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THE UNIVERSITY OF WARWICK

Quantifying ETG Transport In Spherical Tokamaks With Flow Shear

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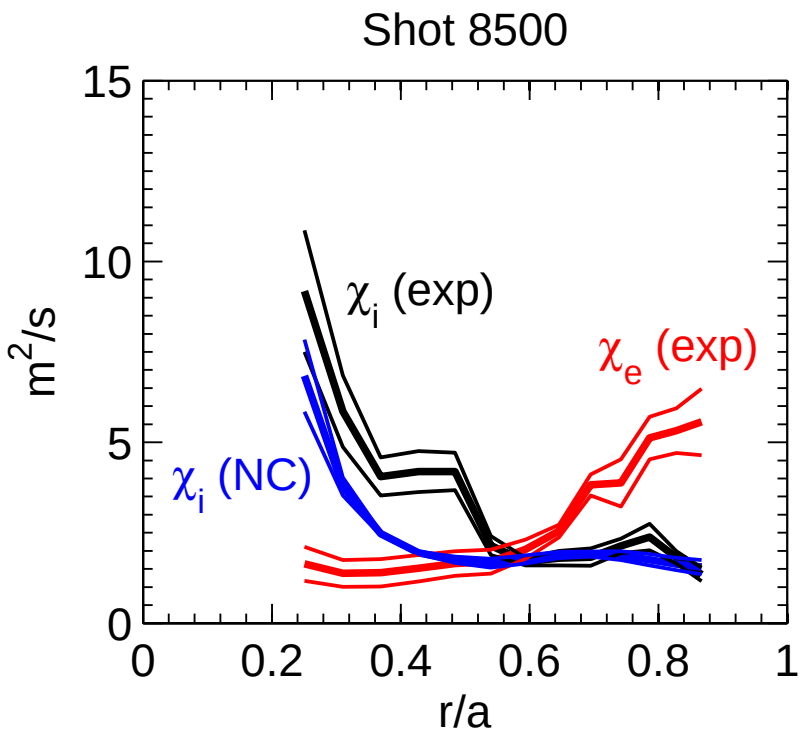
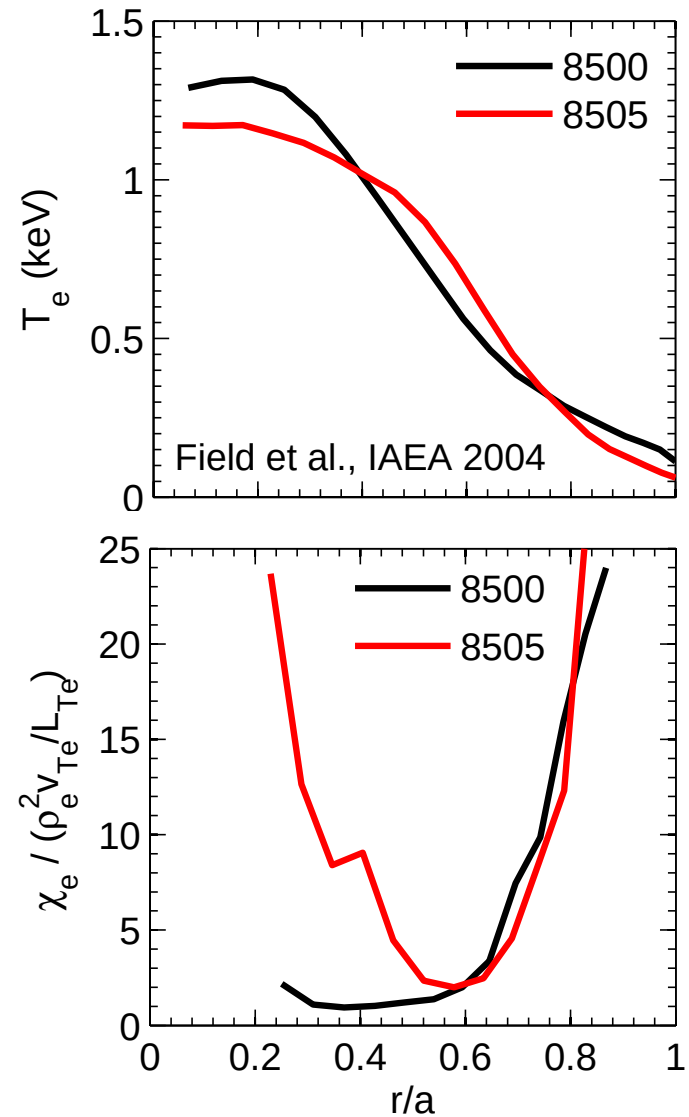
Overview

- Relevance of ETG to spherical tokamaks
- ST-like nonlinear gyrokinetic simulations (low aspect ratio, shaped, *strong $E \times B$ shear*)
- Convergence of simulations using kinetic ions & electrons with grid size, resolution *and boundary conditions* using GYRO (comparison with adiabatic ion model)
- Temperature profile predictions using a model transport expression derived from nonlinear simulations

Experimental Example: MAST Thermal Transport

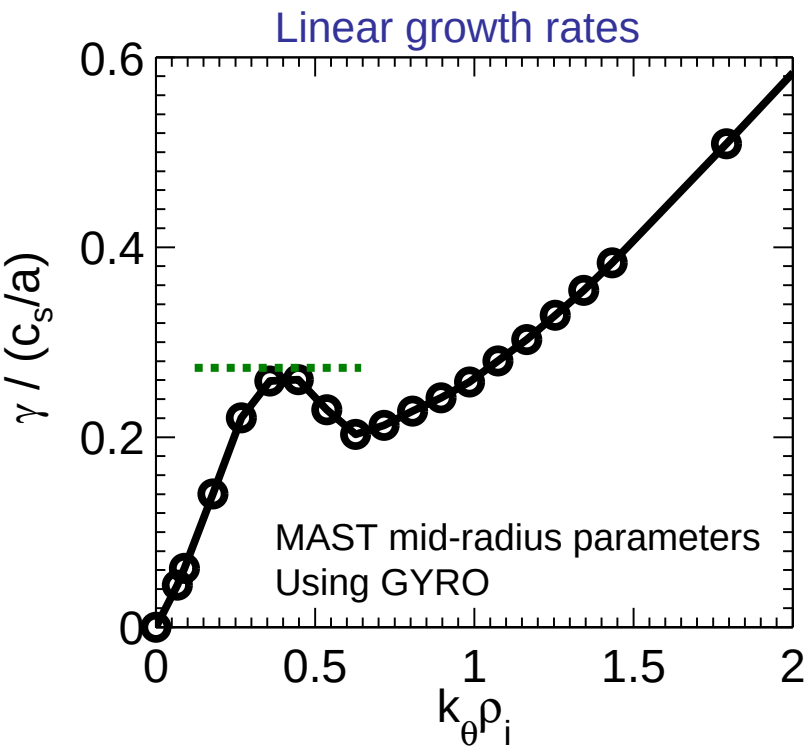
- Considering plasmas $P_{NBI} \leq 2$ MW, $n \sim 3\text{-}5 \times 10^{19}$ m^{-3}, $T_e(0) \sim T_i(0) \leq 1.5$ keV, B~0.45-0.55 T
- χ_i approaches neoclassical
- χ_e anomalous, a few times electron gyroBohm $\chi_e^{GB} = \frac{\rho_e^2 v_{Te}}{L_{Te}}$

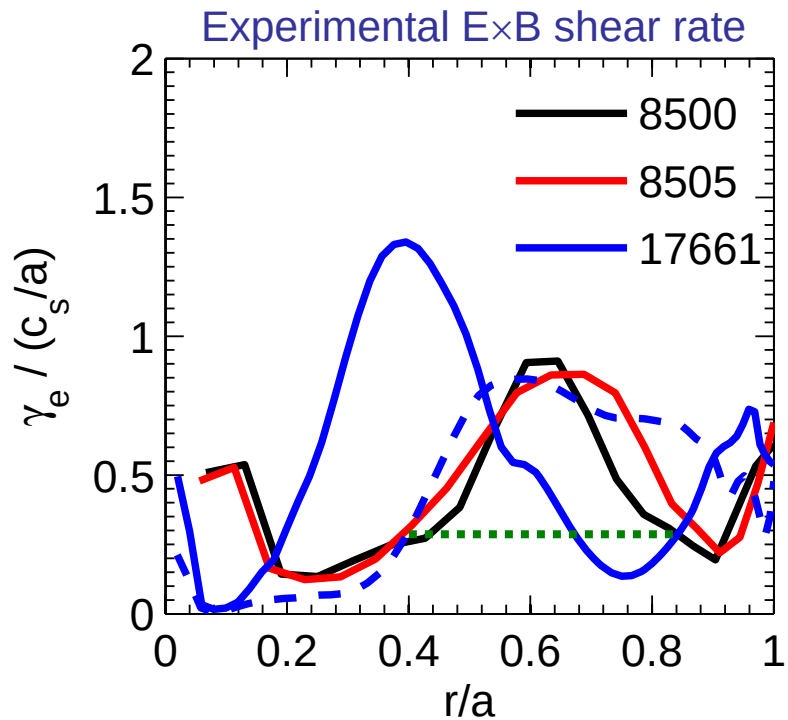




E×B Shear Rates Strong Compared to Ion Scale Linear Growth Rates

- From [Kinsey et al., 2007], complete suppression expected at $\gamma_E \approx 1 \cdot \gamma_{lin,max}$ for R/a=1.6, κ=1.5



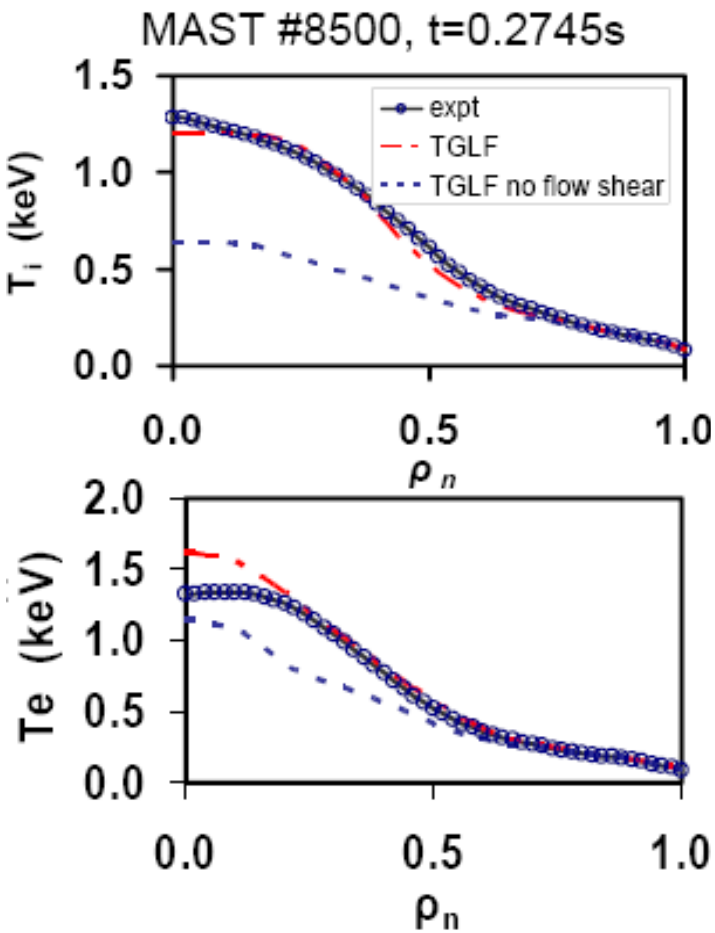


$$\gamma_E = \frac{r}{q}\frac{\partial}{\partial r}\left(\frac{q}{r}\frac{E_r}{B_{unit}}\right)$$

$$B_{unit} = B_0 \frac{\rho}{r}\frac{\partial \rho}{\partial r} \qquad \rho = \sqrt{\frac{\psi_t}{\psi_{t,LCFS}}}$$

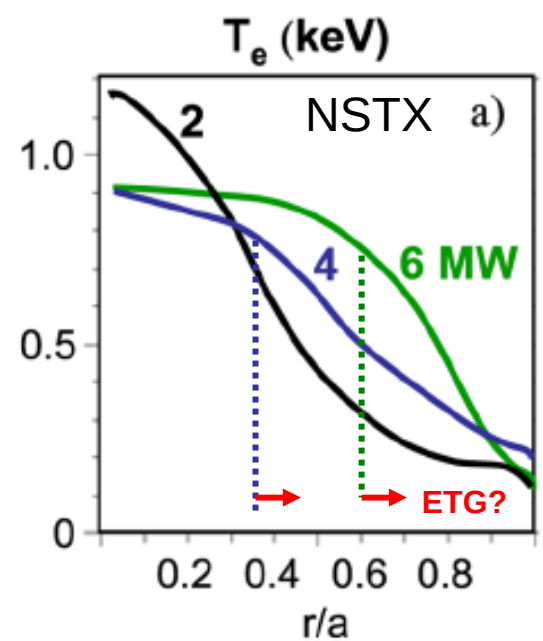
ETG Is Potentially Relevant

- **One** MAST prediction with TGLF works "well", ion transport neoclassical (large E×B shear), electron transport dominated by ETG
- ETG model in TGLF calibrated by **one** non-linear gyrokinetic simulation



Staebler et al. IAEA 2009; Colyer

Obviously can not account for flat profiles in core of high power discharges, but maybe the edge – when and where is ETG valid?



