/3}$$

- Also, stellarator high density regime favors higher <R>/<a>



Outline

- Review of stellarator optimization techniques
- Quasi-symmetry options
- New quasi-poloidal/quasi-isodynamic configurations
- Other ideas

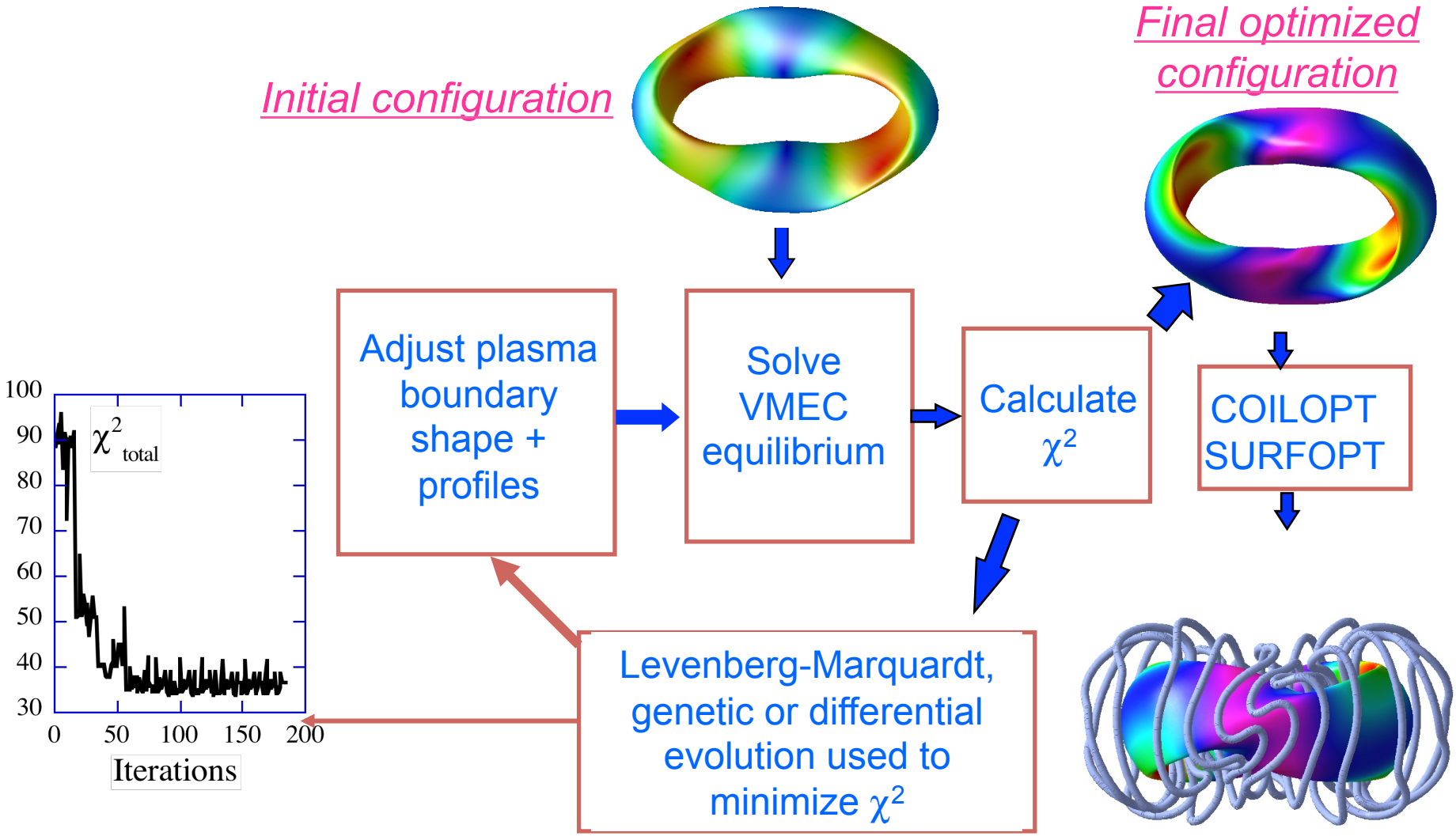
Stellarator optimization/design characteristics

- **Cross-discipline coupling**
  - Theory/modeling, high performance computing
  - Engineering constraints
  - Ongoing insights from experiments
- **Time frames**
  - Long term – identify new configurations
    - Many possibilities
    - High dimensional (50-100) parameter space
  - Short term – converge down to “good enough” devices suitable for construction

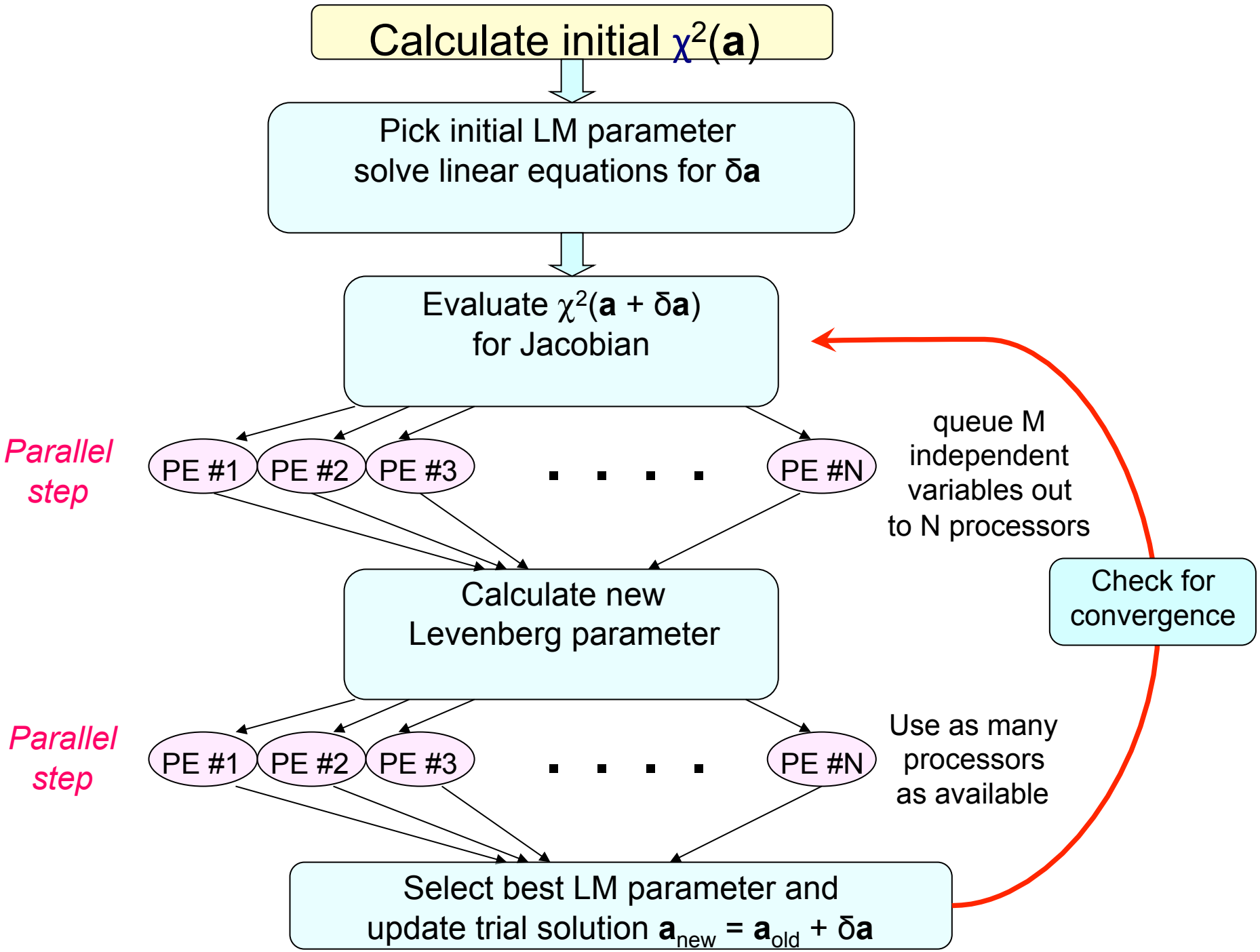
Steps in stellarator optimization

- **Magnetic flux surface shape optimization (STELLOPT)**
  - Vary outer flux surface shape to minimize physics target functions (transport, stability, etc.)
- **Determine coils on fixed winding surface**
  - Two approaches: NESCOIL, SURFOPT/COILOPT
  - Choices: plasma-to-coil winding surface distance, # of coils per period, harmonic content of surface current potential
- **Further optimization of 3D coil shape, but unconstrained by winding surface (COILOPT)**
  - Full 3D flexibility in coil shape
  - Can target both engineering and physics measures of goodness

Stellarator optimization loop determines outer flux surface shape.

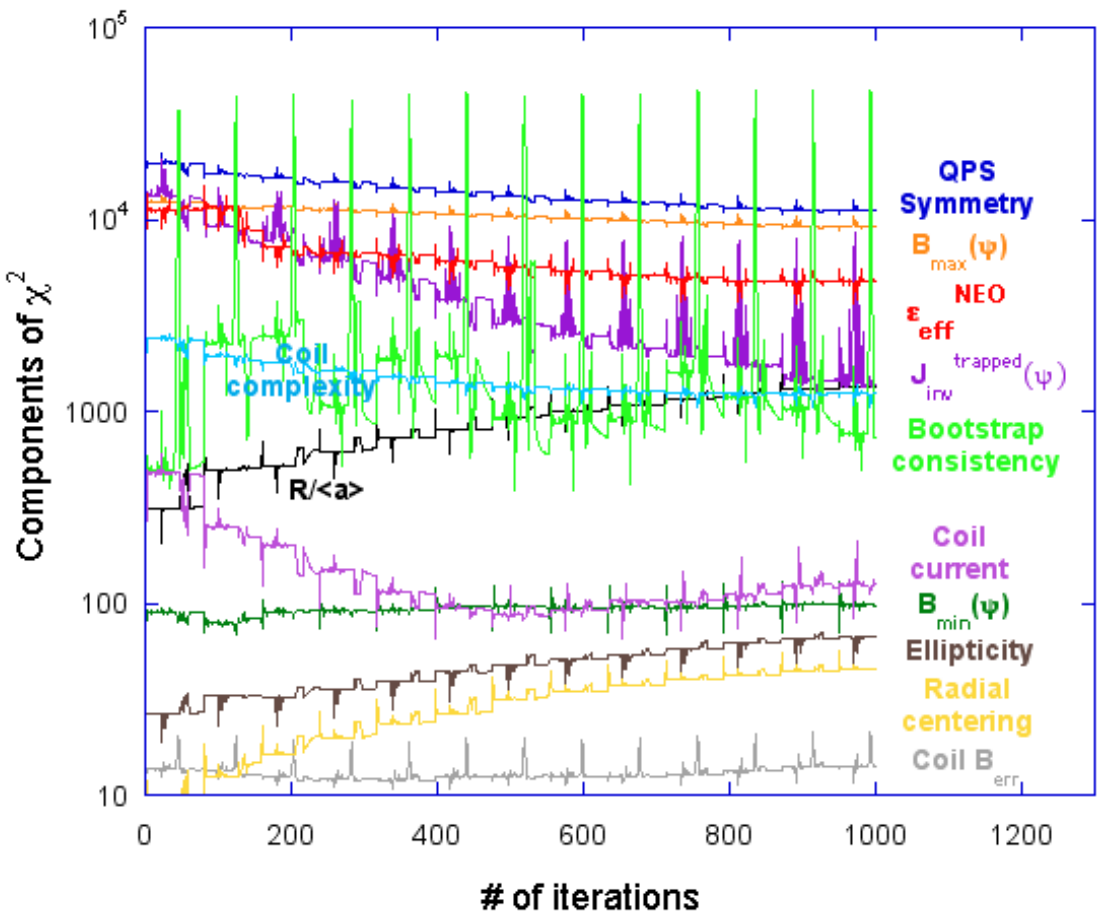


Plasma boundary is characterized by 50 - 100 Fourier harmonics

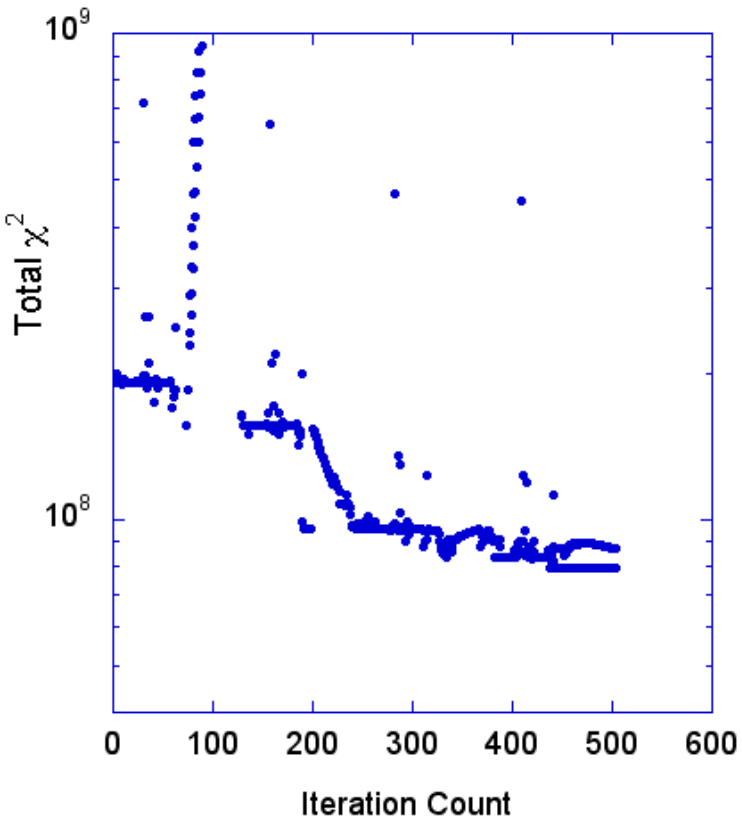

Calculate initial $\chi^2(\mathbf{a})$
Pick initial LM parameter
solve linear equations for $\delta\mathbf{a}$
Evaluate $\chi^2(\mathbf{a} + \delta\mathbf{a})$
for Jacobian
Parallel step
PE #1
PE #2
PE #3
PE #N
queue M independent variables out to N processors
Check for convergence
Calculate new Levenberg parameter
Parallel step
PE #1
PE #2
PE #3
PE #N
Use as many processors as available
Select best LM parameter and
update trial solution $\mathbf{a}_{new} = \mathbf{a}_{old} + \delta\mathbf{a}$