Stutman et al., PRL 2009

ST-ETG Modeling Goals

Linear stability

- Known linear dependence for low beta, modest shaping [Jenko et al. 2001]

$$\left(\frac{R}{L_{Te}}\right)_{crit} = \left(1+\frac{Z_{eff}T_e}{T_i}\right)\cdot\left(1.33+1.91\frac{\hat{s}}{q}\right)\cdot\left(1-1.5\varepsilon\right)\cdot\left(1+0.3r\frac{d\kappa}{dr}\right)$$

- TGLF will hopefully capture modifications at high beta – benchmarking with GS2, GYRO, GKW is underway [motivated by Roach, EPS 2009]

Non-linear transport

- Capture dominant scaling in non-linear transport
- Validate (or recalibrate?) ETG model in TGLF

What are the dominant non-linear scalings?

Interpreting Published Simulations *Assuming* a Critical Gradient Formulation

$$\chi_e = \underline{F(\cdots)} \cdot \underline{\left(\frac{\rho_e^2 v_{Te}}{L_{Te}}\right)} \cdot \underline{\left(\frac{R}{L_{Te}} - \frac{R}{L_{Te}}\bigg|_{crit}\right)^{1.0}}$$

Non-linear effects

gyroBohm → local effects only

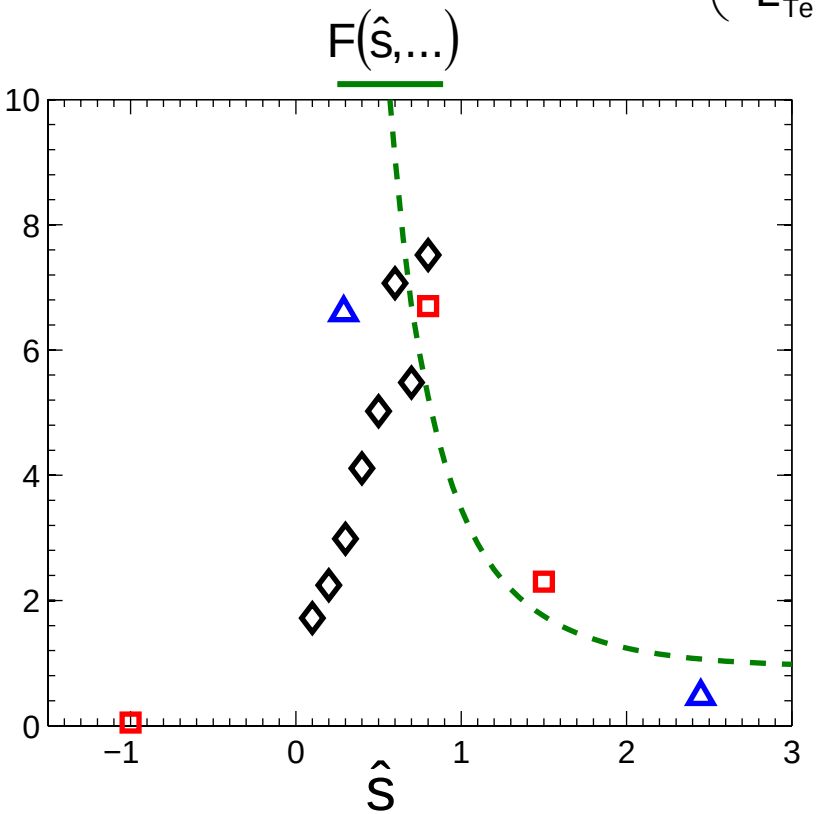
Linear threshold scaling
[Jenko et al. 2001]

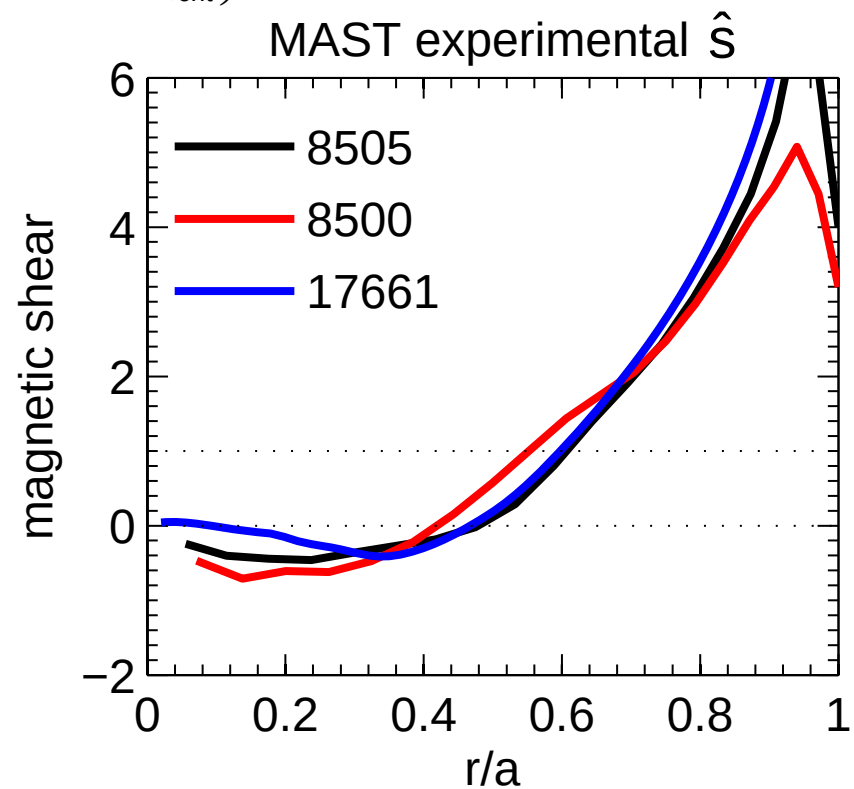
$$\left(\frac{R}{L_{Te}}\right)_{crit} = \left(1 + \frac{Z_{eff} T_e}{T_i}\right) \cdot \left(1.33 + 1.91\frac{\hat{s}}{q}\right) \times$$
$$(1 - 1.5\varepsilon) \cdot \left(1 + 0.3r\frac{d\kappa}{dr}\right)$$

Strong Dependence Of Nonlinear ETG Transport On Magnetic Shear Expected

- Interpreting three published sets of simulations *assuming* a critical gradient model

$$\chi_e = \underline{F(\ldots)} \cdot \left(\frac{\rho_e^2 v_{Te}}{L_{Te}} \right) \cdot \left(\frac{R}{L_{Te}} - \frac{R}{L_{Te}}\bigg|_{crit} \right)$$

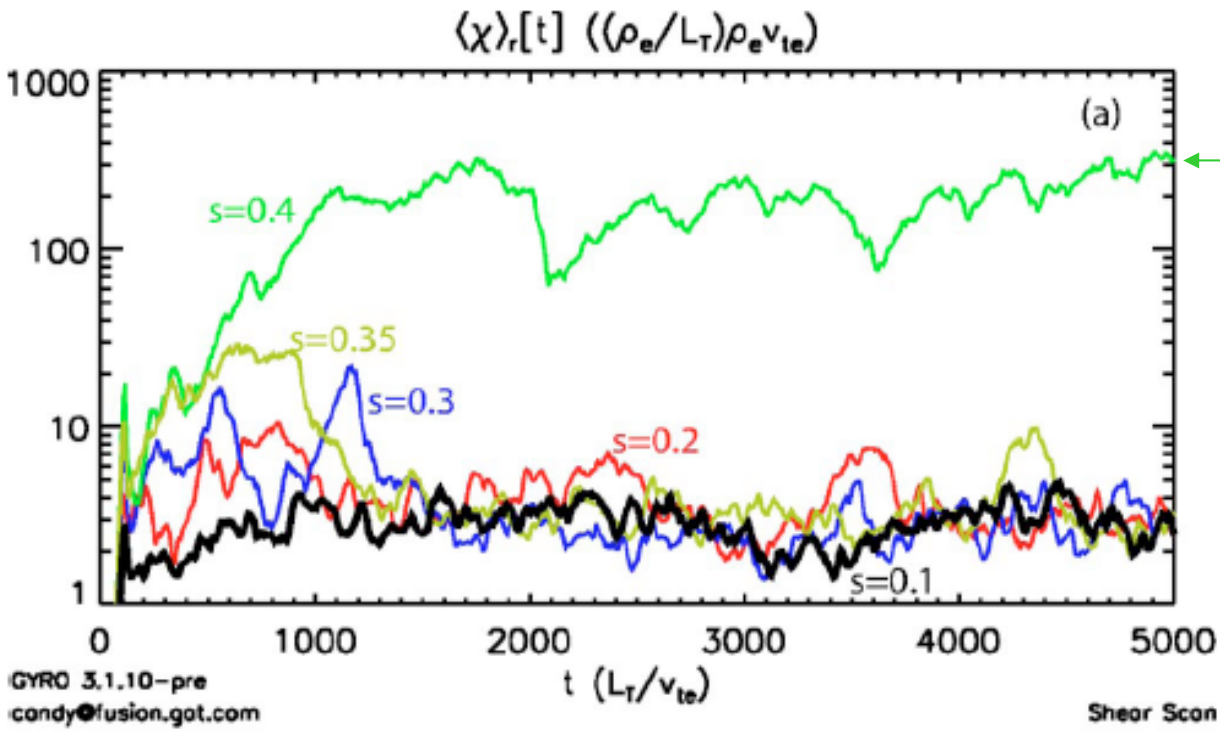




- Nearly all simulations at low safety factor (q=1.4) – want to expand (s,q) parameter space for STs with a consistent set of simulations

ETG Saturation Problems

- Convergence problem in ETG simulations using adiabatic ions for magnetic shear > 0.4 [Nevins et al., 2006]



Unphysical, monotonic spectra

Simulation Issues

- Convergence problem in ETG simulations using adiabatic ions for magnetic shear > 0.4

- Evidently kinetic ions (ETG-ki), strong E×B shear (ETG-ai-exb), or finite electron collisionality can provide long wavelength cut-off/physical saturation mechanism in the simulations

  [Nevins et al. 2006; Mikkelsen; Candy et al. 2007; Roach et al., 2009]

- Following simulations for MAST-like mid-radius parameters:

  $R/a=1.6$, $r/a=0.5$
  $\kappa=1.5$, $\delta=0.2$
  $q=1.4$, $s=0.8$
  $a/L_T=3$, $a/L_n=1$
  BUT
  $\beta=0$ (electrostatic), $\nu=0$

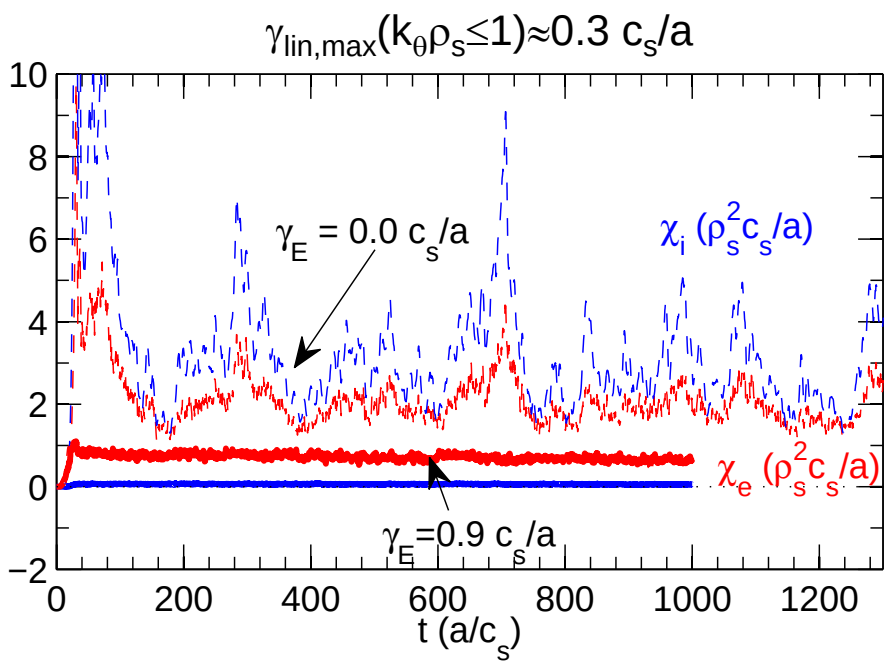
  Kinetic ions and electrons ($m_i/m_e=3600$)

  Using GYRO but in the *local* limit (no n, T, ∇n, ∇T profile variation→ no adaptive source)