Reduction in target functions with iterations

Individual targets

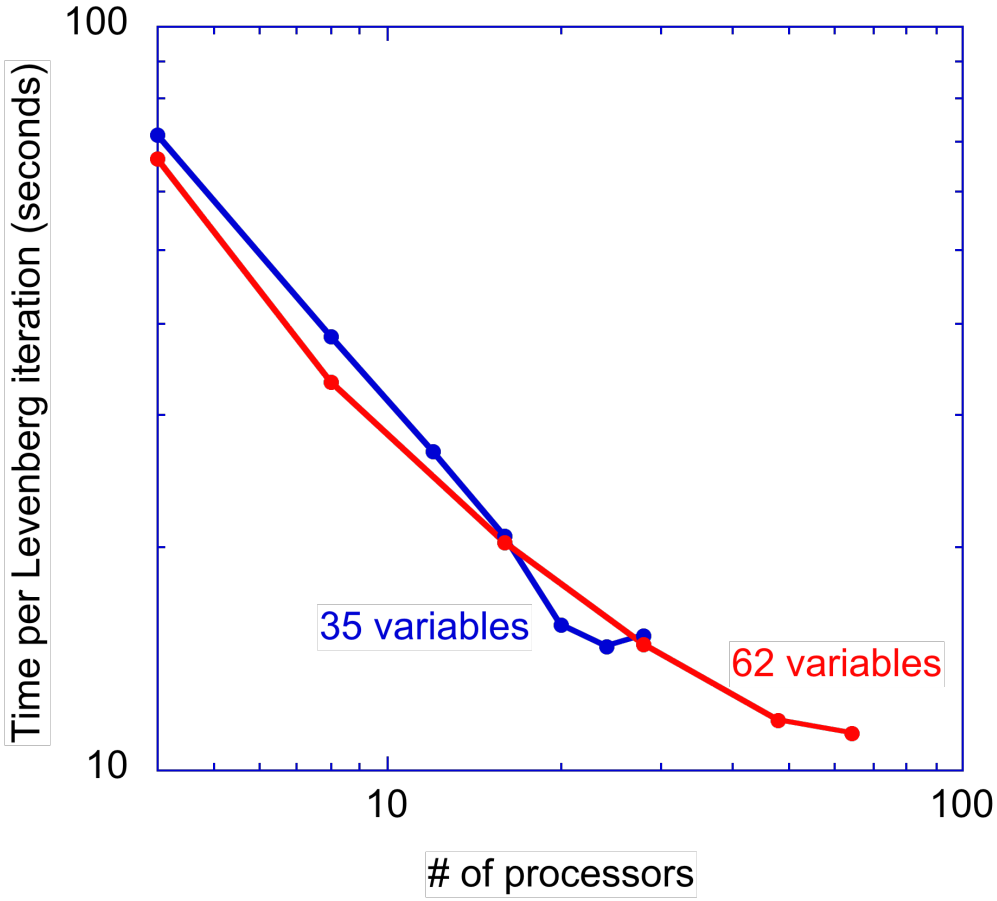


Overall χ^2



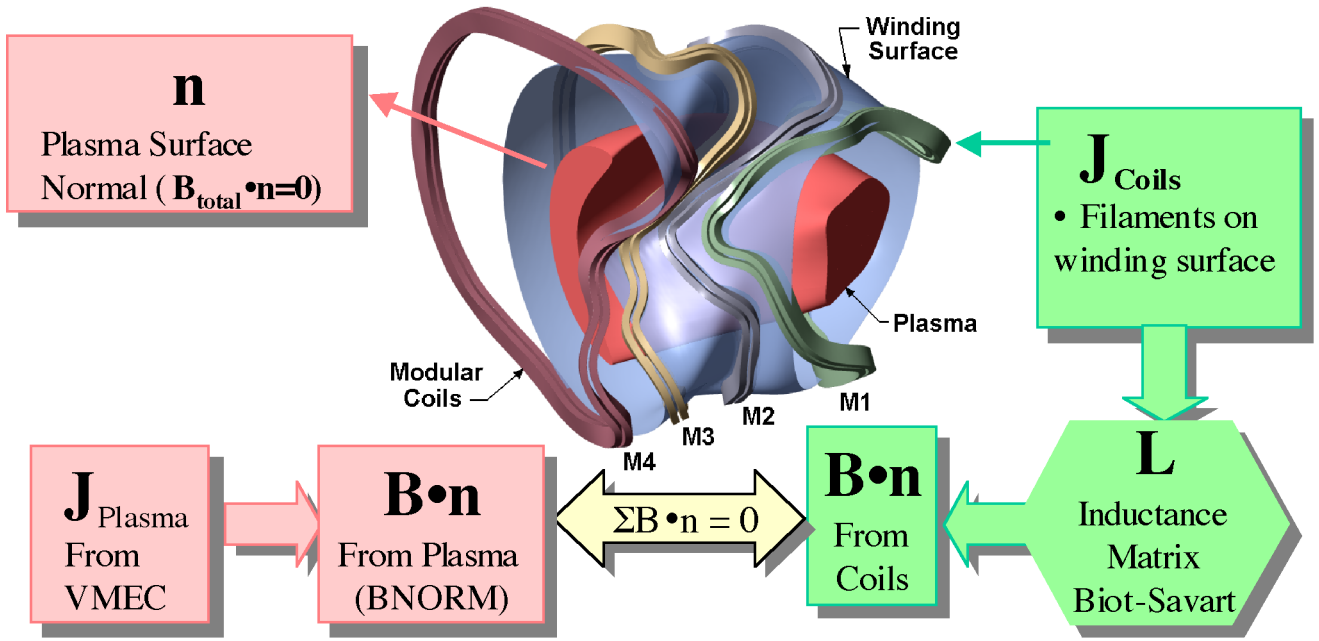
Parallelization speeds up stellarator optimization (but STELLOPT can only consume # of processors ~ # of independent variables)

- Allows more physics targets to be included.
- Parallel speedup saturates
  - as processors ≥ (0.5 to 1) x (# of independent variables)
- # of parallel tasks = # of independent variables + 1
- Communication overhead



Coils to produce the physics optimized shape are "reverse–engineered":

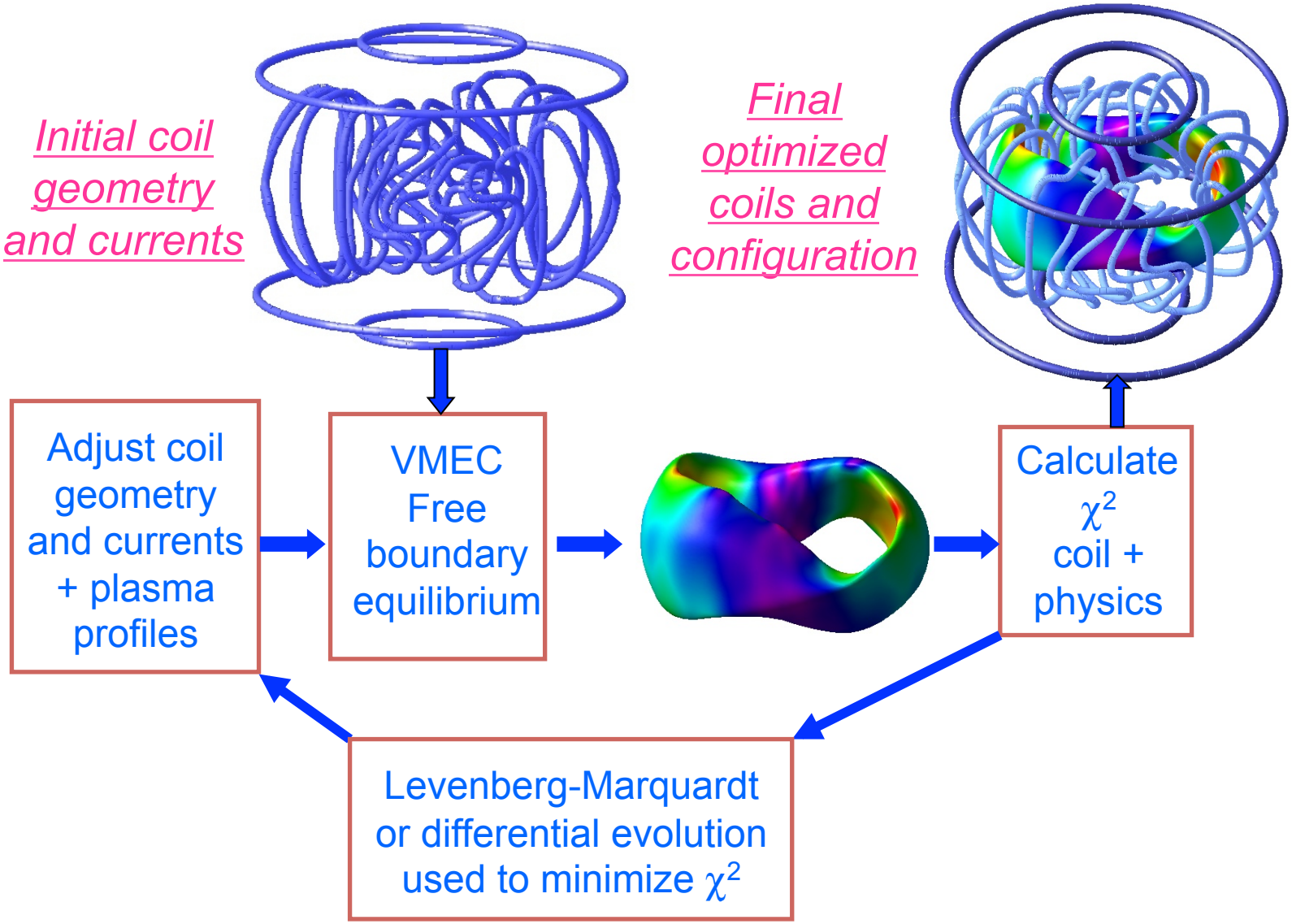
Coil Design Process



Vary "L" until L• J_{Coils} = B •n(Coils) ≈ -B •n(Plasma)

Merged coil/plasma stellarator optimization

"Development of a Robust Quasi-Poloidal Compact Stellarator," D. J. Strickler, S. P. Hirshman, D. A. Spong, M. J. Cole, J. F. Lyon, B. E. Nelson, D. E. Williamson, A. S. Ware, Fusion Science & Technology, 45(1), 15 (2004); "Designing Coils for Compact Stellarators," Fusion Science & Technology 41(2), 107 (2002).



Coil geometry is typically characterized by several hundred parameters

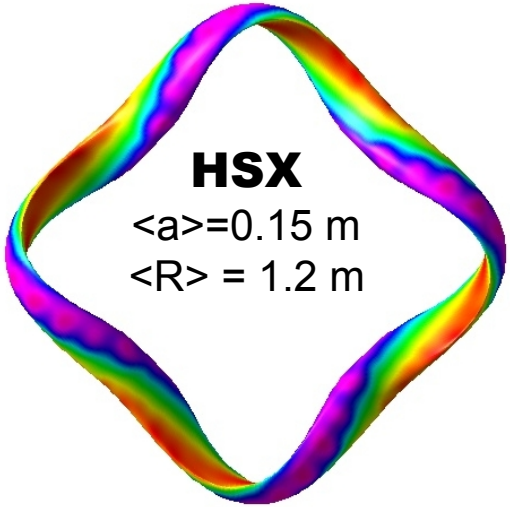
STELLOPT targets include: transport, stability, configuration, coils/engineering, and equilibrium:

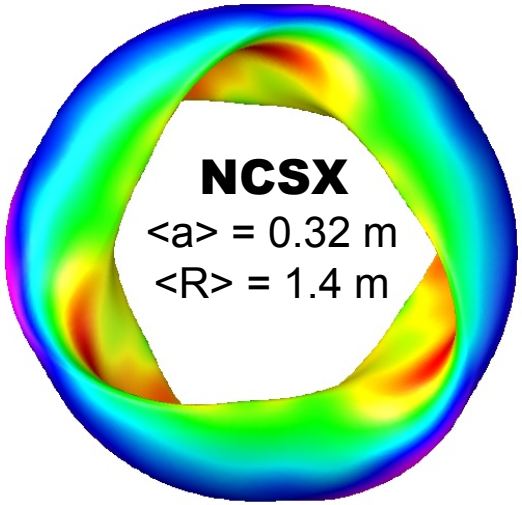
TARGET	EXAMPLE	
Bounce-averaged omnigeneity	$J = J(\psi)$ $B_{min} = B_{min}(\psi)$ $B_{max} = B_{max}(\psi)$	Transport
Nearby quasi-symmetries	Minimize B_{mn} if $m \neq 0$ (QP) Or if $n \neq 0$ (QA)	
Collisional transport coefficients	L_{11} coefficient from DKES at $\nu^* \sim 1$	
Effective ripple ε_{eff}	$\varepsilon_{eff}^{3/2}$ from NEO code	
Bootstrap consistent current profile	Match I_{BS} (BOOTSJ), $I(\psi)$ goes to 0 at edge	Iota/current profiles
Limit maximum plasma current	e.g., $I_{max} < 60$ kAmps at $\langle\beta\rangle \sim 2\%$	
Iota profile	$i(\psi) = 0.2\ (\psi{=}0)\ 0.3\ (\psi{=}1)$	
Magnetic Well, Mercier	$V'' < 0$, $D_M > 0$ over cross section	Stability
Ballooning stability	$\langle\beta\rangle \sim 2\text{-}3\%$	
Aspect ratio	$R_0/a \approx 2.5$ to 3.5	Coils, engineering, configuration
NESCOIL targets/feasible coil design	Complexity, B_{err}, Max. current density	
Adequate shielding of neutrals	Minimum "waist" thickness	
Fit within vacuum bell jar	$R_{max} < 1.5$ meter	
Suppress islands	Iota profiles that avoid resonances Residue reduction	Equilibrium quality
Limit outer surface curvature	avoid strong elongation/cusps	

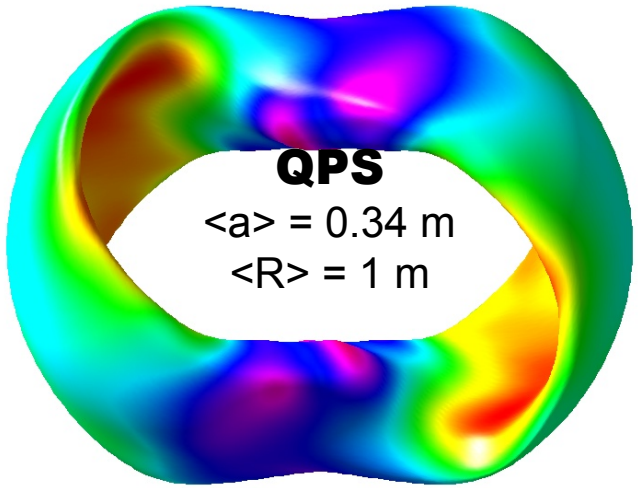
Quasi-symmetry options

- **Perfect symmetry in Boozer coordinates not realizable – goal is to obtain "good enough" symmetry for transport, energetic particle confinement**
  - i.e., dictionary definition: quasi = hidden, approximate, being partly or almost
  - QA = tokamak-like, but built-in transform breaks symmetry
  - QH = helical symmetry, but at best on one surface
  - QP = poloidal symmetry, but toroidal curvature breaks symmetry
  - Quasi-isodynamic, quasi-omnigenous – minimize deviation of drift orbits from flux surfaces

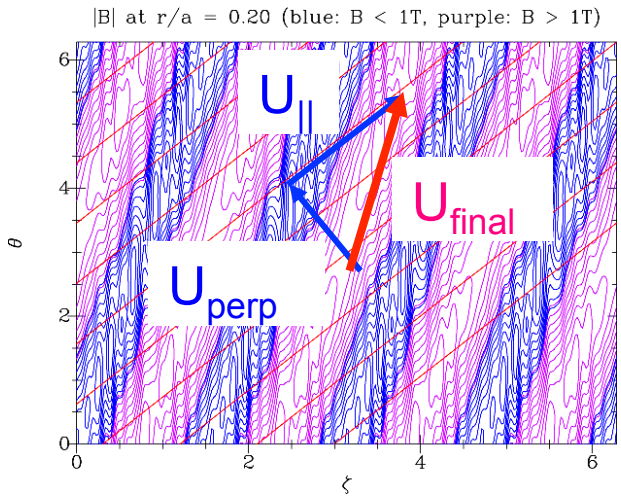
Advances in stellarator optimization have allowed the design of 3D configurations with magnetic structures that approximate: straight helix/tokamak/or connected mirrors:



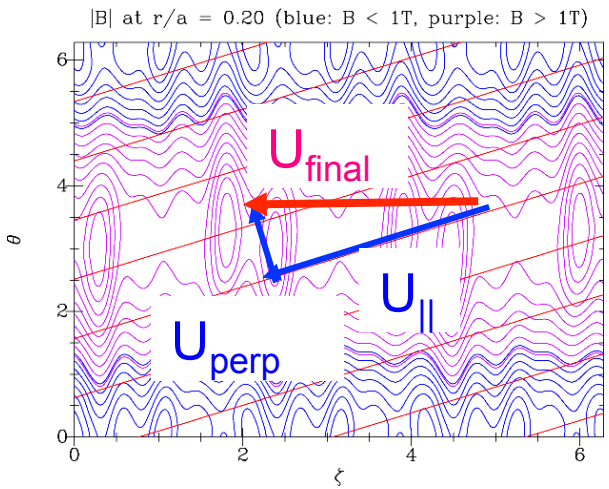




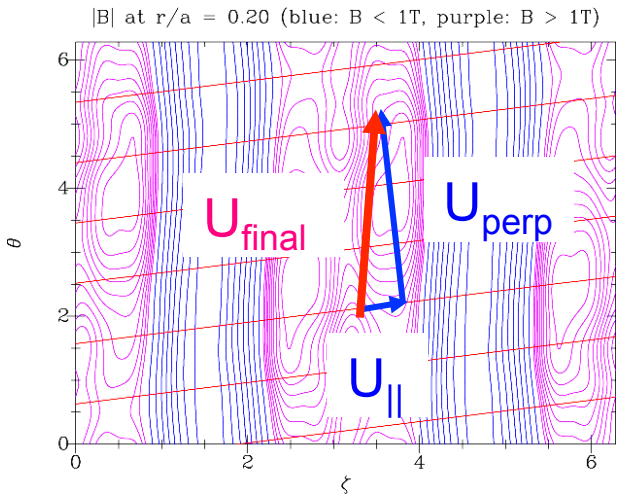
Quasi-helical symmetry
|B| ~ |B|(mq - nz)



Quasi-toroidal symmetry
|B| ~ |B|(q)



Quasi-poloidal symmetry
|B| ~ |B|(z)



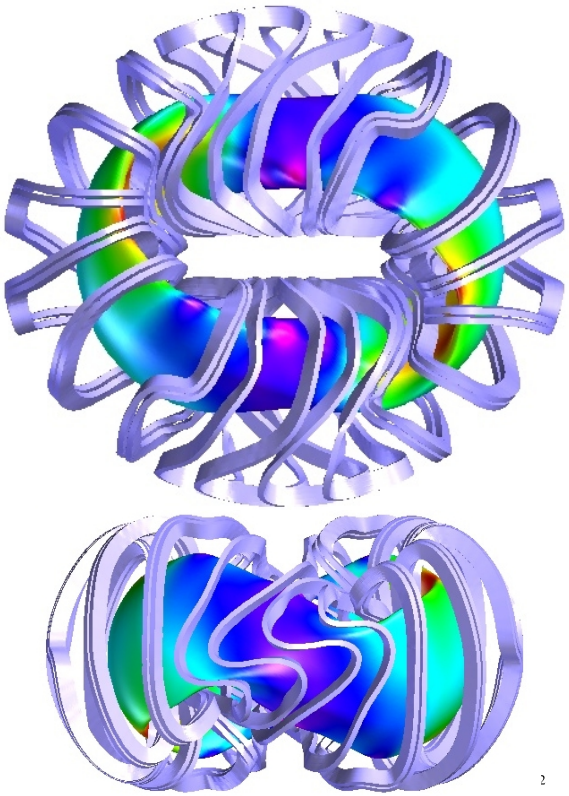
Features of quasi-poloidally (QP) symmetric stellarators

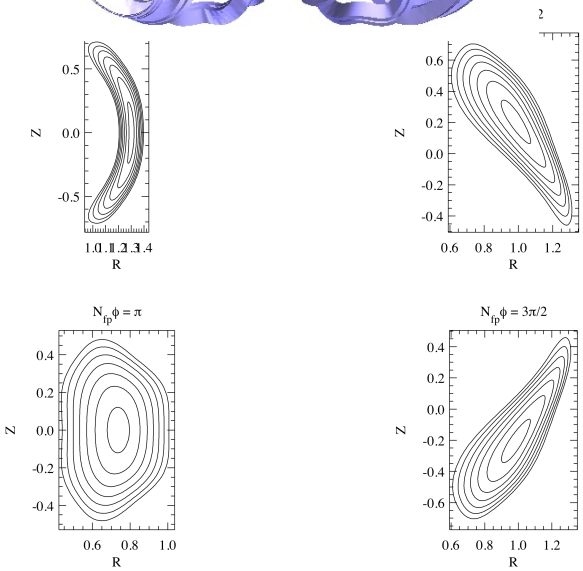
- Closer **B** and ∇ **B** alignment than with other forms of symmetry
  - For exact QP symmetry, P_θ is constant of the motion rather than P_ϕ
  - reduces radial drift; banana thickness ~ $\rho_{toroidal}$ rather than $\rho_{poloidal}$
- Minimum flow damping in the direction of $\mathbf{E_r}$ x **B**
  - Flow shear potentially self-sustained
  - Via internally generated $\mathbf{E_r}$ driven by plasma ambipolar diffusion
- Second stability access and improved omnigeneity at high β
  - S. Hudson, C. Hegna, R. Torasso, A. Ware, Plasma Phys. Contrl Fusion **46** (2004) 869.
- Trapped particle localization in low curvature regions
  - potential improvements to DTEM (dissipative trapped electron mode) stability [e.g. see G. Rewoldt, L.-P. Ku, W. M. Tang, Phys. of Plasmas **12** (2005) 102512; A. Kendl, H. Wobig, Plasma Physics **6**, 4714 (1999)]

New directions for QP configurations

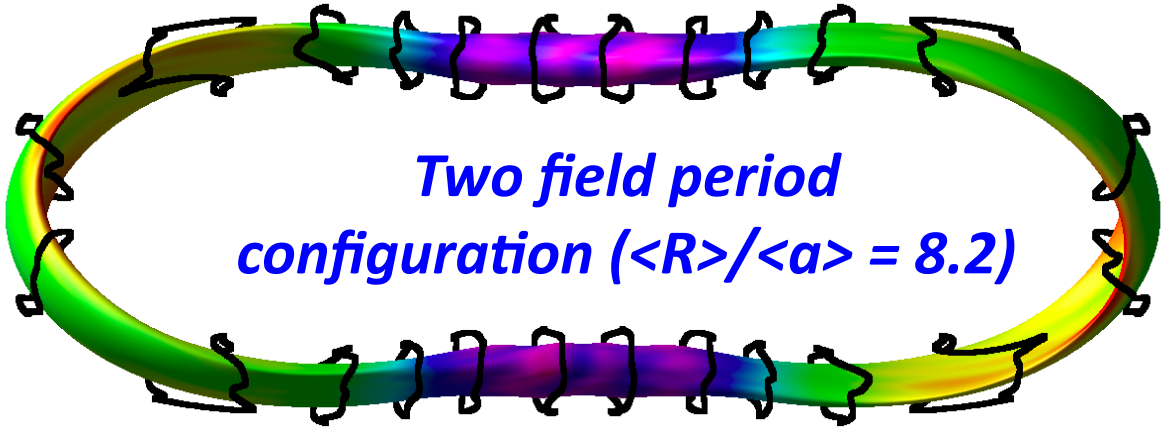
- Goal: explore the design space of quasi-isodynamic, quasi-poloidal stellarators similar to QPS, but at higher aspect ratios/field periods
- Search for: simplified coil geometry, improved symmetry,
- Higher surface-to-volume ratio can reduce wall heat fluxes in reactor configurations
- Compatibility with divertors
- Accommodate radial build from coils + structural elements
- Approaches
  - New surface optimization
  - Remove outer flux surfaces from existing QPS equilibrium and re-develop coils

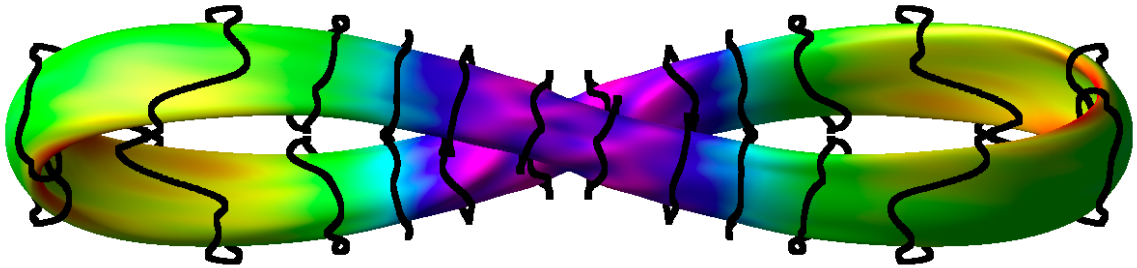
QPS (<R>/<a> = 2.7)

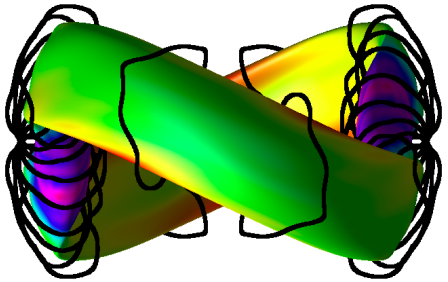


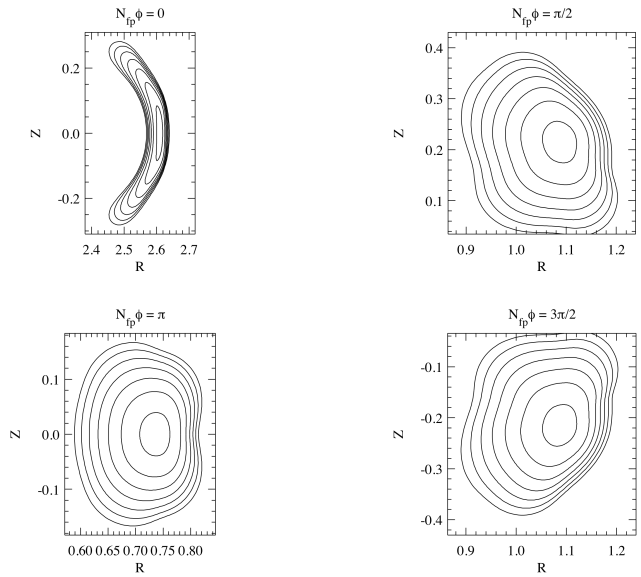


Two field period configuration (<R>/<a> = 8.2)