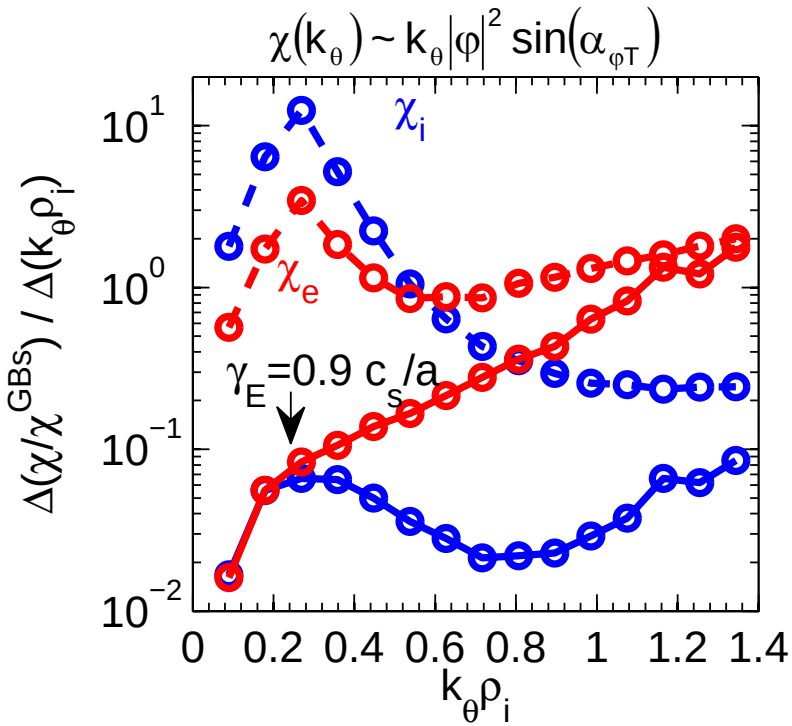
  E×B shear → E_r profile, $\omega_E(r)$
  Non-periodic, fixed BCs

Ion Scale Transport Suppressed In *Local* Simulations With Experimental Level of E×B Shear

- Kinetic ions and electrons
- $70\rho_i \times 70\rho_i$ box, 16/64 toroidal/radial modes, 128 velocity space



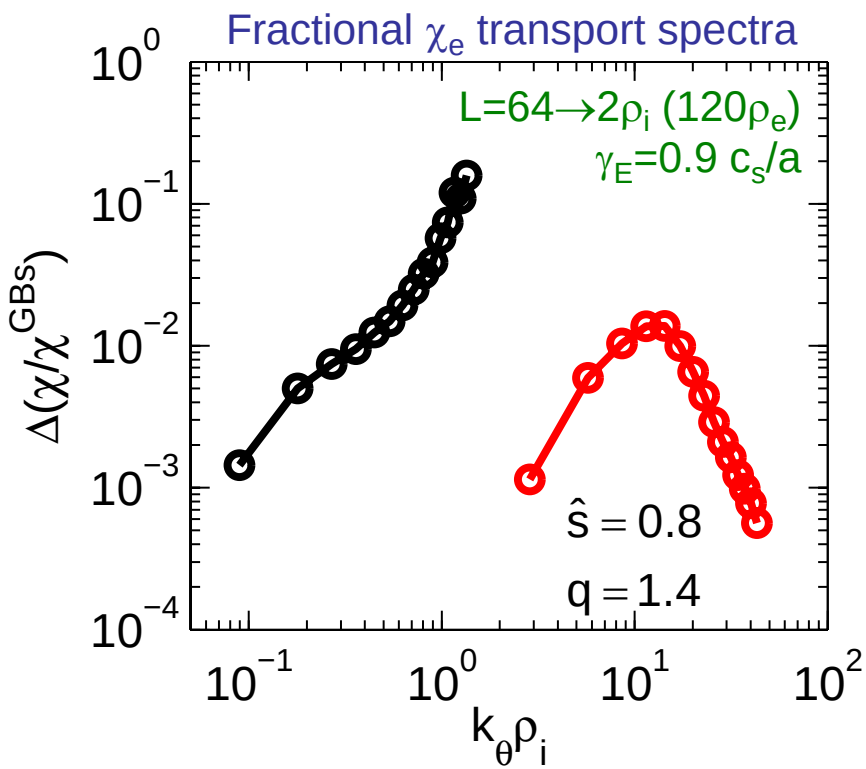
Fractional transport spectra

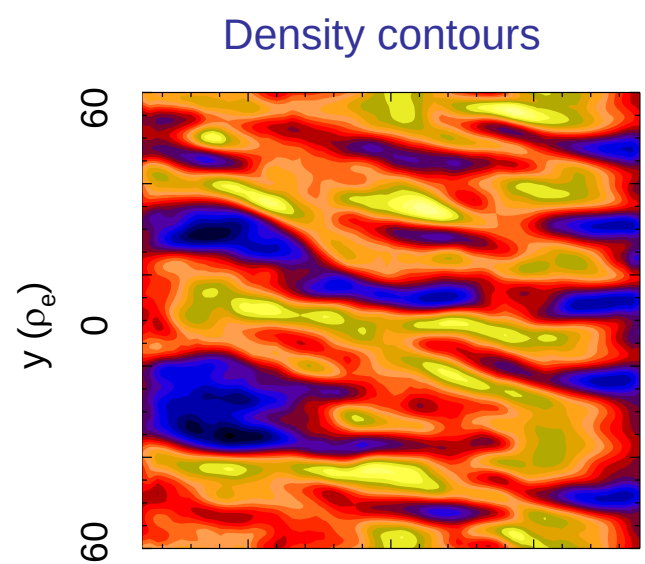


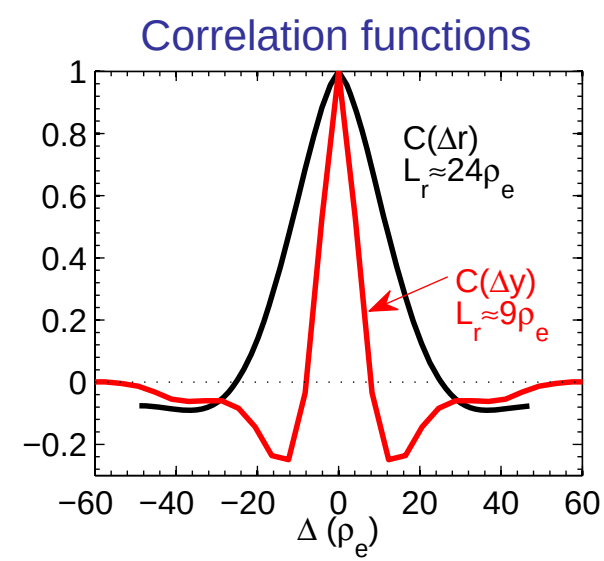
- Neither simulation well resolved → transport contributions at high k_θ not falling off rapidly (or at all)

With Reduced Box Size Spectrum Peak Recovered At Electron Scales

- Transport peak occurs at $k_\theta \rho_e \approx 0.15$

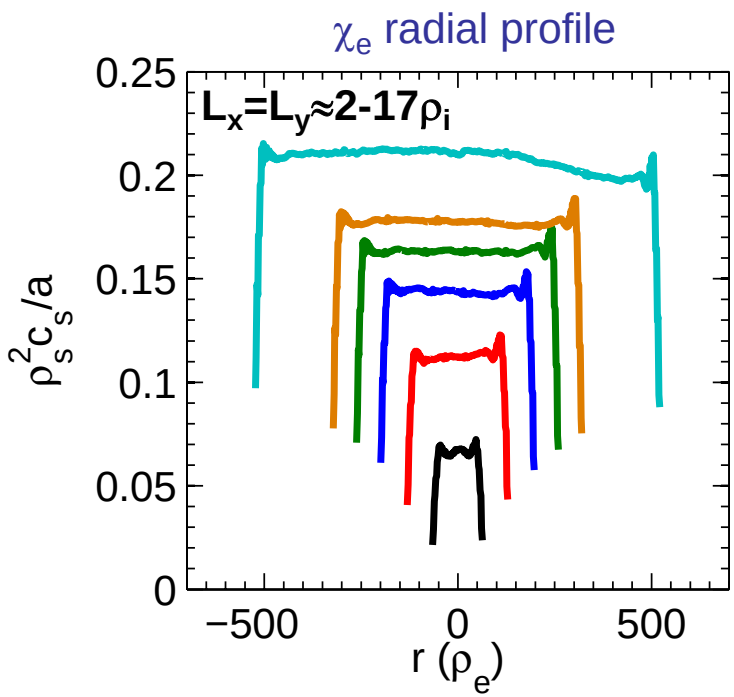


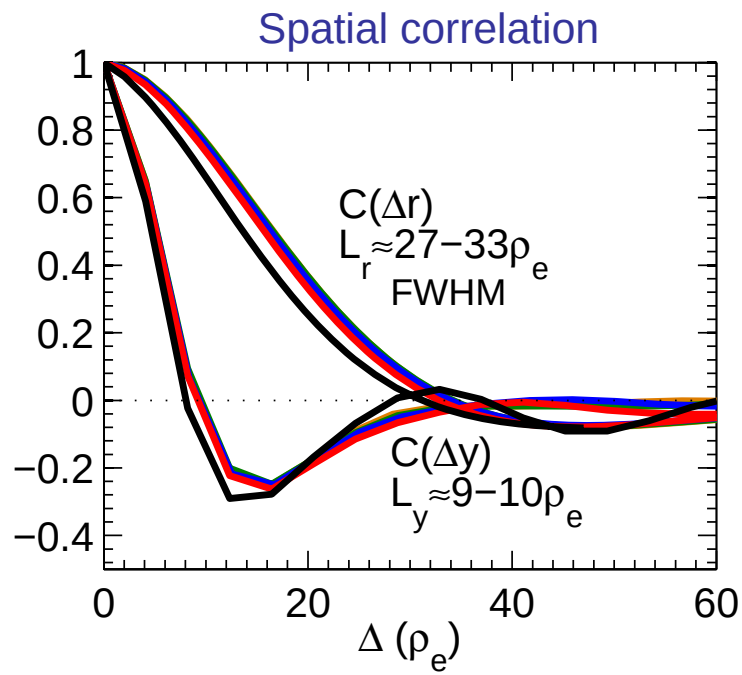




Increased Box Size Alone Does Not Lead To Convergence

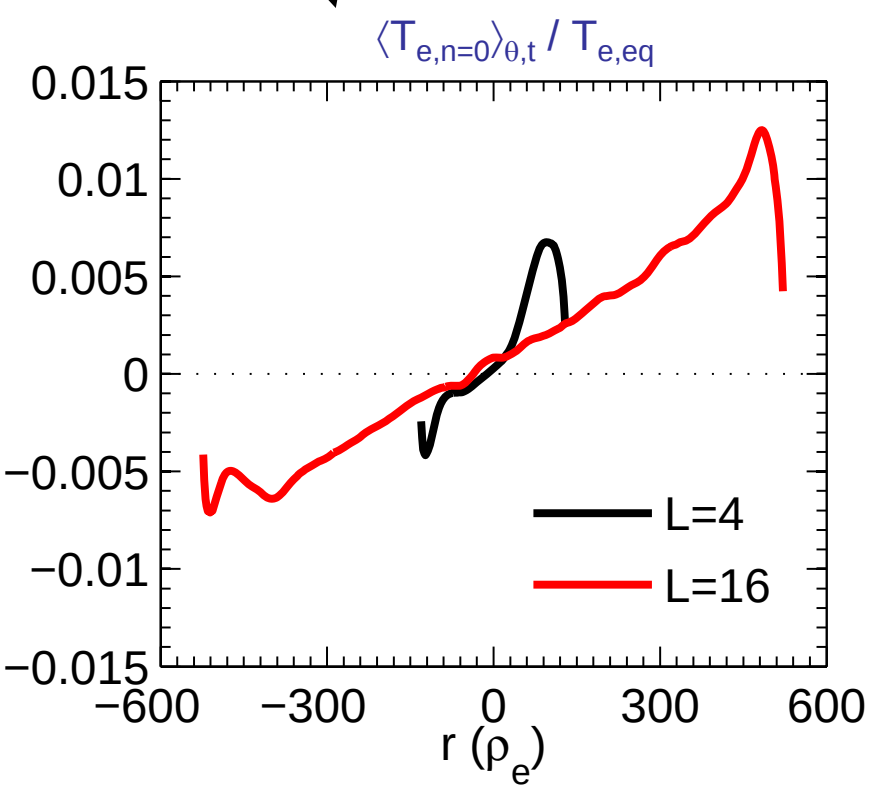
- Toroidal/radial grid 16/64→128/512
- Turbulence characteristics (spectra, correlation lengths) remain relatively constant

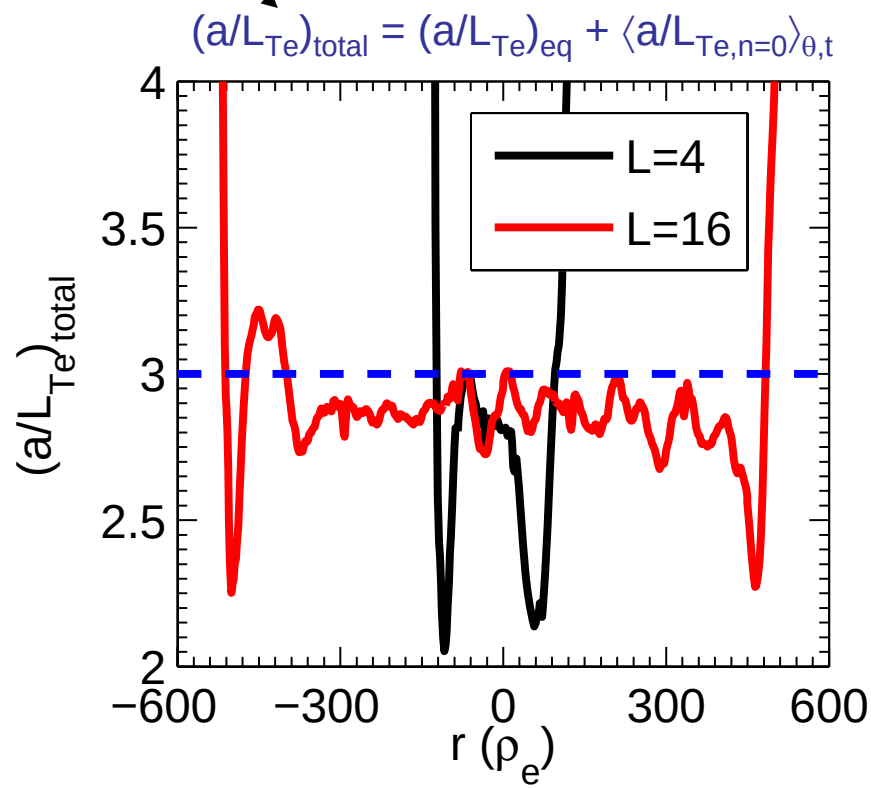




Equilibrium Gradient Perturbations Are Significant

- Even in *local* simulations (flat profiles, a/ρ_e=7500), perturbations grow ($\delta T_{e,n=0}/T_e$~0.01) to cancel equilibrium driving gradients