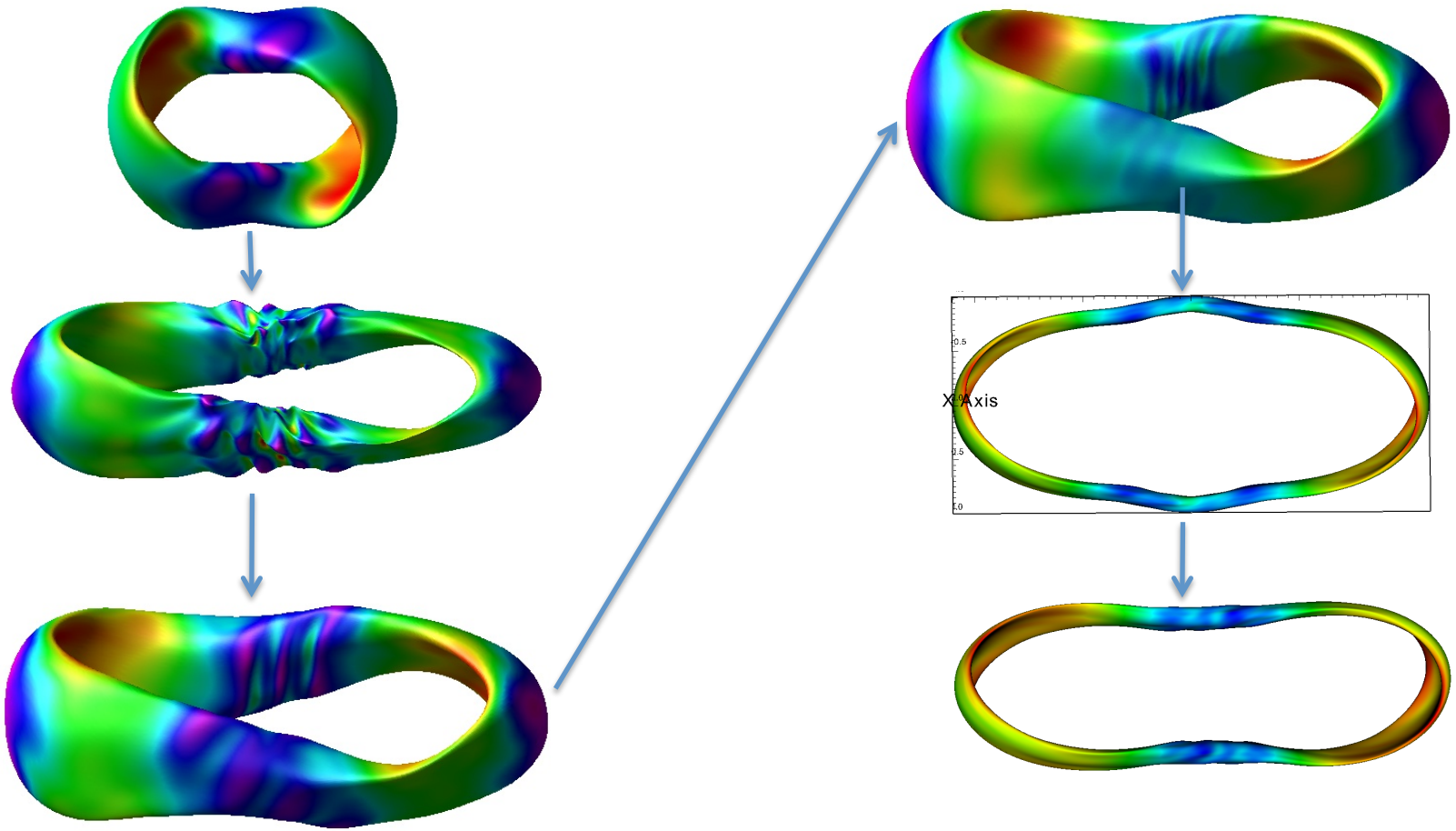




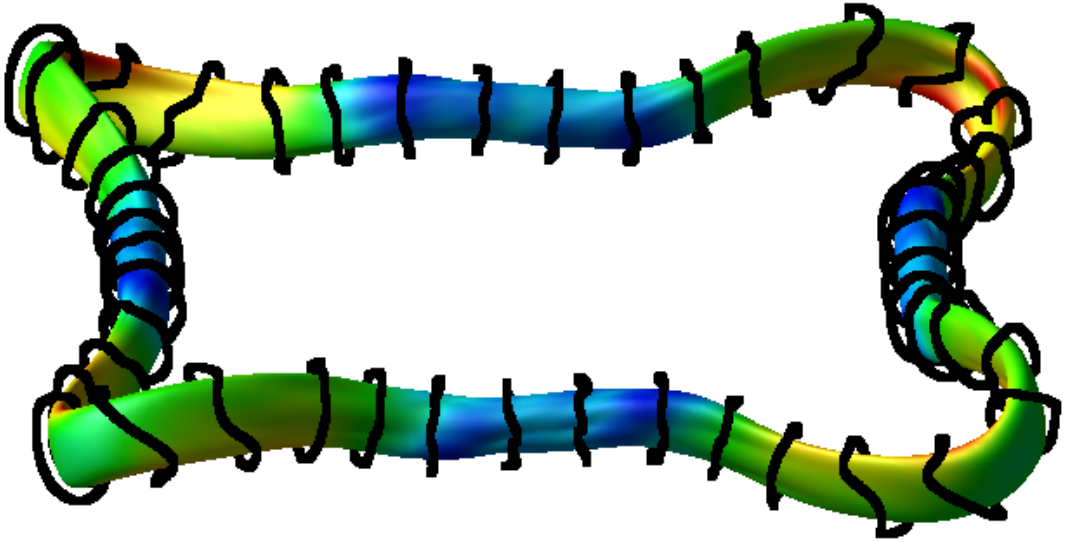


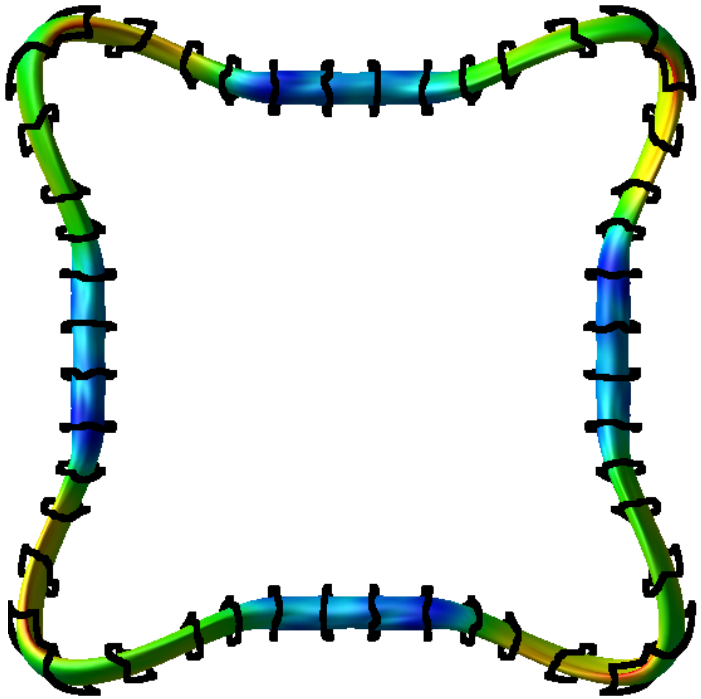


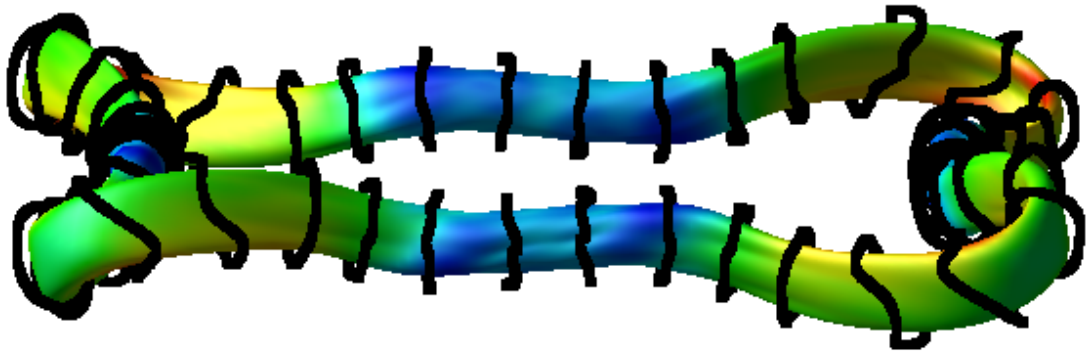
Evolution of racetrack configurations

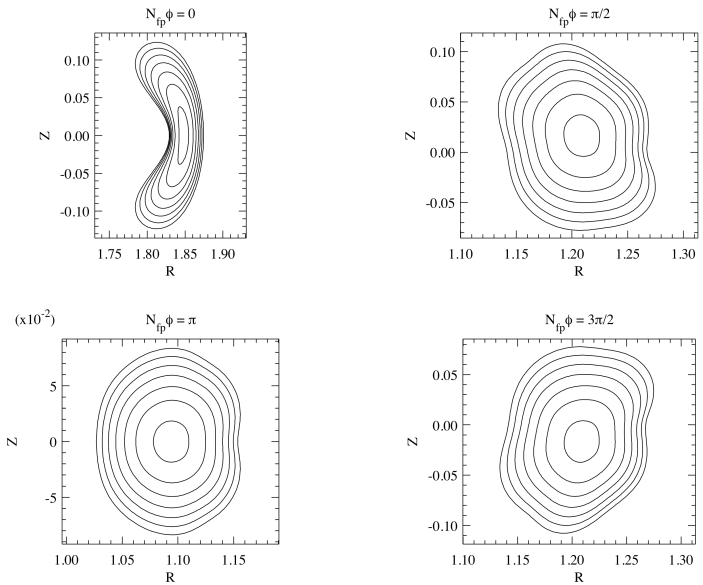


Four field period configuration (<R>/<a> = 16)



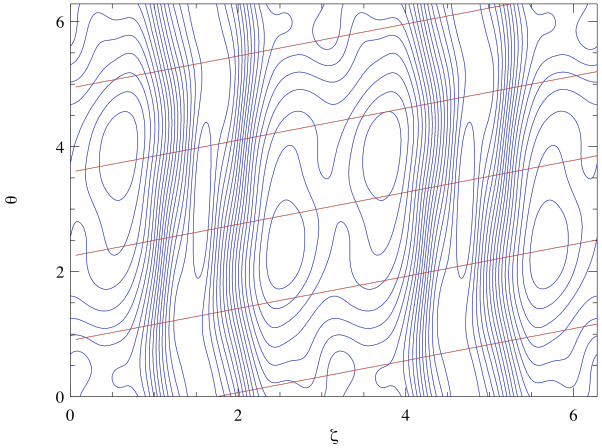


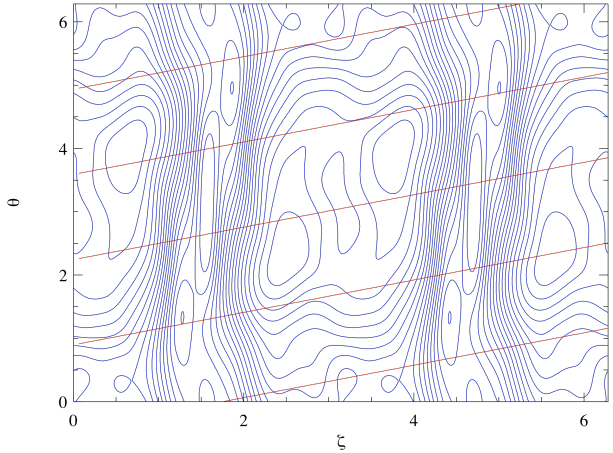




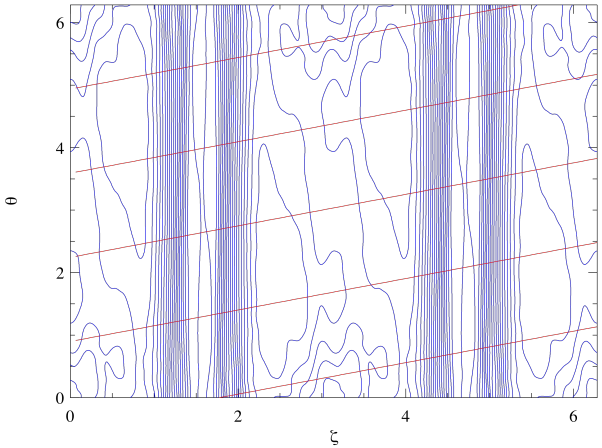
|B| contours at <r>/<a> = 0.5

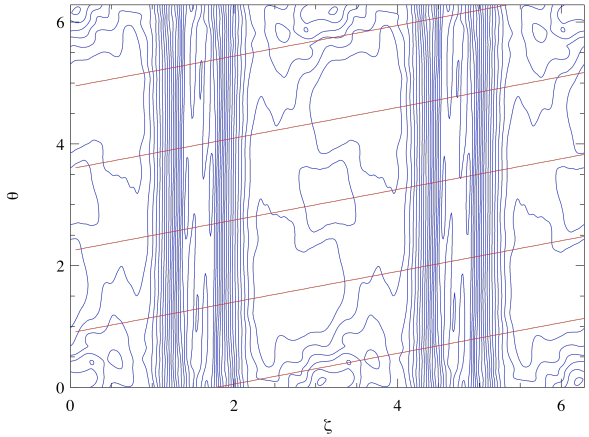
|B| contours at <r>/<a> = 0.7



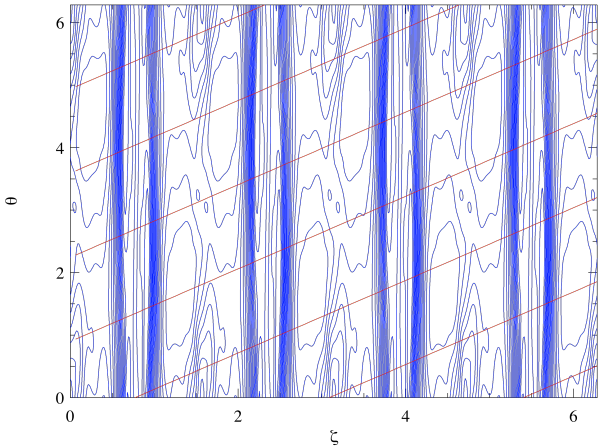


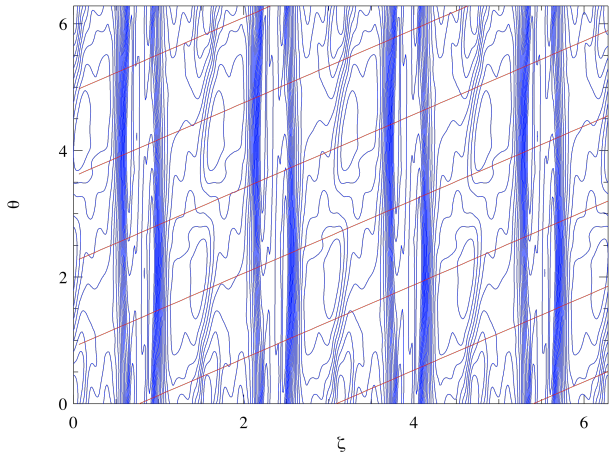
QPS





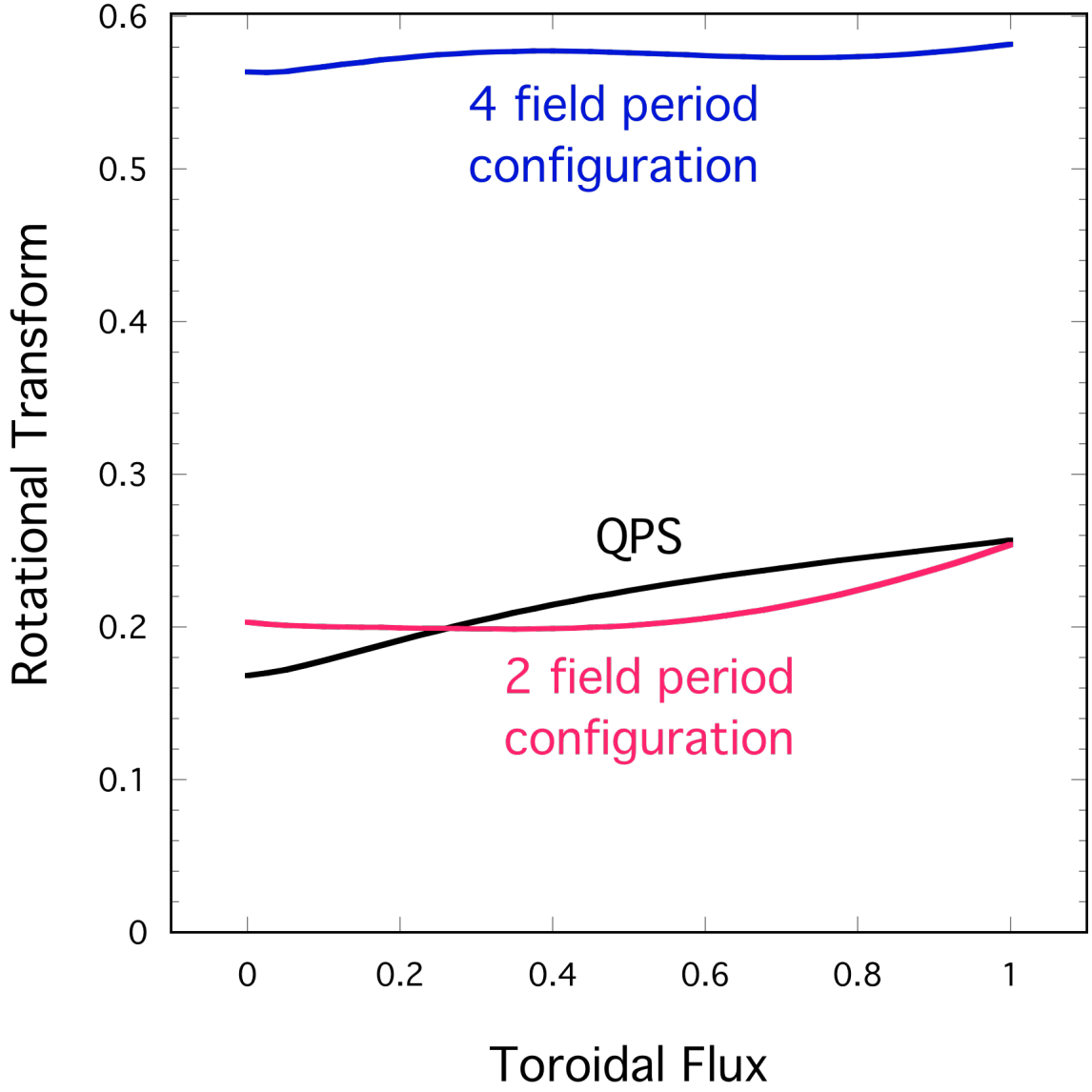
$N_{fp} = 2$
$A = 8$





$N_{fp} = 4$
$A = 16.6$

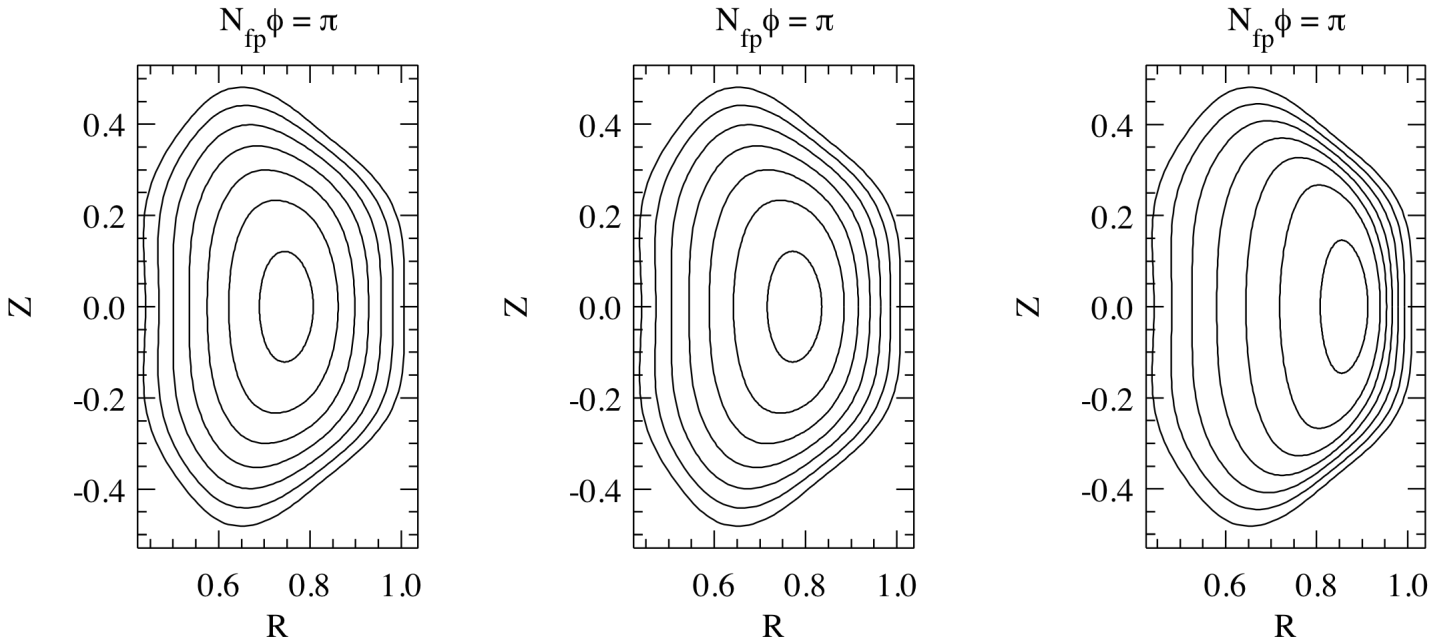
Rotational transform profiles



QPS $\Delta \propto \frac{<R>/<a>}{i^2(a)}$

Shafranov shift variation with β

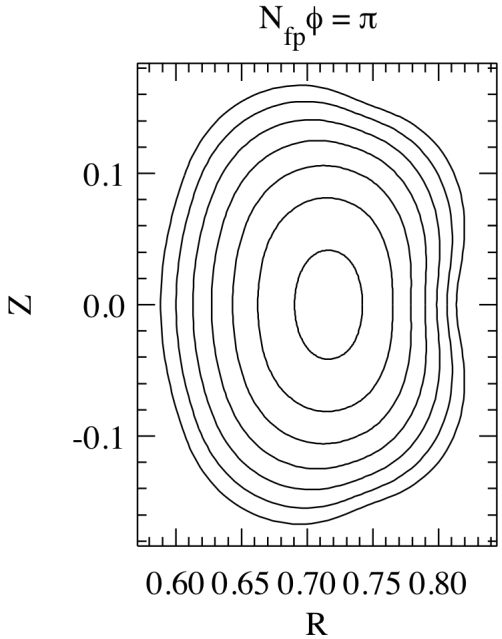
<β> = 0.01 0.02 0.05



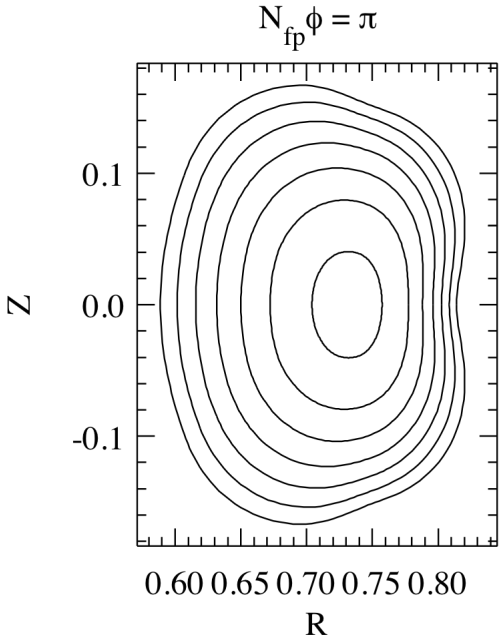
$N_{fp} = 2, A = 8.2$

Shafranov shift variation with β

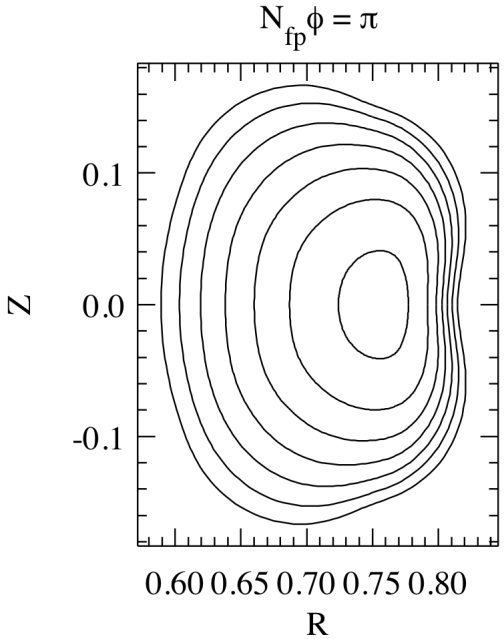
<β> = 0.005