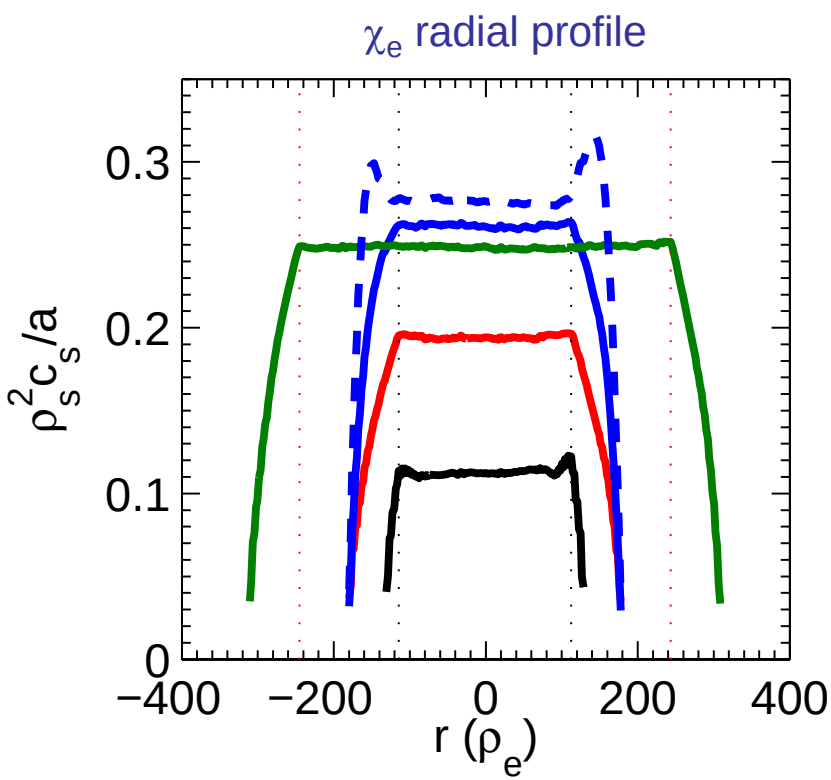




- Need to optimize boundary regions for these electron scale simulations (new territory)

Treatment Of Simulation Boundary

- Fixed boundaries lead to large variation at domain edges (e.g. $\nabla\cdot q$)
- Damping n=0 components of distribution counteracts relaxation



Damping of axisymmetric mode in boundary

$$\delta f_s(\vec{x}) = -\frac{z_\sigma e}{T_\sigma} F_{Ms}\left[\delta\phi(\vec{x}) - \langle\delta\phi(\vec{x})\rangle\right] + h_s(\vec{R})$$

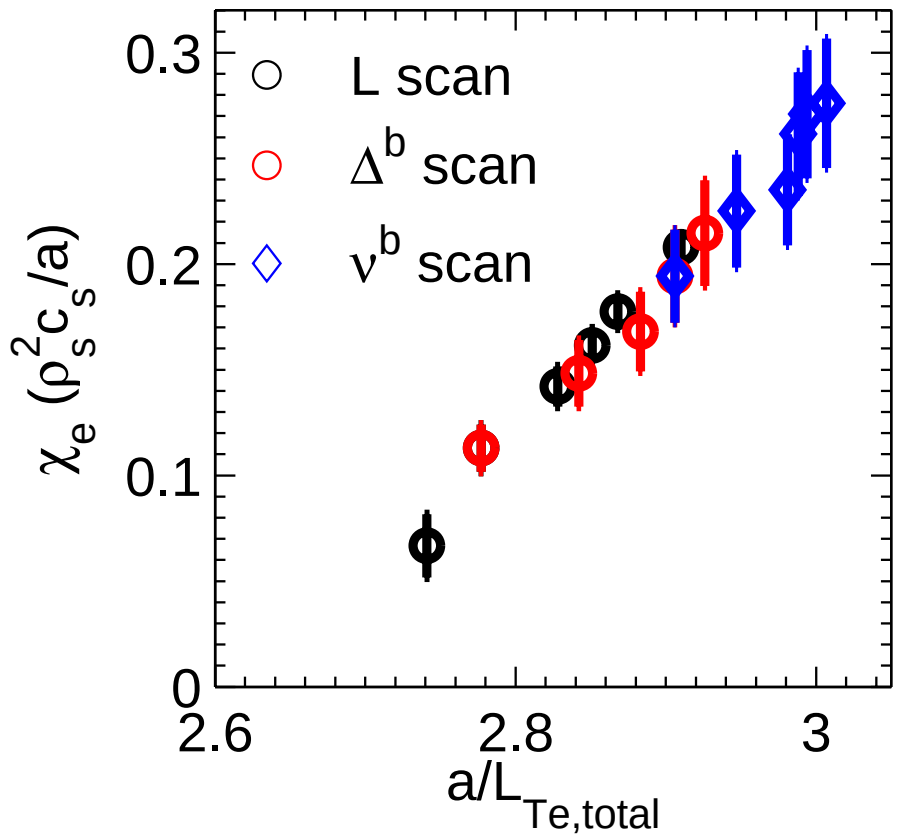
$$\frac{\partial h_{n=0}}{\partial t} = (\mathrm{RHS})_{n=0} - \nu^{\mathrm{buffer}} \cdot \left(h + \frac{Ze}{T_\sigma} F_M \langle\delta\phi\rangle\right)_{n=0} \quad |r| > |r_{\mathrm{buffer}}|$$

$\Delta^b=16\rho_e$, $\nu^b=1\ c_s/a$
$\Delta^b=64\rho_e$, $\nu^b=1\ c_s/a$
$\Delta^b=64\rho_e$, $\nu^b=6{,}30\ c_s/a$
$\Delta^b=64\rho_e$, $\nu^b=2\ c_s/a$

$\Delta^b \sim 10^2\ \rho_e > 2\cdot L_r$ seems okay, but will likely depend on magnitude of transport & correlation lengths

Resulting Convergence Problem Is Mostly Explained By Perturbed Equilibrium Gradient

- Exacerbated by resulting stiff transport

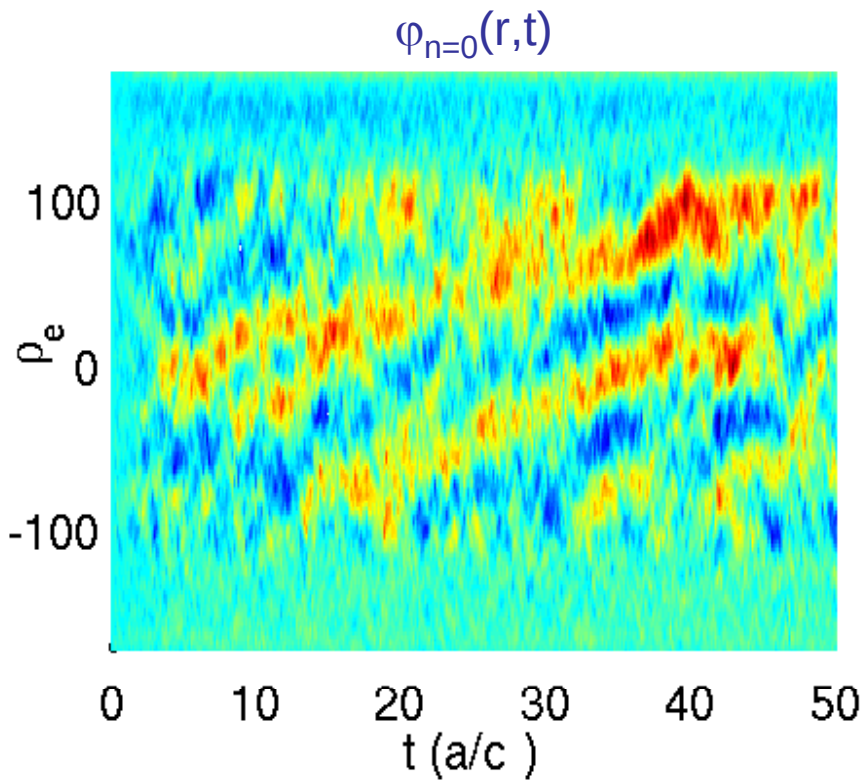


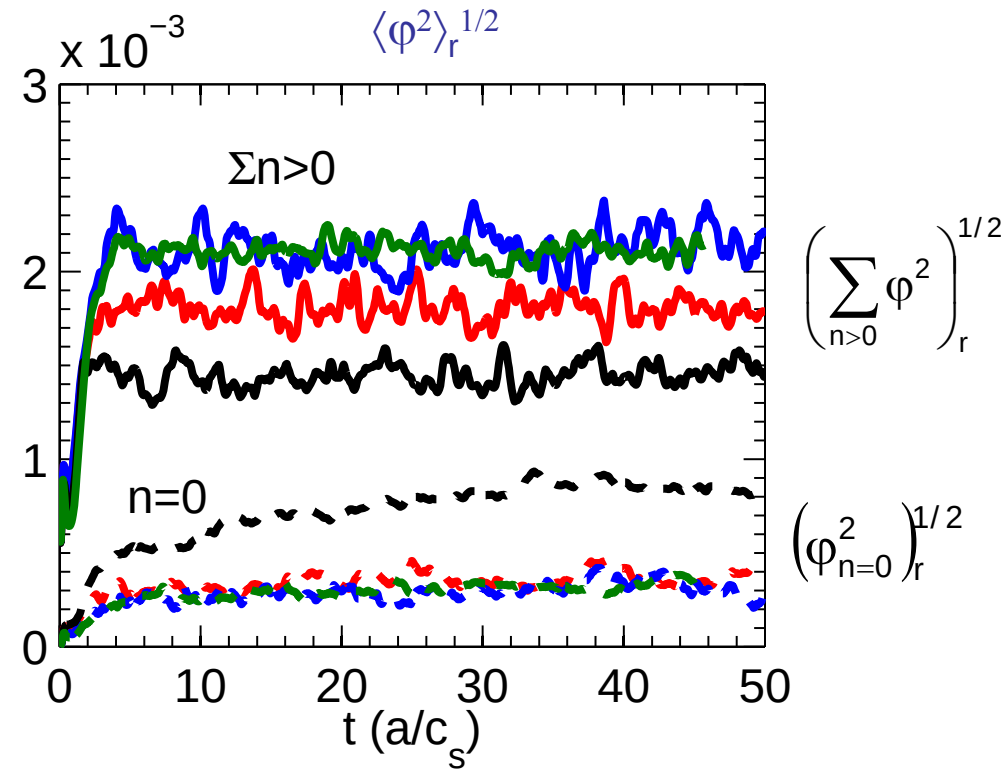
$$\left(\frac{a}{L_{Te}}\right)_{lin,crit} \approx 1.5$$

$$(a/L_{Te})_{total} = (a/L_{Te})_{eq} + \langle a/L_{Te,n=0}\rangle_{\theta,t}$$

Strength Of Zonal Potential Converges And Reaches Steady State

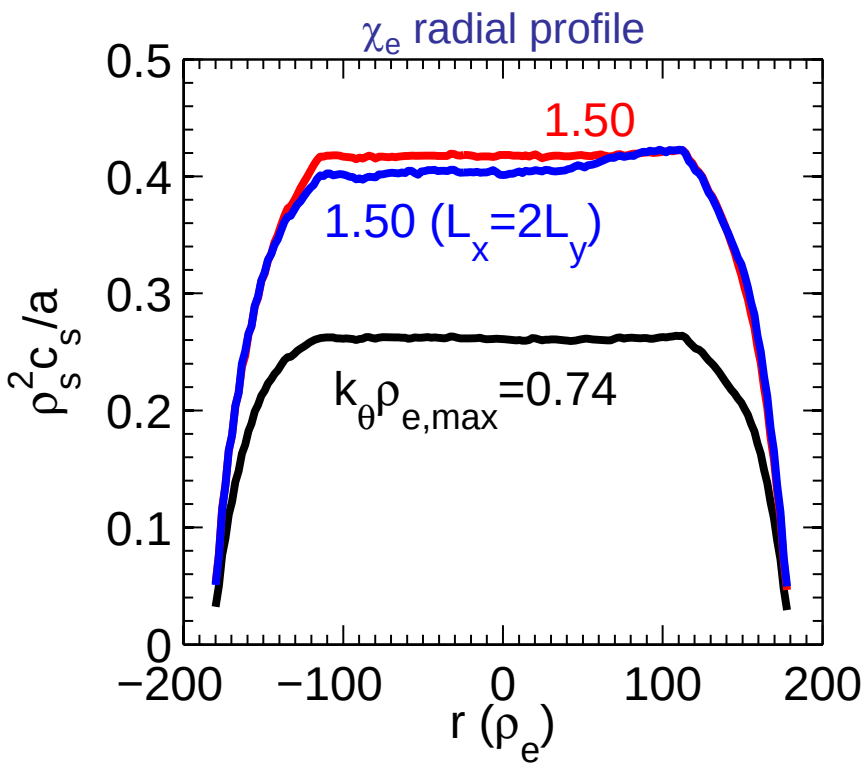
- Radial modes (zonal/axisymmetric) tend to evolve on slower time scales, but they appear to have converged and reached steady state