0.01



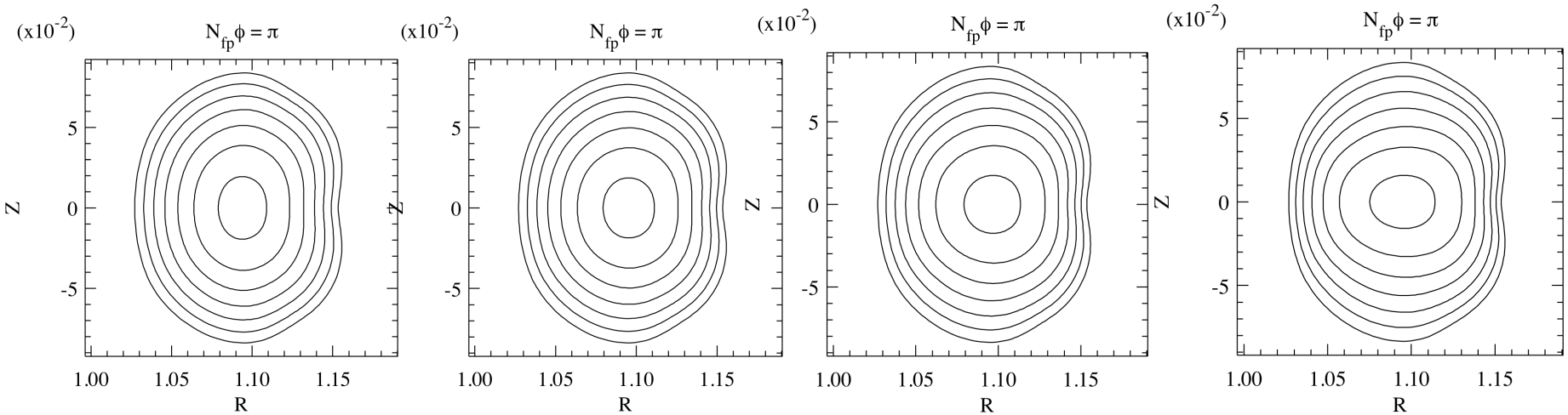
0.02



$N_{fp} = 4, A = 16.6$

Shafranov shift variation with β

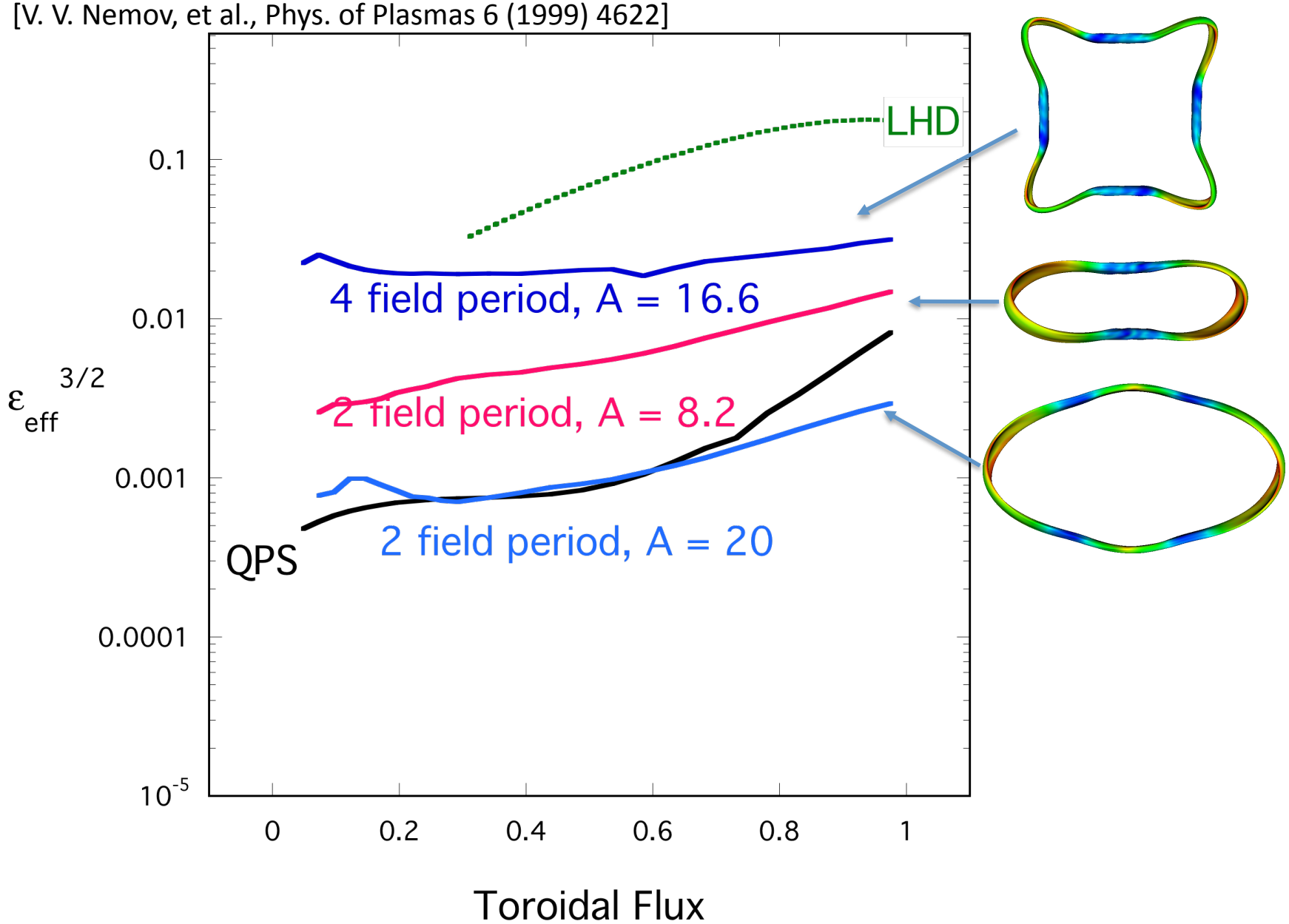
<β> = 0.01 0.02 0.03 0.05



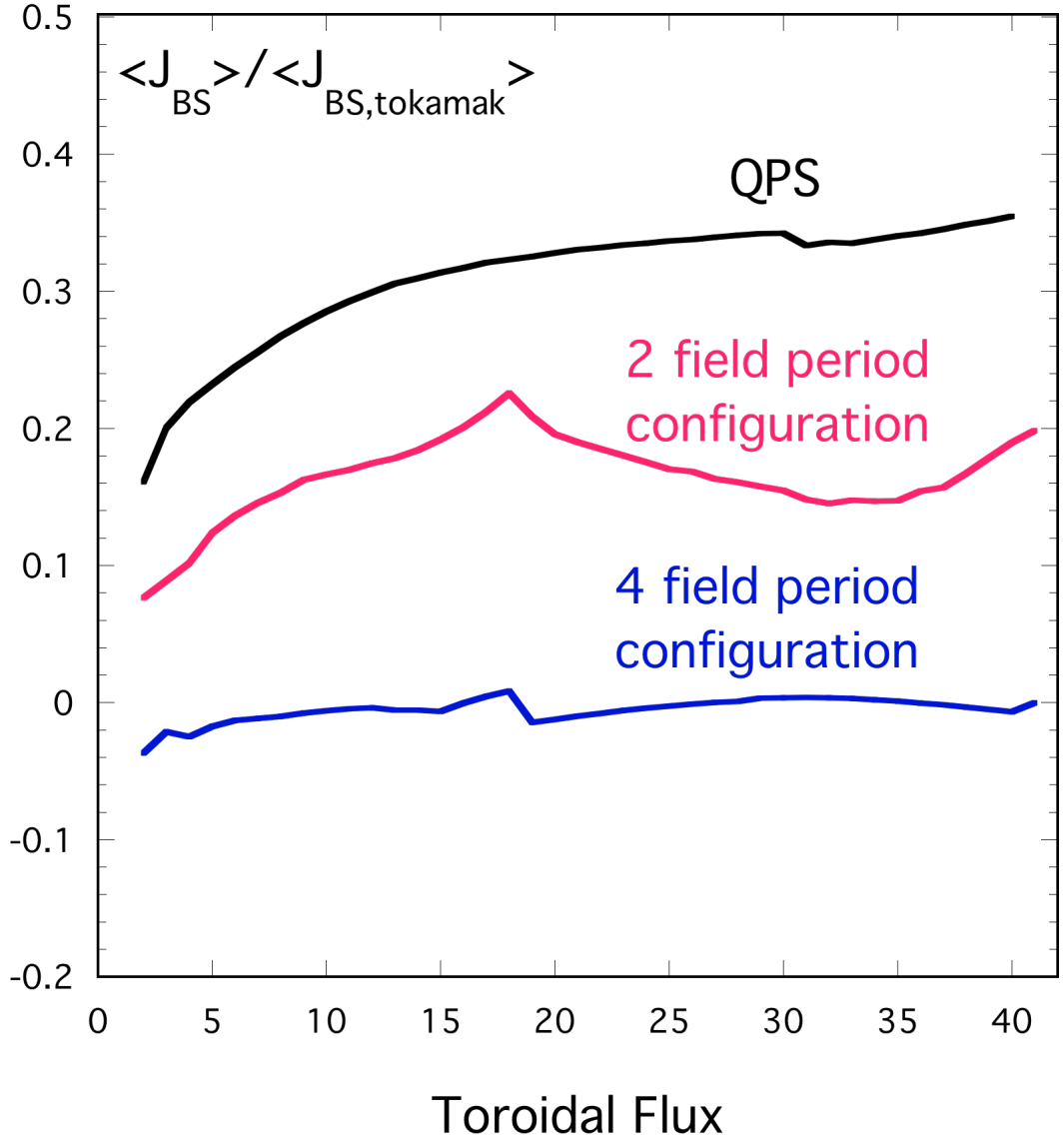
Effective ripple: measure of 1/ν ripple transport

$$D(\nu \to 0) \propto \frac{n v_{th}^2 \rho_L^2}{\nu R^2} \varepsilon_{eff}^{3/2}$$

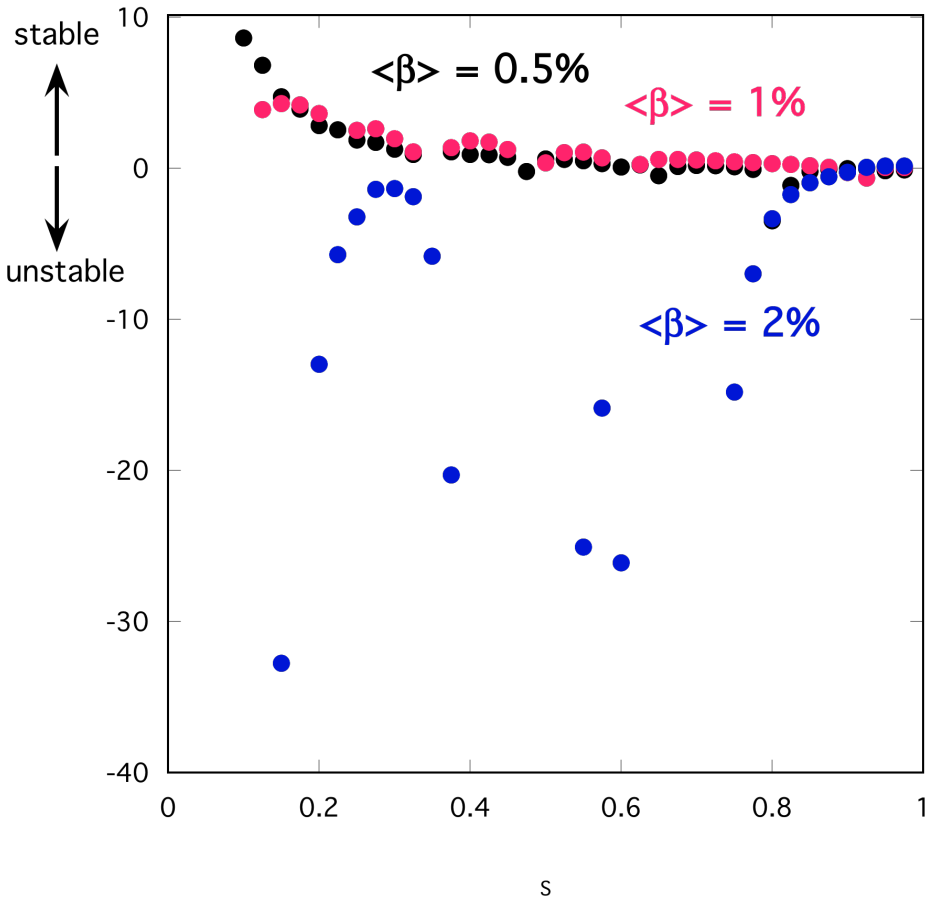
[V. V. Nemov, et al., Phys. of Plasmas 6 (1999) 4622]



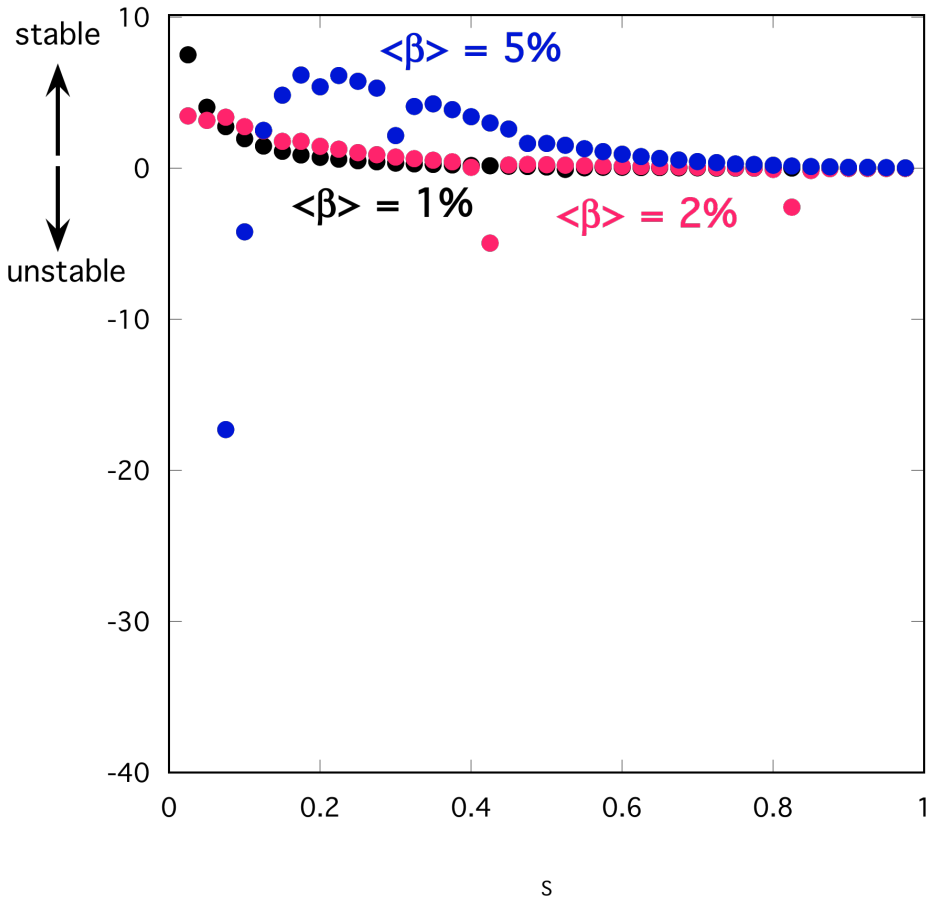
Bootstrap Current levels relative to tokamak



Mercier stability – N_{fp} = 2

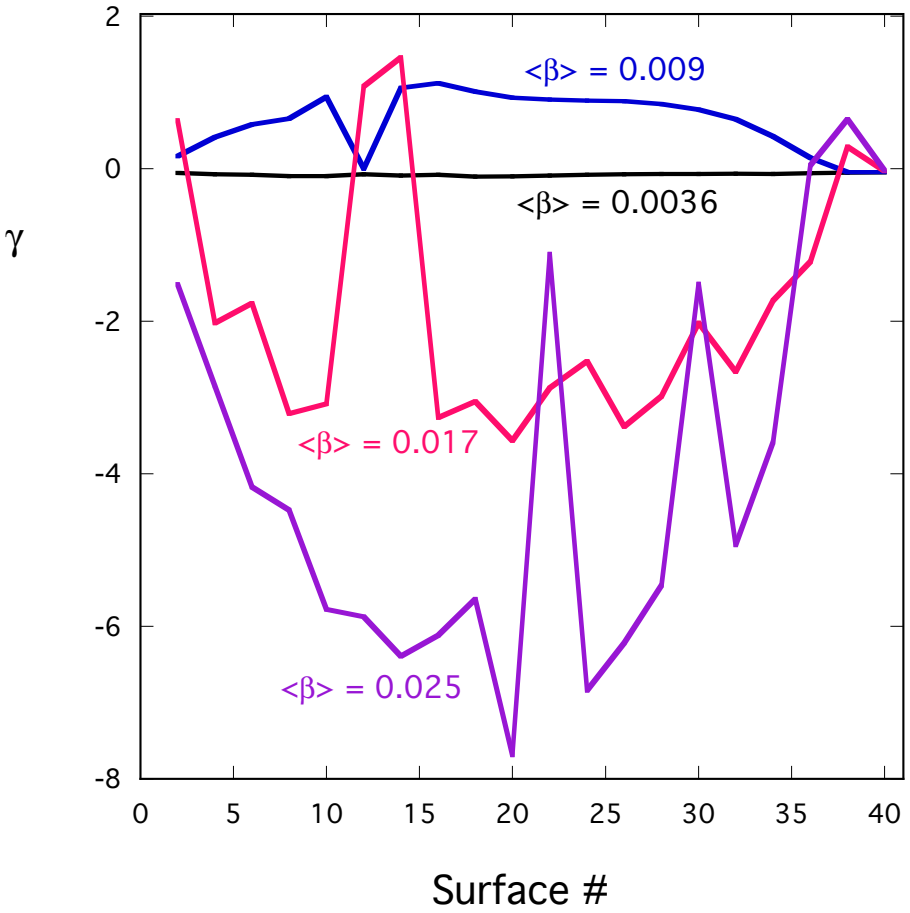


Mercier stability – QPS

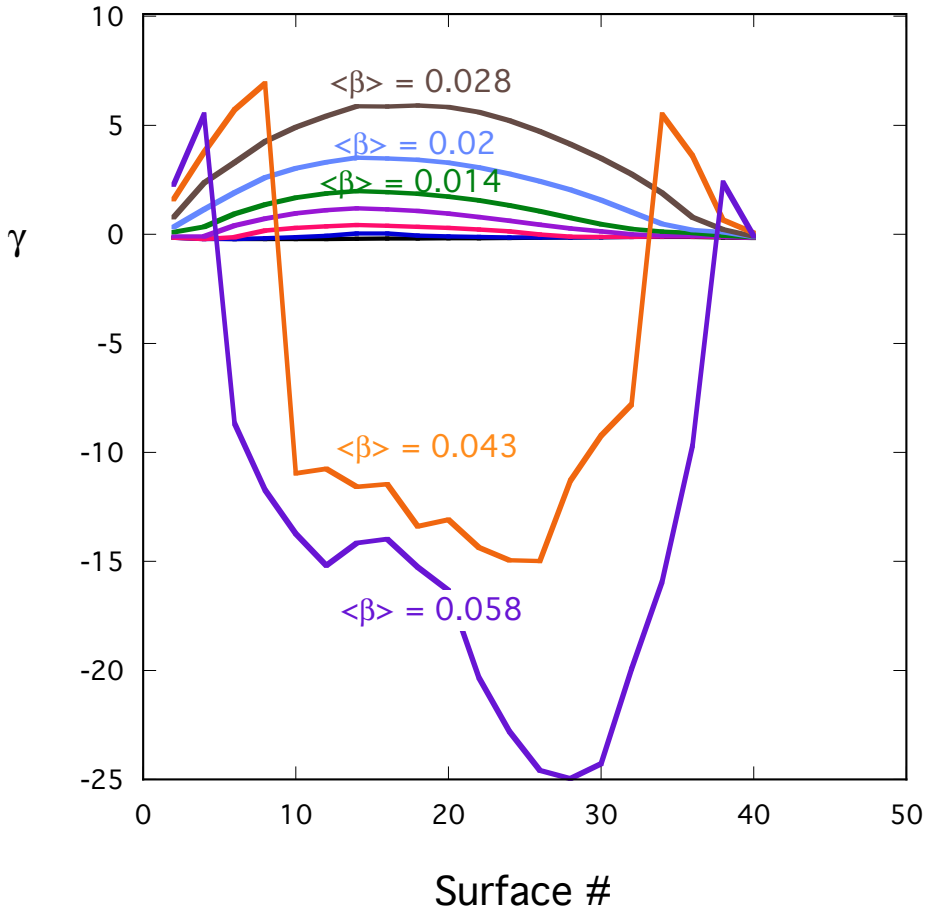


Both 2 and 4 field period configurations have access to ballooning second stability regimes:

2 period device enters second stability at <β> ≈ 1.5%

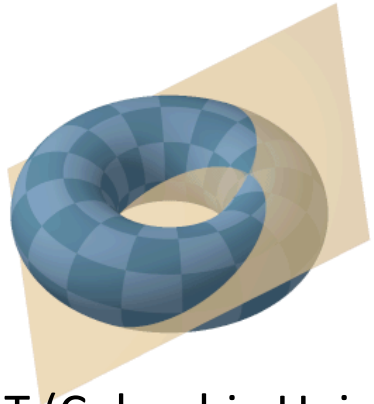


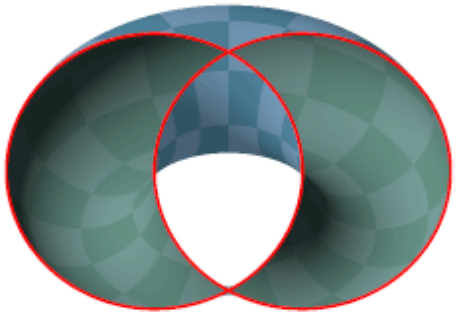
4 period device enters second stability at <β> ≈ 3.5%

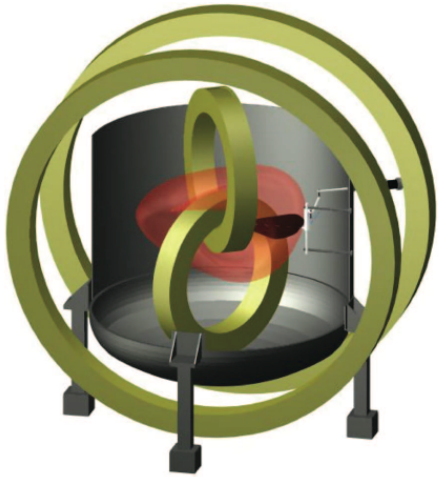


Other ideas

- Villarceau coils
  - Yvon Villarceau (1813-1883)French mathematician, astronomer

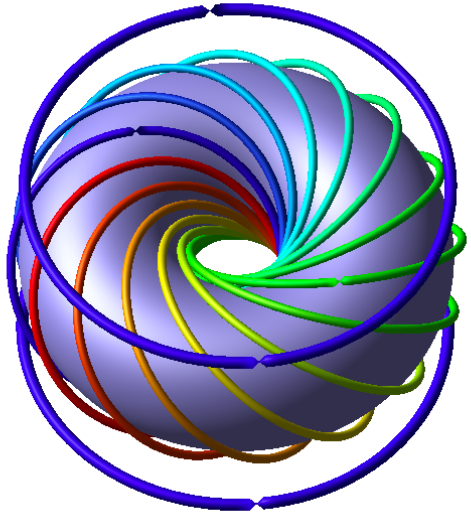


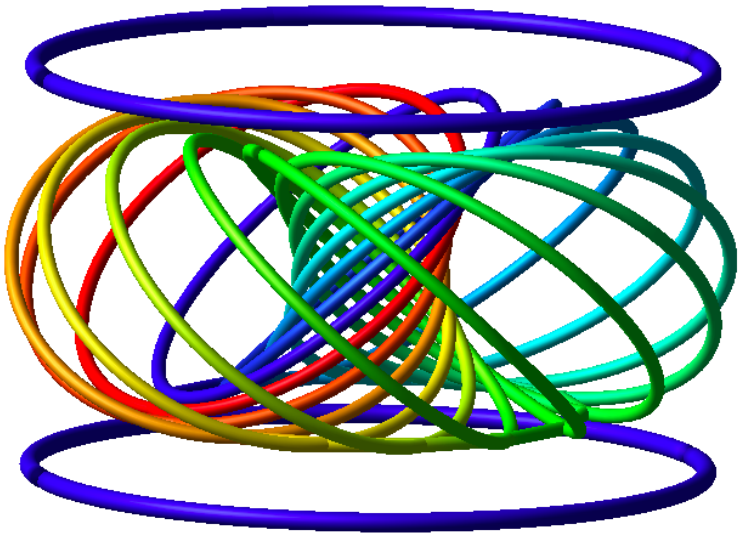




CNT device

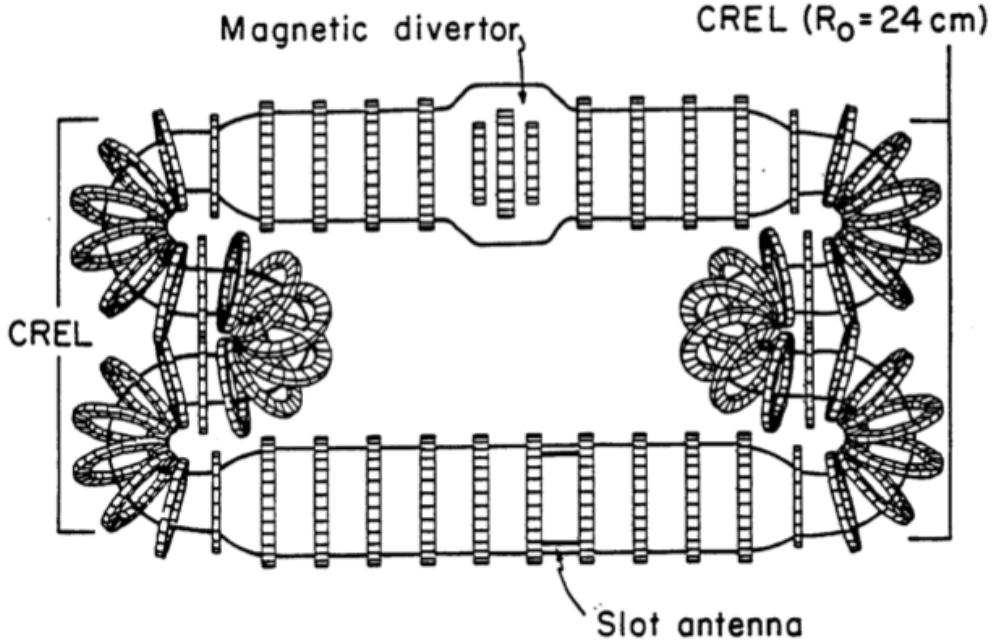
  - CNT (Columbia Univ.) is a 2 coil version
  - Coils can force free (J x B ~ 0) at center post (Wayne Reiersen)
  - No vacuum transform – requires toroidal current





Other ideas (contd.): Drakon

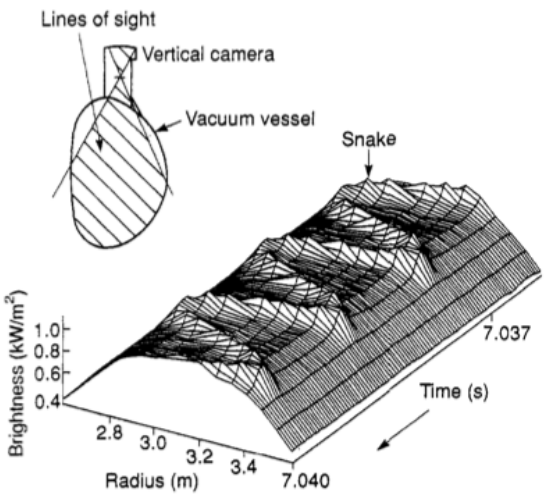
(Russian acronym for long equilibrium configuration)

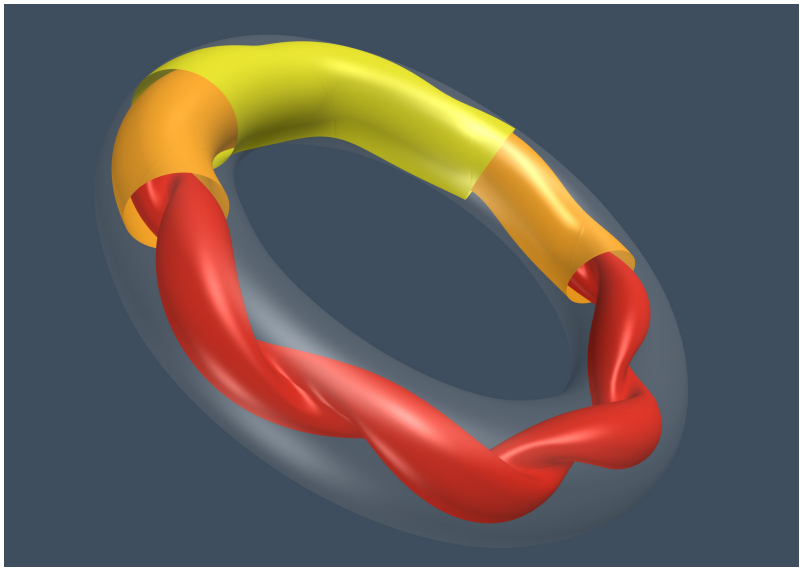


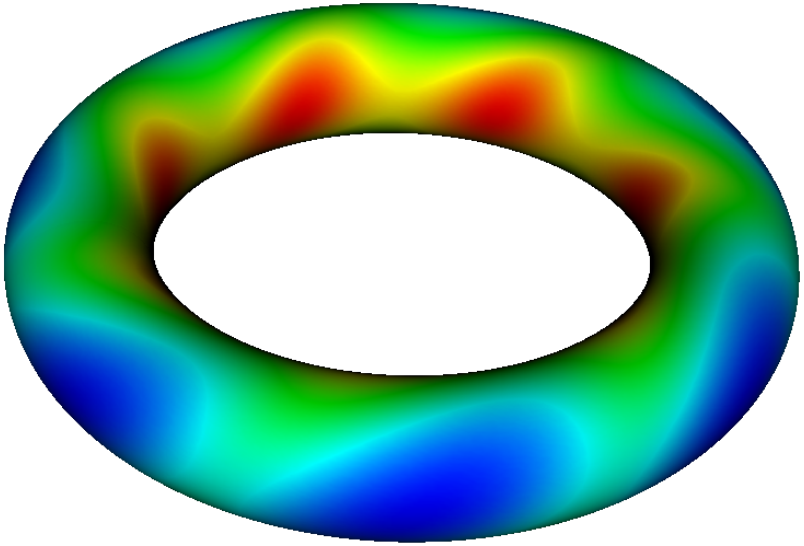
- Pfirsch-Schlüter currents confined to end sections
- Drift-optimization in end sections
- Allows arbitrarily long straight sections
- Circular coils, simple blanket design

Other ideas (contd.)

- Natural stellarators, self-organized helical states
  - "Snakes" in JET: helical m = 1 islands with very high confinement times (A. Weller, et al., Phys. Rev. Lett. ***59 (1987) 2303***)
  - n = 7 single helicity states in RFX (Piero Martin, 2009 APS talk)







Other ideas (contd.)

- Stellarator without stellarator symmetry

$$R = \sum \left[R_{mn,c}(\rho)\cos(m\theta - n\phi) + R_{mn,s}(\rho)\sin(m\theta - n\phi) \right]$$

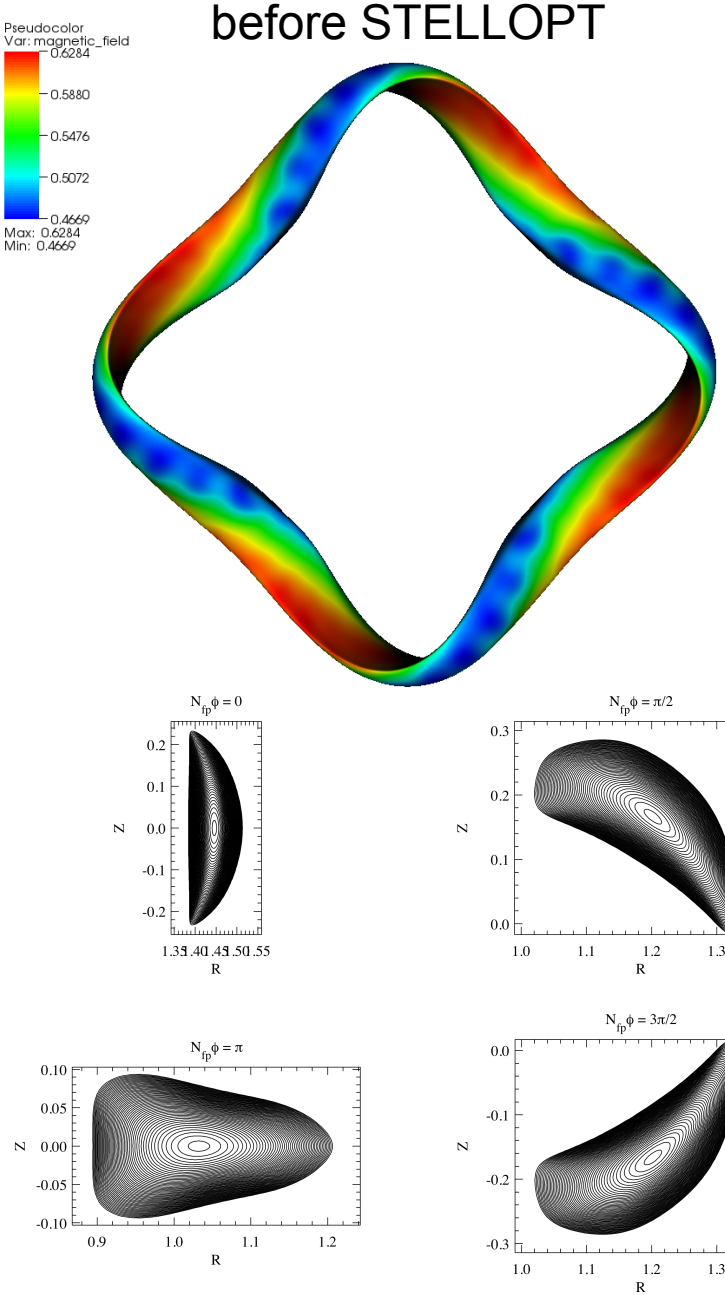
instead of $R = \sum R_{mn}(\rho)\cos(m\theta - n\phi)$