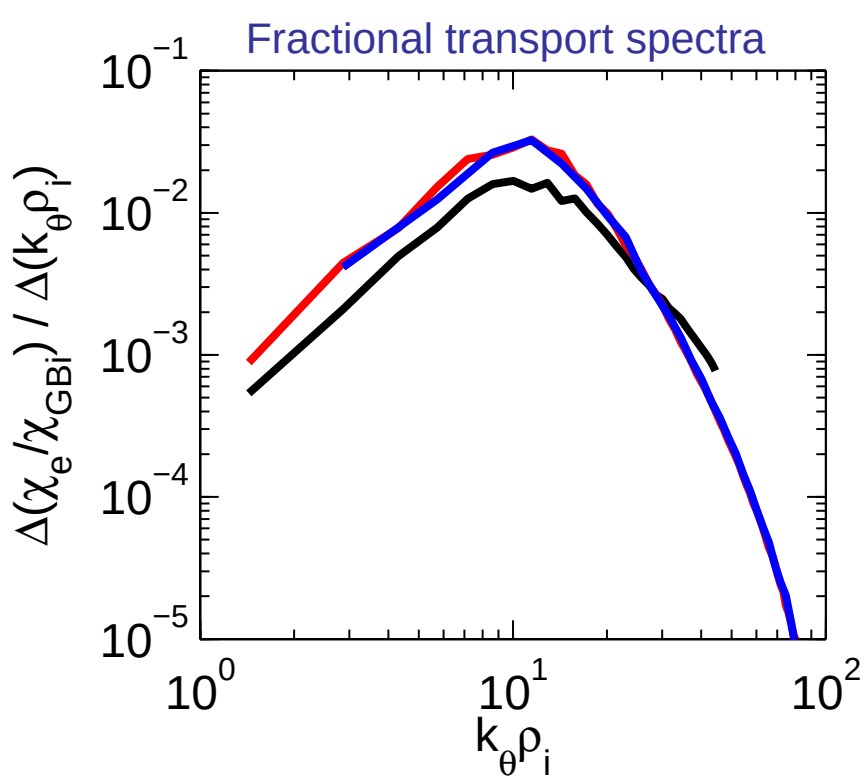




Transport Nearly Doubles With Increased Binormal Resolution

- Similar to ETG-ai benchmark paper [Nevins et al. 2006]
- With such strong shear suppression, can reduce binormal width (L_x=2L_y)





- Additional runs demonstrate convergence with radial resolution (dx → 1.33dx), velocity space (n_{vel}=128→288), and parallel orbit time (n_τ=10→14)

Including Toroidal Rotation & Parallel Flow Shear Does Not Affect These ETG Scale Simulations

- At this level of shear and box size, there is little difference when including toroidal flow (M_{tor}) and parallel flow shear (γ_p)

γ_E=0.9

$\gamma_p=(q/\varepsilon)\gamma_E$=4.32

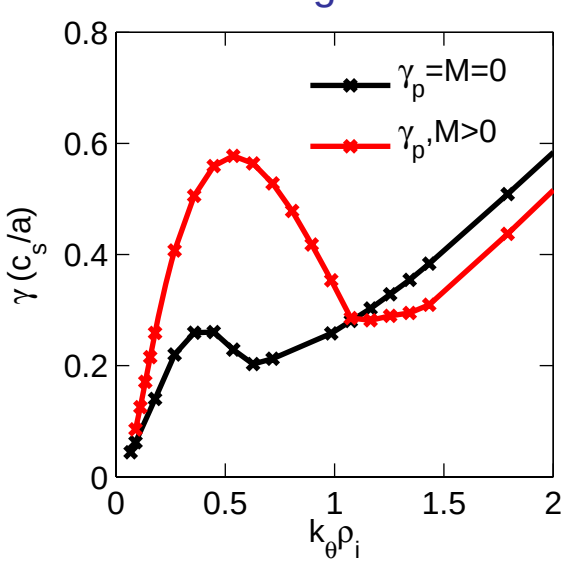
M_{tor}=0.6

$$\gamma_E = \frac{r}{q}\frac{\partial}{\partial r}\left(\frac{\partial \Phi_0}{\partial \psi}\right)$$

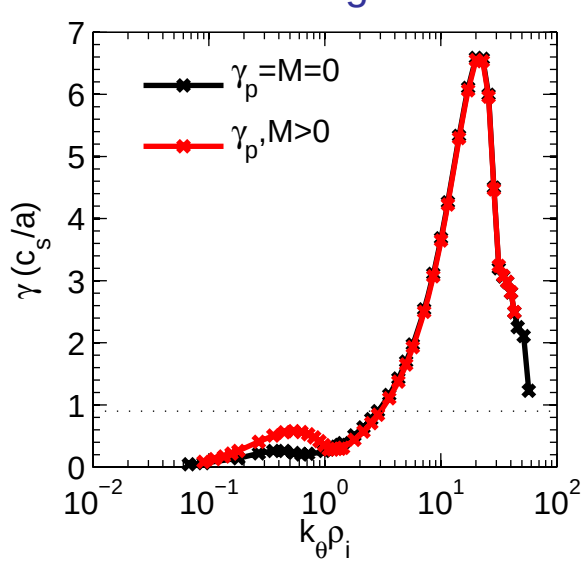
$$\gamma_p = R_0 \frac{\partial}{\partial r}\left(\frac{\partial \Phi_0}{\partial \psi}\right) = \frac{qR_0}{r}\gamma_E$$

$$M = \frac{R_0}{c_s}\frac{\partial \Phi_0}{\partial \psi}$$

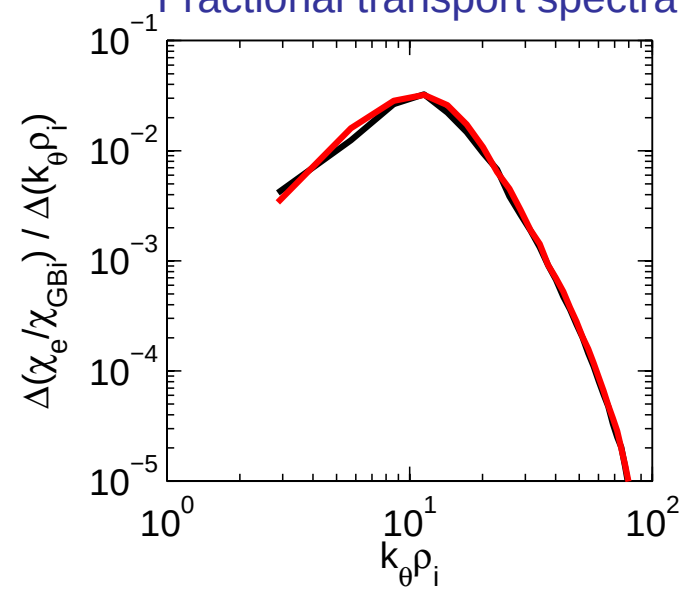
Ion scale growth rates



Electron scale growth rates



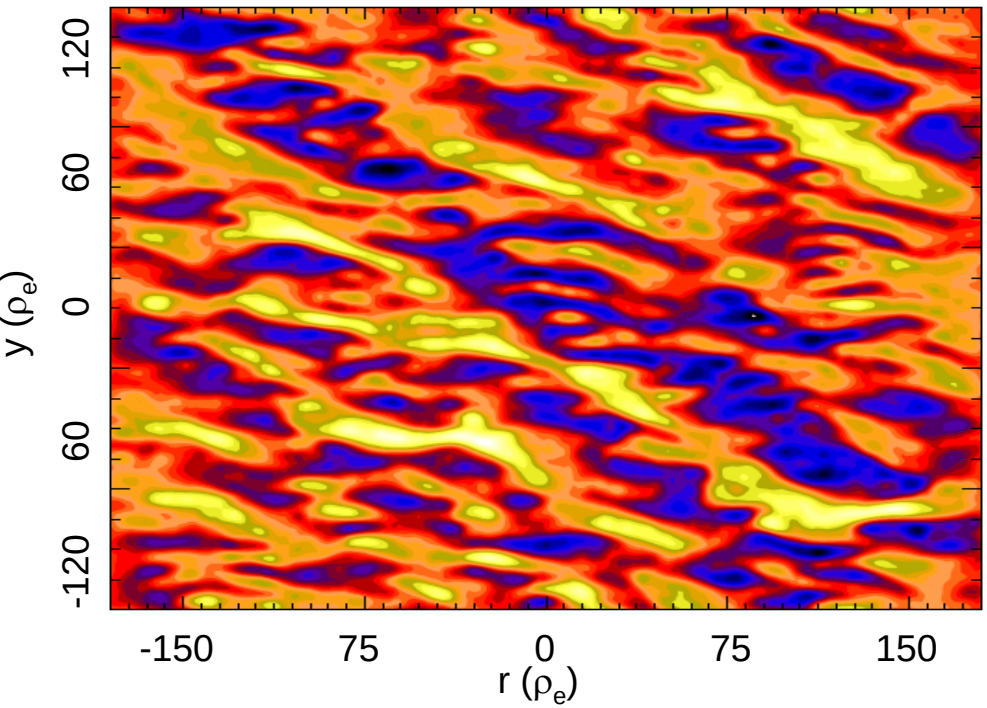
Fractional transport spectra



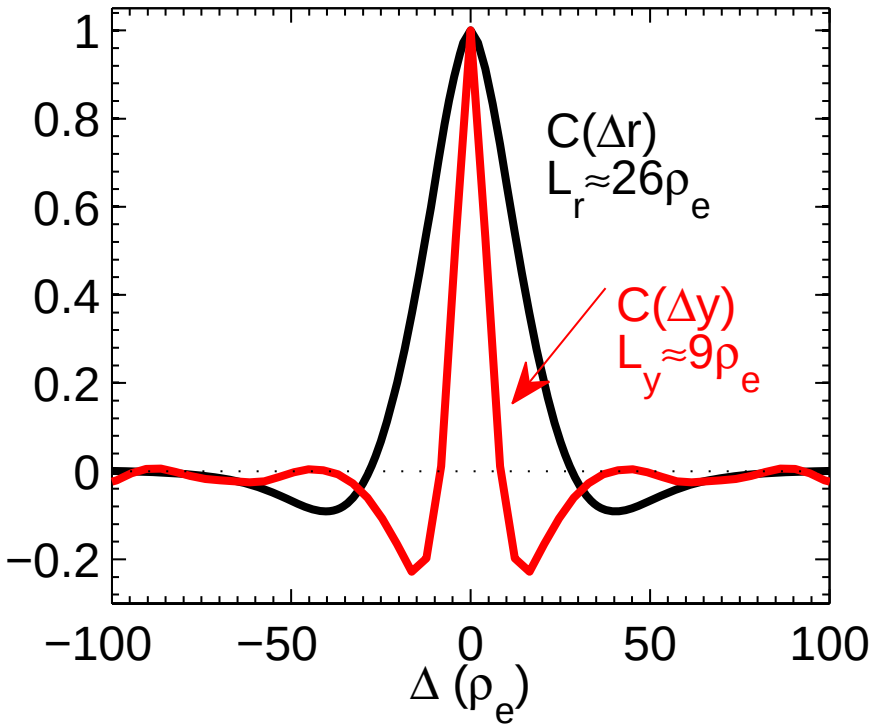
Turbulence Characteristics Qualitatively Constant

- While quantitative transport has varied considerably (χ_e=2-8$\rho_e^2 v_{Te}/L_{Te}$), qualitative turbulence characteristics are largely unchanged
- In the presence of E×B shear, eddies remain elongated (but tilted) with $L_r \approx 3L_y$

Potential contours



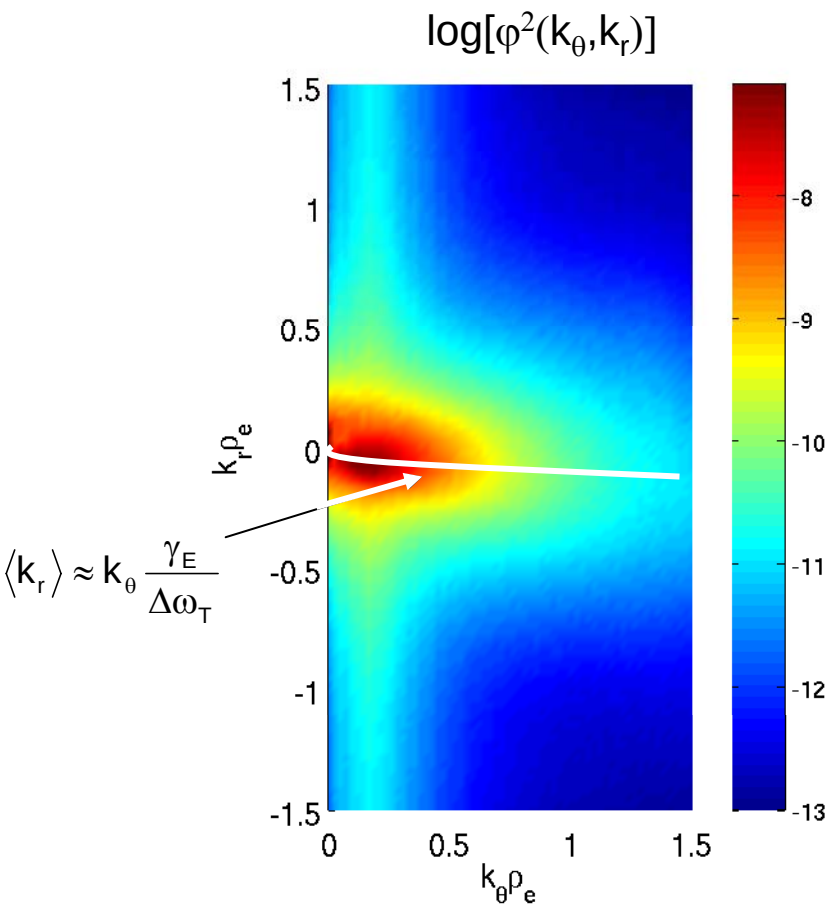
1-D correlation functions

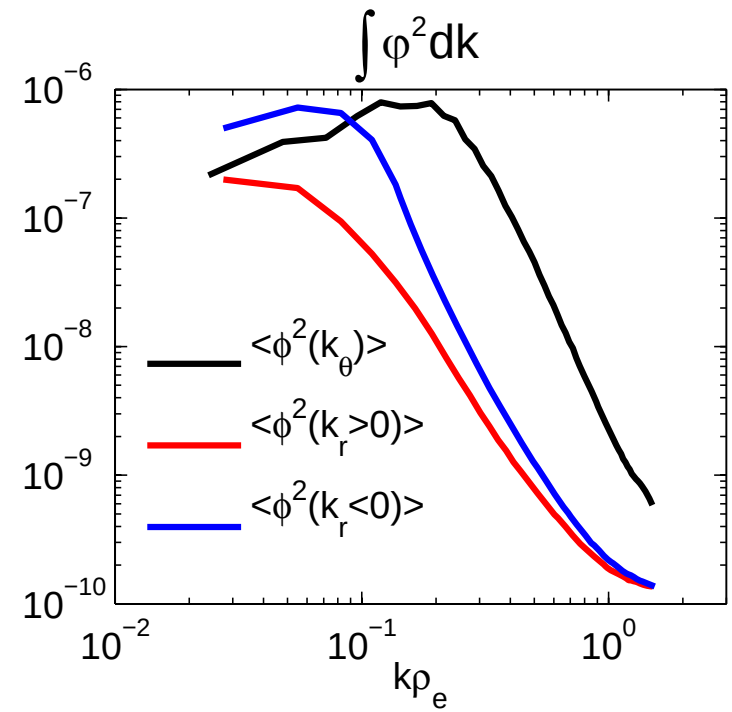


$L_{total} \approx 36\rho_e$

Spectra Are Anisotropic In The Presence Of E×B Shear

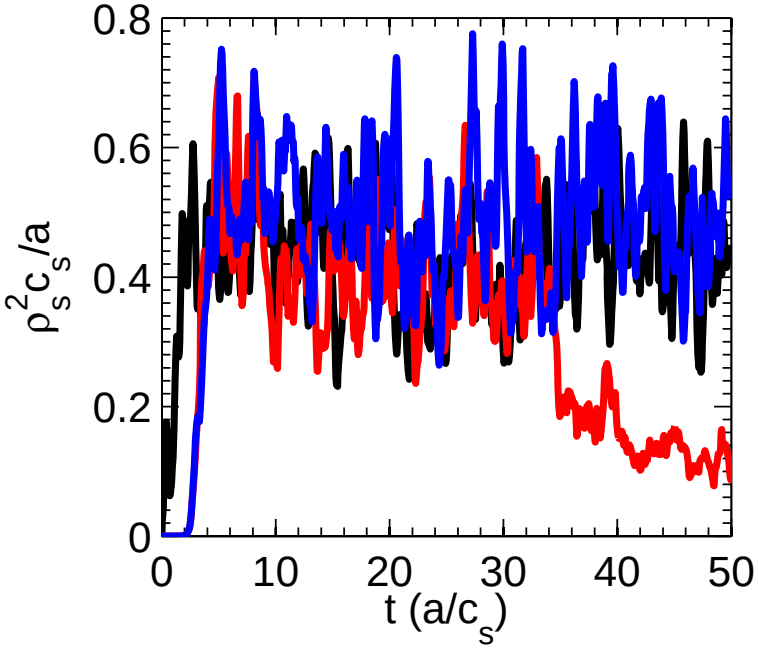
- Spectra are anisotropic at high $k\rho_e$>0.2, contrary to:
  - ETG simulations without E×B shear [Nevins et al., 2006]
  - Multi-scale simulations without E×B shear [Waltz et al., 2007]

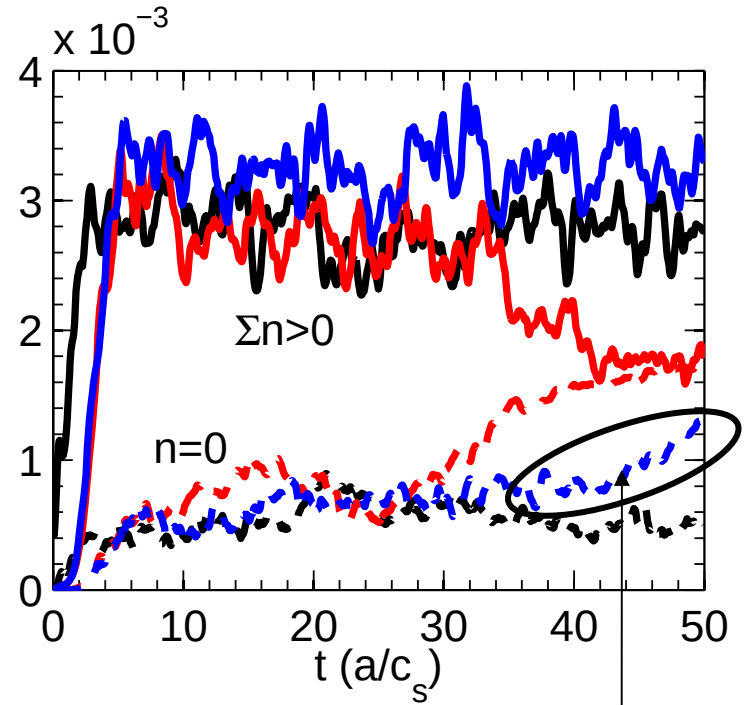




Consideration for interpretation of high-k turbulence measurements & synthetic diagnostics?

Adiabatic Ion Model Can Be Unreliable Even With E×B Shear





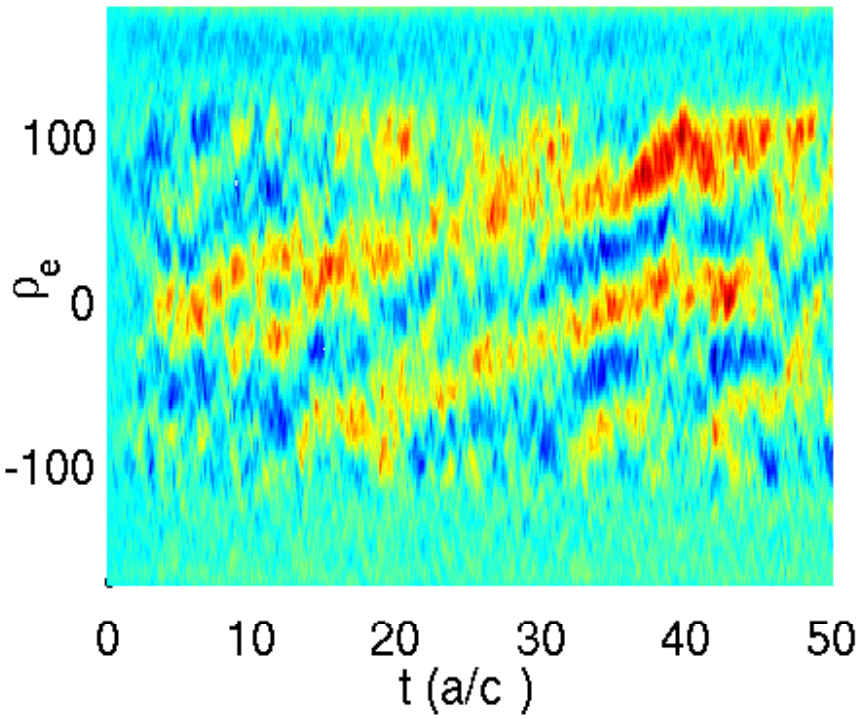
kinetic ion ν^b=6 c_s/a
adiabatic ion ν^b=6 c_s/a
adiabatic ion ν^b=60 c_s/a

Stronger buffer damping only delays onset

Strong Radial Mode Develops

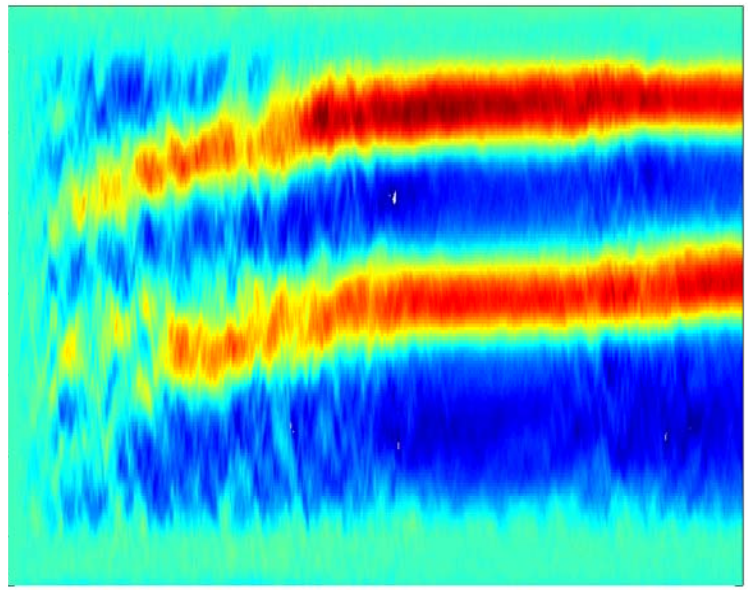
Kinetic ions

$\varphi_{n=0}(r,t)$



Adiabatic ions

$\varphi_{n=0}(r,t)$



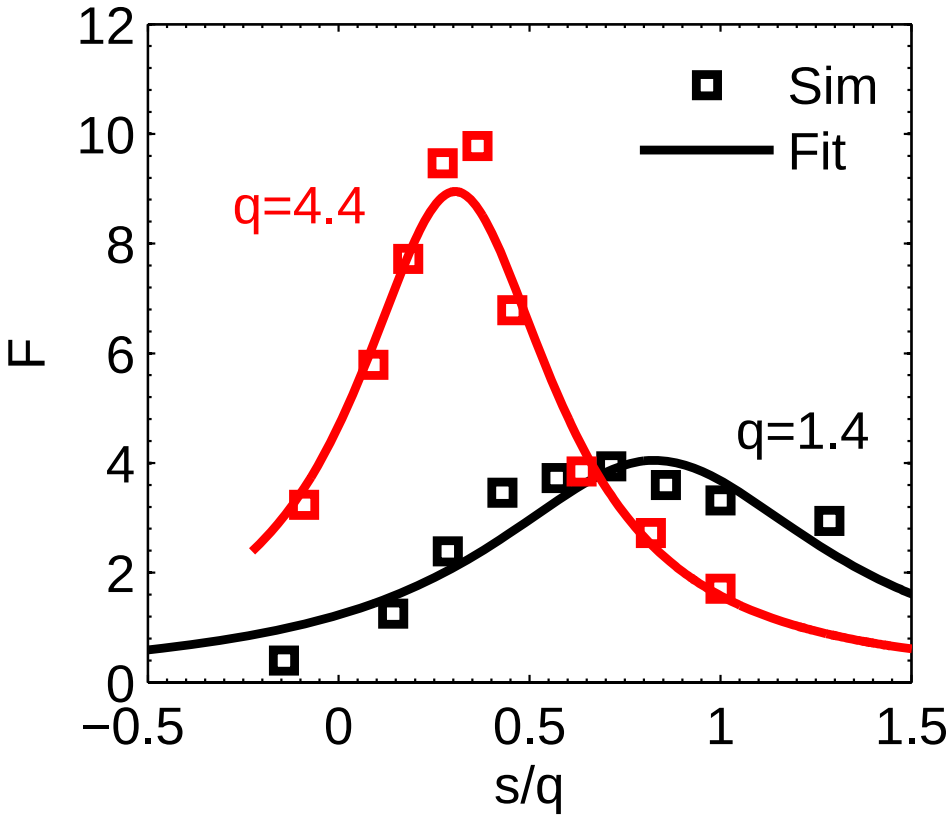
t=100 (a/c_s)

While The Adiabatic Ion Model Can Be Unreliable It Has Been Used For An Initial Scan in s & q