  – Doubles number of degrees of freedom for optimization

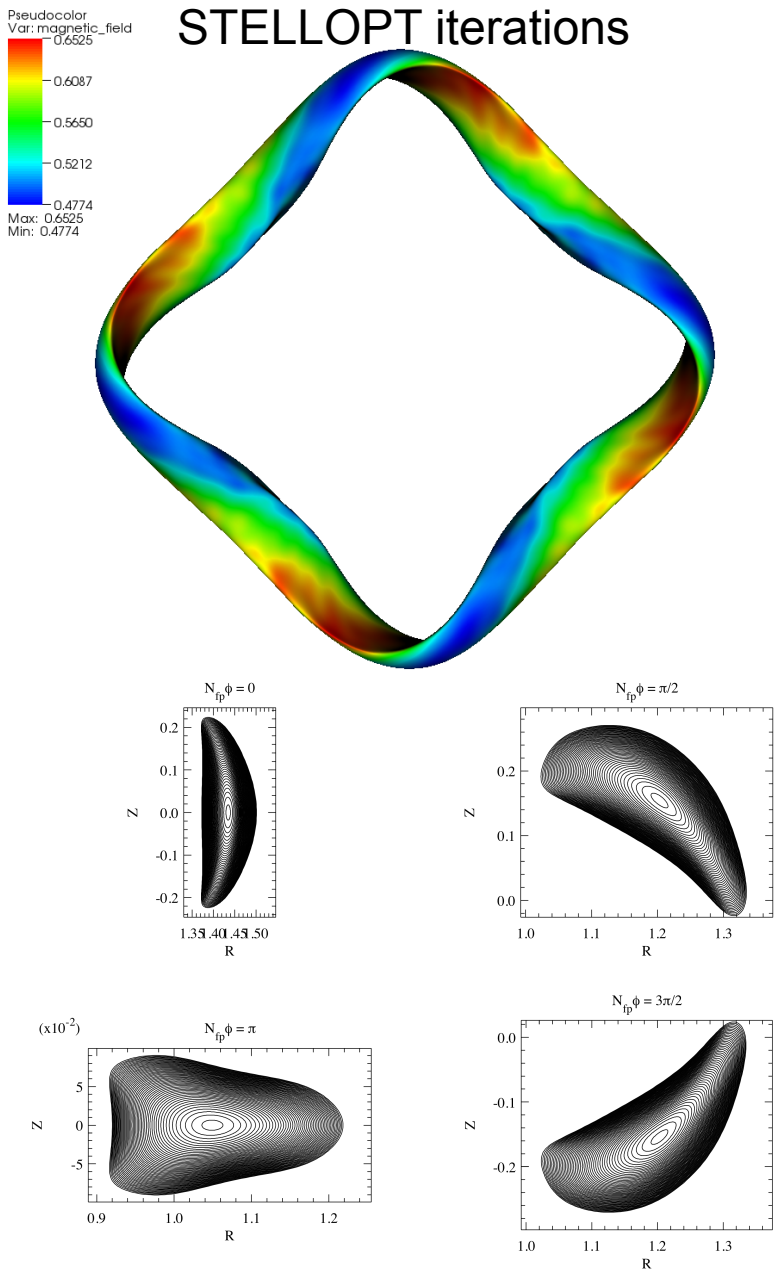
- Helical symmetry for |B| helicity other than (1,1)
  – Continuous transition from QA (m >> n) to QP (n >>m)

HSX re-optimized

before STELLOPT



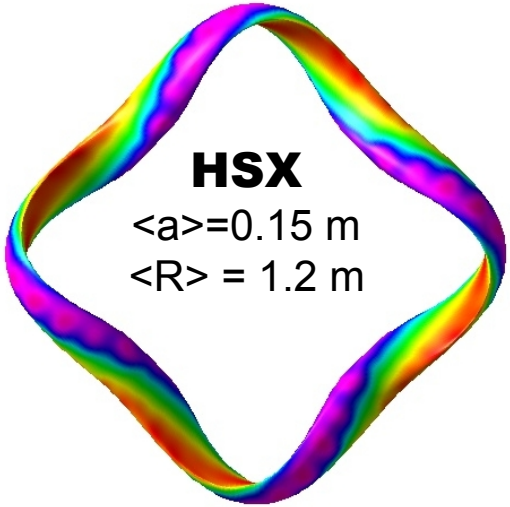
After ~a few 100 STELLOPT iterations

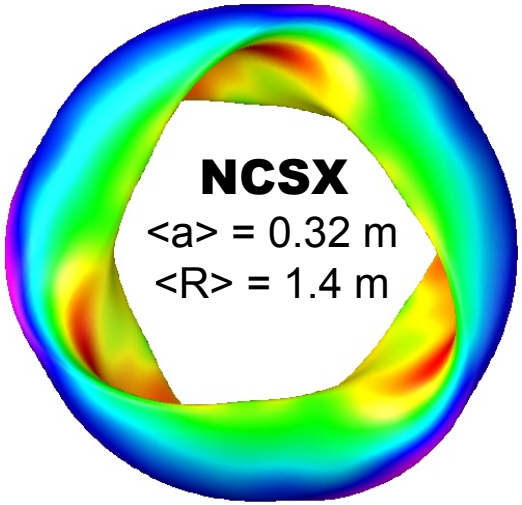


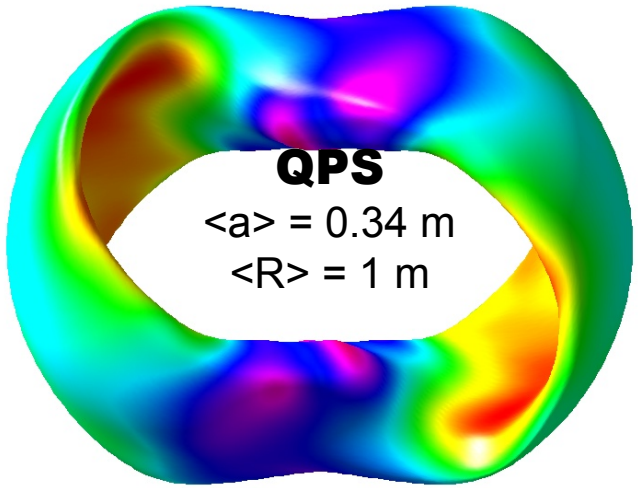
Conclusions

- The design space of quasi-isodynamic, quasi-poloidal stellarators similar to QPS, but at higher aspect ratios/field periods has been explored
- 2 and 4 field period configurations have been found with:
  - simplified coil geometry, improved symmetry,
  - Higher surface-to-volume ratio - reduced wall heat fluxes in reactor configurations
  - Straight sections facilitate toroidal bundle divertor designs (field null in annular outer region of plasma)
  - Open central region can better accommodate radial build from coils + structural elements
  - Good accessibility for magnetic beach ICRF heating techniques –> direct ion heating

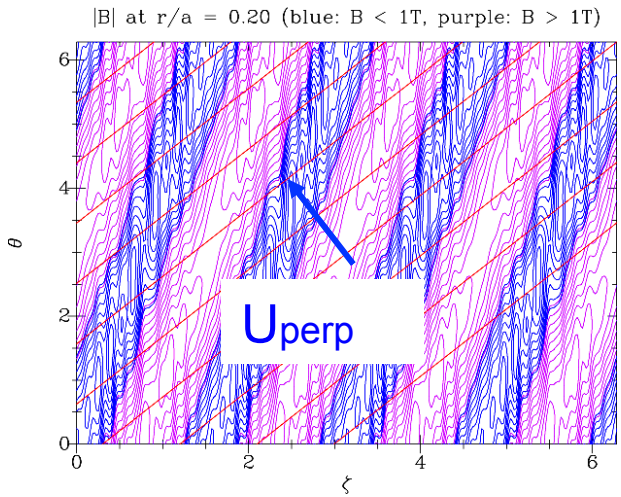
Advances in stellarator optimization have allowed the design of 3D configurations with magnetic structures that approximate: straight helix/tokamak/or connected mirrors:



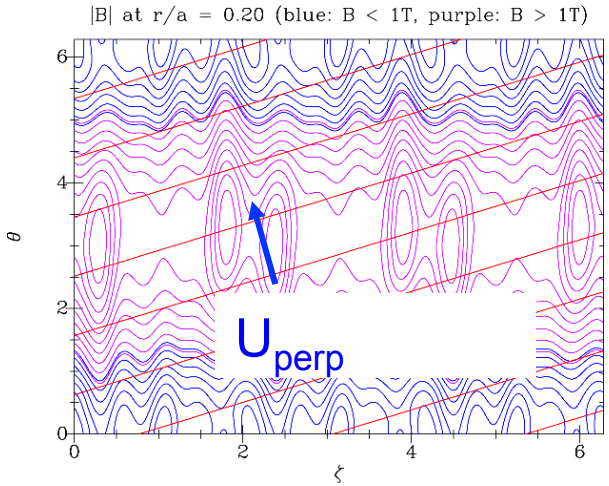




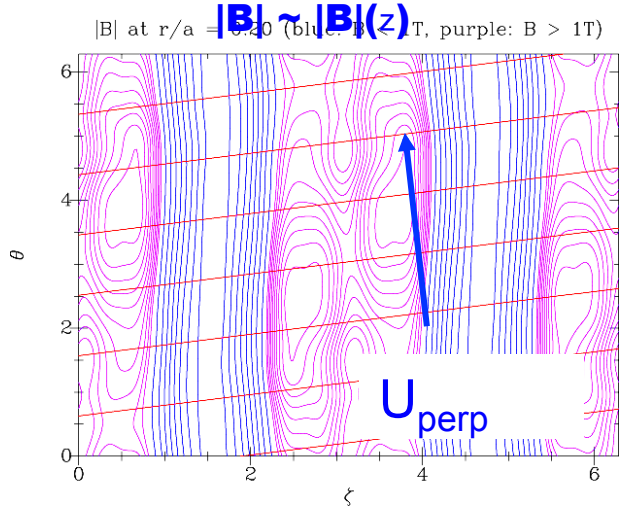
Quasi-helical symmetry
$|B| \sim |B|(mq - nz)$



Quasi-toroidal symmetry
$|B| \sim |B|(q)$

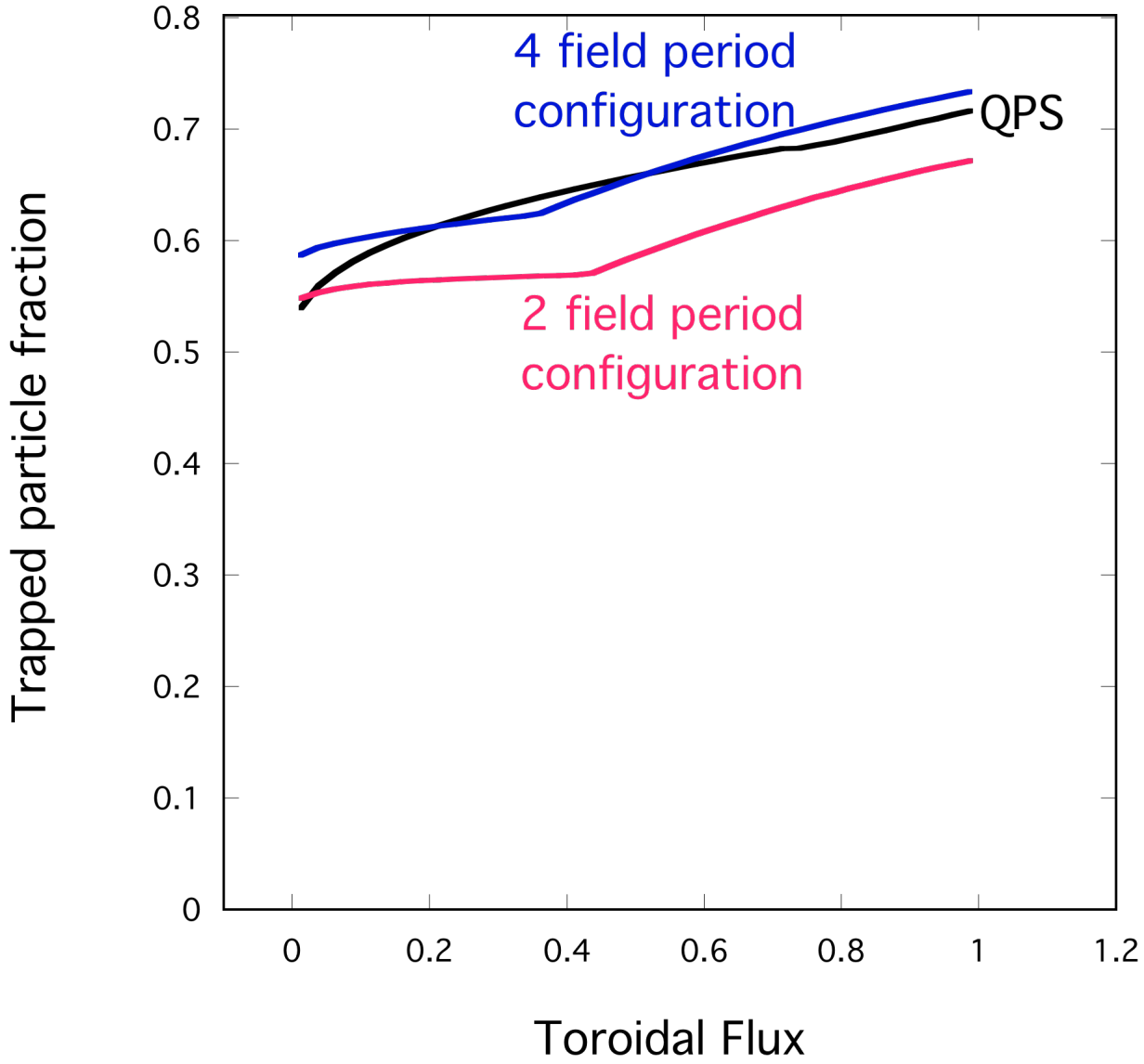


Quasi-poloidal symmetry
$|B| \sim |B|(z)$



New directions in stellarator optimization/design

Don Spong
Oak Ridge National Laboratory

University of Wisconsin Plasma Seminar
Madison, Wisconsin, November 16, 2009

Trapped particle fraction