- Earlier (naïve) simulations for comparable box size and resolution were run for varying magnetic shear and safety factor
- They used different boundary conditions and adaptive source (relatively strong, small perturbed gradients)
- *None of the simulations exhibited the transition*

Significant Transport Occurs For Window Of s/q

- Higher safety factor recovers larger transport
- Simple fit to simulations for comparison with experiment



$$\chi_e = F(\cdots)\cdot\left(\frac{\rho_e^2 v_{Te}}{L_{Te}}\right)\cdot\left(\frac{R}{L_{Te}} - \frac{R}{L_{Te}}\bigg|_{crit}\right)$$

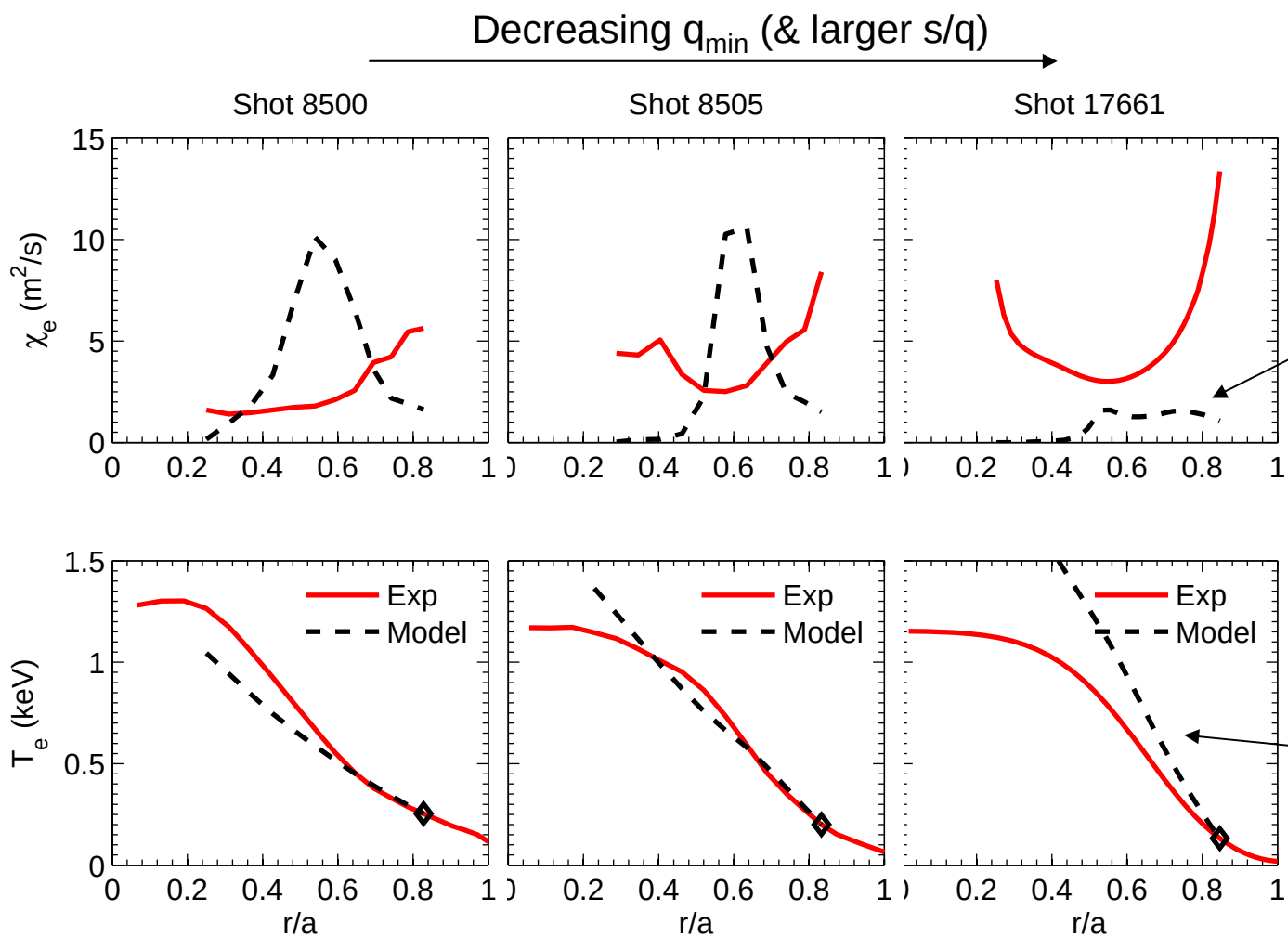
$$F(\hat{s},q) = c_1 \cdot \frac{q^{c_q}}{1+\left|\dfrac{\hat{s}/q - c_2(1+c_3 q)}{c_4(1+c_5 q)}\right|^{c_s}}$$

$c_q \approx 0.7$
$c_s \approx 1.9$

For comparison, χ_i from IFS-PPPL $\quad \chi_i^{IFS-PPPL} \sim \dfrac{q^{1.13}}{1+\hat{s}^{0.84}} \quad \begin{pmatrix} 0.7 < q < 8 \\ 0.5 < \hat{s} < 2 \end{pmatrix}$

Model ETG Transport Significant Over Midradius

- Sensitive to safety factor profile (and *derivative*) – experimental uncertainties?



Direct comparison of χ_e using experimental profiles

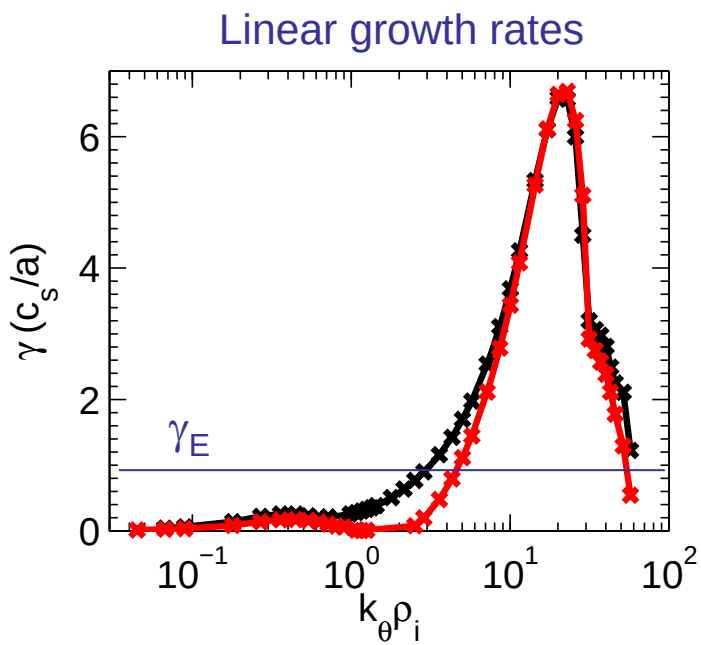
$$\chi_e = F(\cdots)\cdot\left(\frac{\rho_e^2 v_{Te}}{L_{Te}}\right)\cdot\left(\frac{R}{L_{Te}} - \frac{R}{L_{Te}}\bigg|_{crit}\right)$$

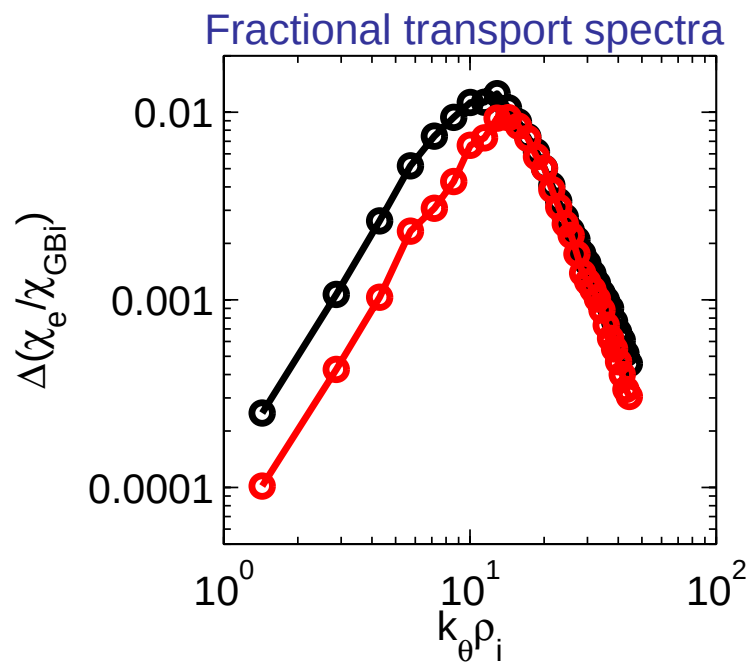
Predicting T_e profile using experimental heat flux and transport model

$$T_e^{model} = -\int \nabla T_e\left(q_e^{exp}, \chi_e^{model}\right) + T_{e,BC}$$

Additional Scaling Expected From Varying Collisionality

- Collisionality in MAST stabilizing to trapped electron contributions
- Transport reduced ~25%





- Additional scaling expected with γ_E and $\beta, \nabla\beta$

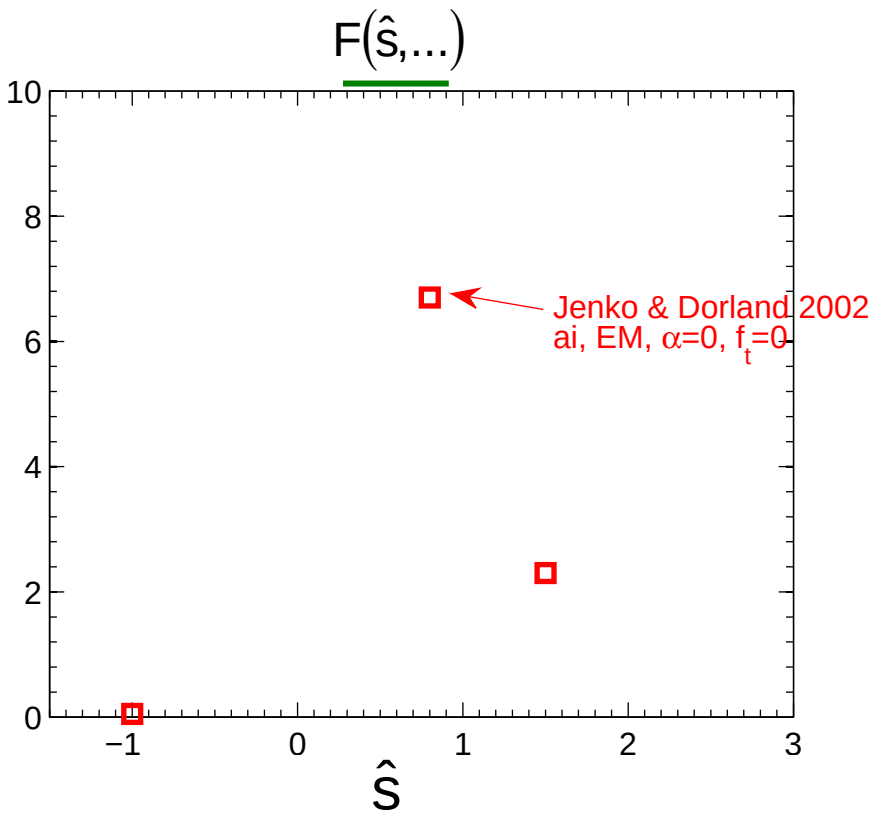
Summary

- Demonstrating convergence of ETG simulations for large magnetic shear, important for spherical tokamak parameters
- Sensitive to simulation boundary (not using spectral flux tube representation)
- Adiabatic ion model with E×B shear can reproduce kinetic ion results, at least in some cases – expect larger disparity for reduced E×B shear (increased drive at ion scales)
- Initial simple transport model seems to recover similar behaviour to TGLF predictions (in **one** instance)

Strong Dependence Of Nonlinear ETG Transport On Magnetic Shear Expected

- Interpreting three published sets of simulations *assuming* a critical gradient model

$$\chi_e = \underline{F(\ldots)} \cdot \left(\frac{\rho_e^2 v_{Te}}{L_{Te}} \right) \cdot \left(\frac{R}{L_{Te}} - \frac{R}{L_{Te}}\bigg|_{crit} \right)$$



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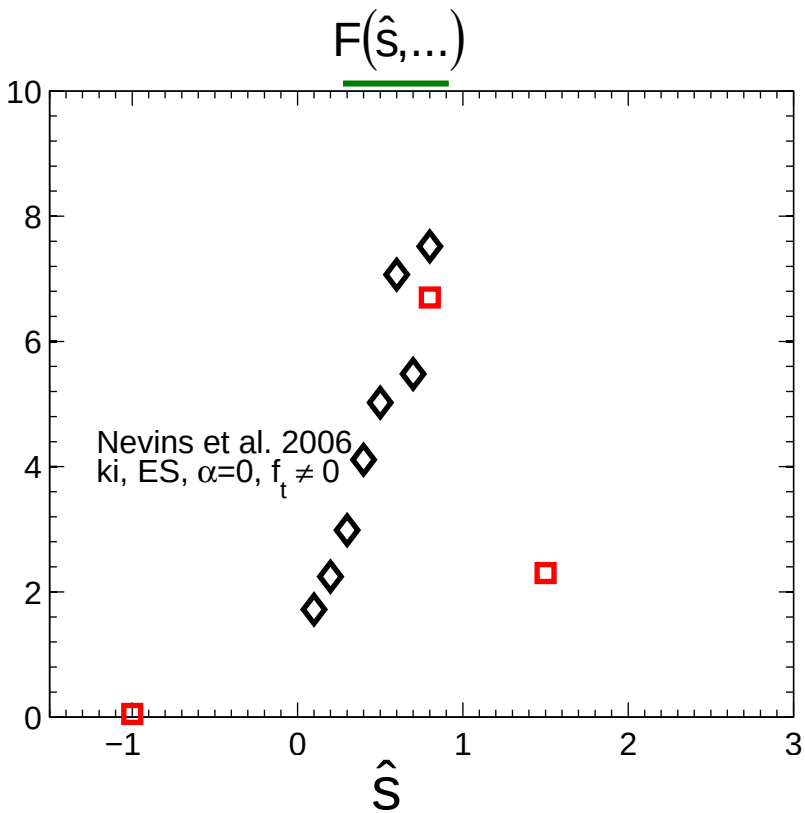

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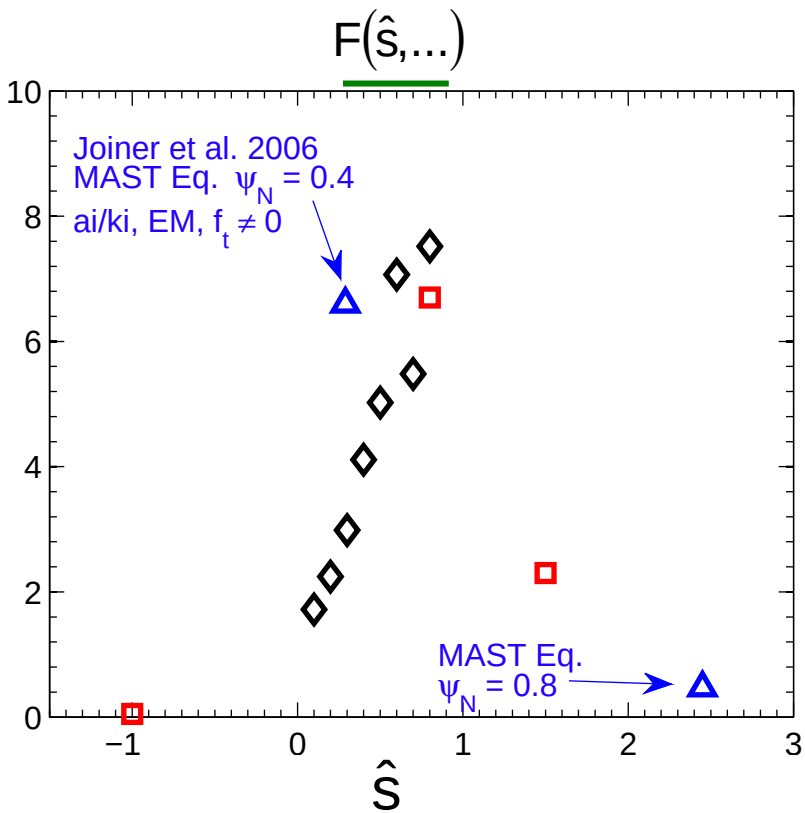

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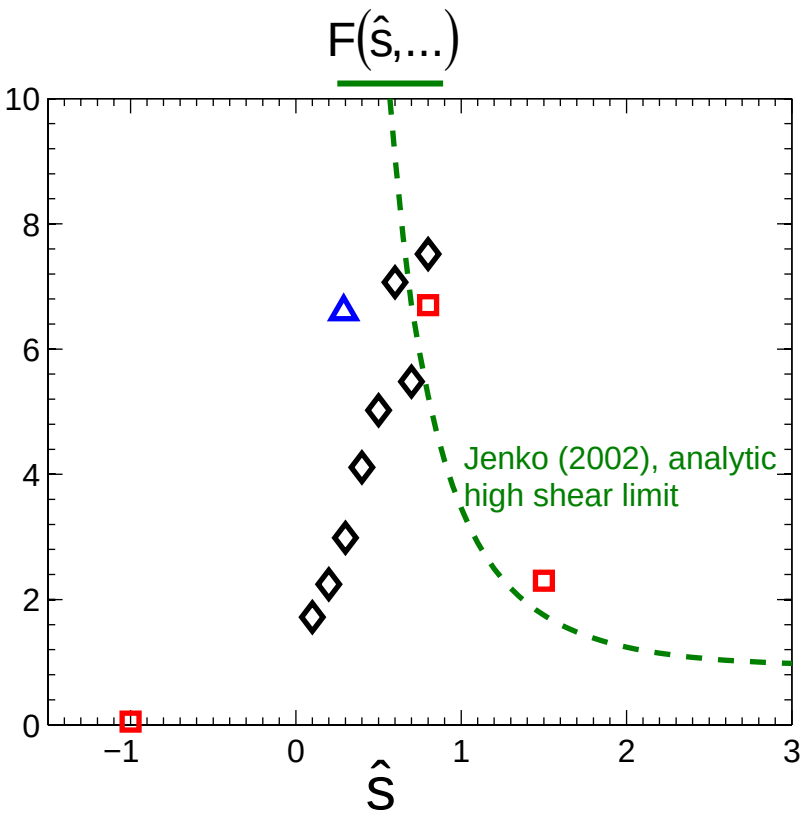

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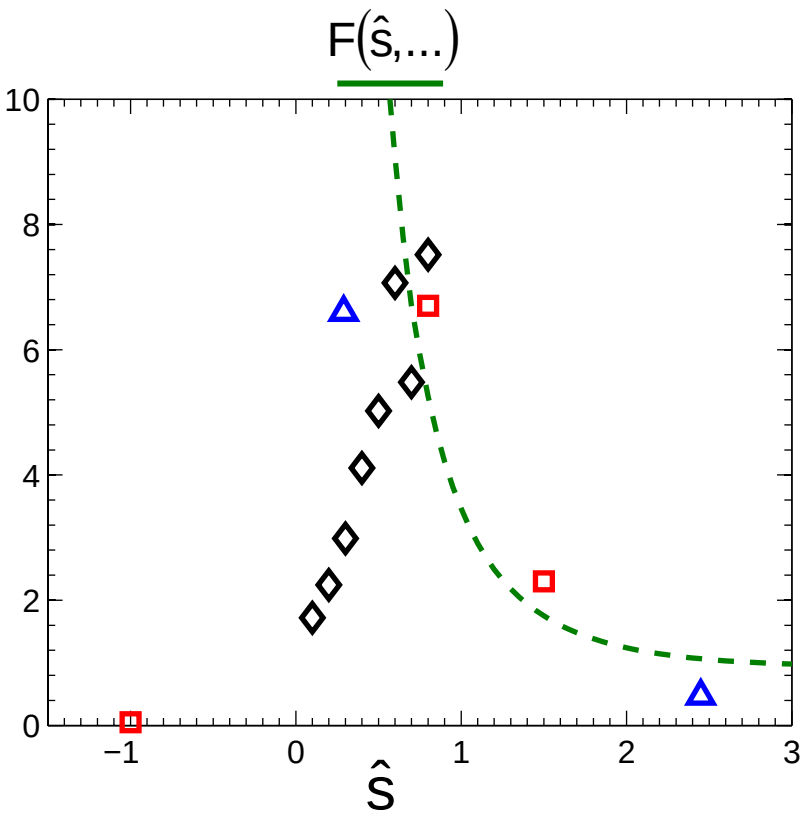
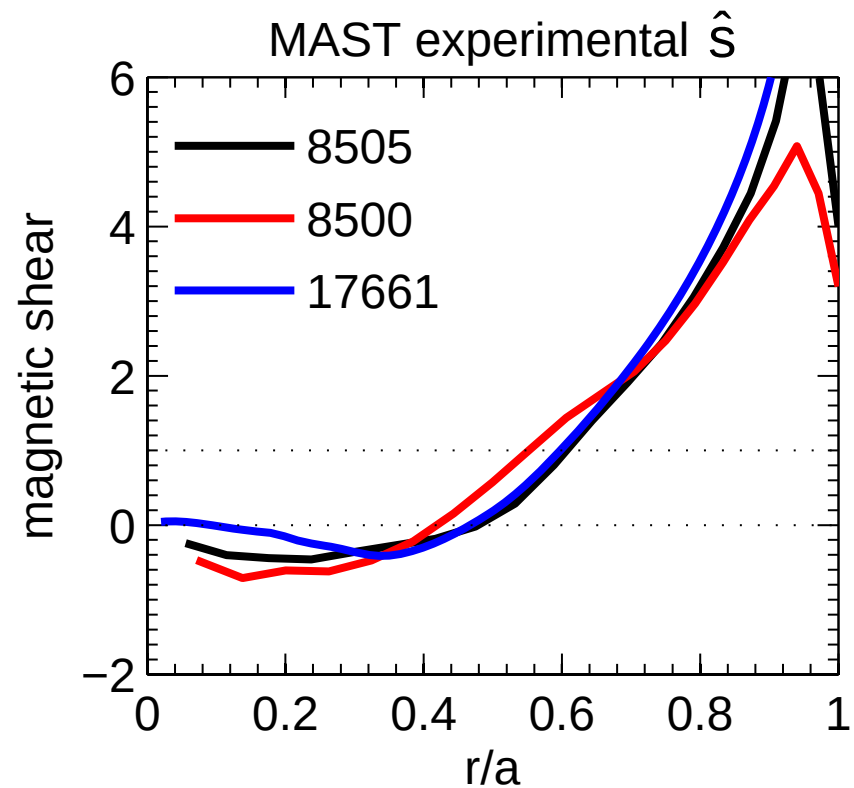
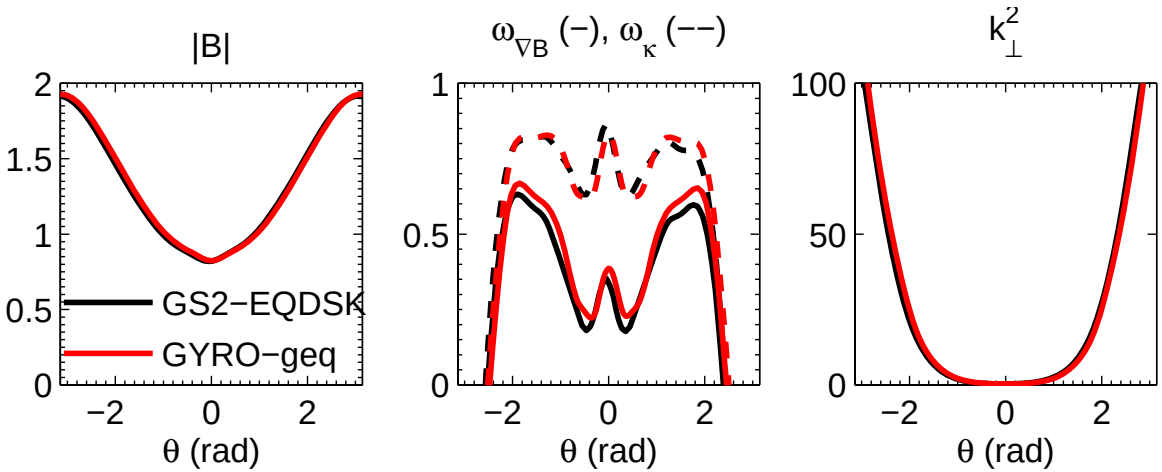




ST-ETG Modeling Goals

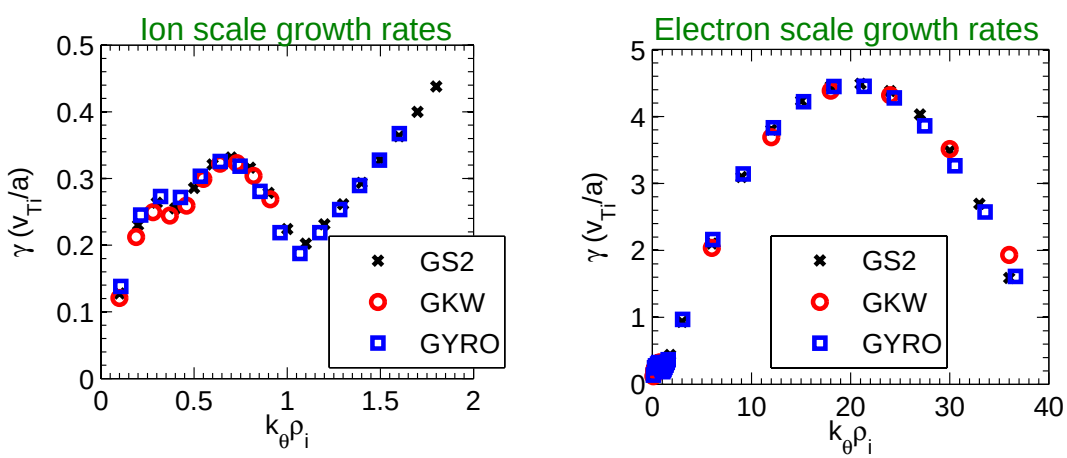
<u>Linear stability</u>

- Known linear dependence for low beta, modest shaping [Jenko et al. 2001]
- TGLF will hopefully capture modifications at high beta – benchmarking with GS2, GYRO, GKW is underway [motivated by Roach, EPS 2009]



Geometry Verification

MAST experimental case, mid-radius, EFIT equilibrium specification



Gyrokinetic Verification